

(A Not So Technical) Introduction to Quantum Computation

What does it take to successfully use quantum computers?

Harold Ollivier

Outline

- Quantum Computing
- 2 Current impacts
- Looking into the future





Quantum Computing

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Set of axioms used to describe reality (at the microscopic scale)



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2 Systems can be combined via tensor products $ec{u}, ec{v} o ec{u} \otimes ec{v}$	Scaling
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The Hadamard gate

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Two successive H gates behave like identity

$$ec{u}_0 \xrightarrow{H} rac{1}{\sqrt{2}} (ec{u}_0 + ec{u}_1) \xrightarrow{H} rac{1}{2} (ec{u}_0 + ec{u}_1 + ec{u}_0 - ec{u}_1) = ec{u}_0.$$

A different view of the Hadamard gate



We can compactly represent the computation of amplitudes

	a = 0	a = 1
x = 0	$1/\sqrt{2}$	$1/\sqrt{2}$
$\mathbf{x} = 1$	$1/\sqrt{2}$	$-1/\sqrt{2}$

which we can rewrite $(-1)^{a.x}/\sqrt{2}$.



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And its power



Because contributions (amplitudes) can be negative,

- Some paths add-up (constructive interference)
- Some paths cancel each other (destructive interference)

Toffoli gate



The amplitudes can also be computed in a very compact way:

$$\delta_{x_1,a_1} \times \delta_{x_2,a_2} \times \delta_{x_3,a_3 \oplus (a_1.a_2)}$$

i.e. is 1 when the input-output relation is satisfied, and 0 otherwise



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The amplitudes are written:

$$\delta_{x,a} \times (-1)^a$$







For CZ the amplitude is $(-1)^{a_1.a_2} \delta_{a_1,x_1} \delta_{a_2,x_2}$

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CCZ gate



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The power of superpositions

Computing amplitudes for small circuits (recursively applying the formulas)





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The transition amplitude from $a = \vec{u}_0^{\otimes n}$ to $y = \vec{u}_0^{\otimes n}$ corresponds to:

$$(\vec{u}_0^{\otimes n}, C_P \vec{u}_0^{\otimes n}) = \frac{1}{2^n} \sum_{x = (x_i)_i \in \{0,1\}^n} (-1)^{P(x)} = \frac{1}{2^n} (\#\{x : P(x) = 0\} - \#\{x : P(x) = 1\})$$



Quantum computers "compute" transition amplitudes

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Defining gap(P) for P degree-3 polynomial

$$gap(P) = \#\{x : P(x) = 0\} - \#\{x : P(x) = 1\}$$

where $P = \sum \alpha_{i,j,k} x_i . x_j . x_k + \sum \beta_{i,j} x_i . x_j + \sum \gamma_i x_i$, and $\alpha_{i,j,k}, \beta_{i,j}, \gamma_i \in \{0,1\}$.



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Hardness

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- Classically computing gap(P) is hard (in $PP \supset NP$)
- Computing $ngap(P) = gap(P)/2^n$ is also hard
- Quantum computers seem to do it with few gates: $ngap(P) = (\vec{u}_0^{\otimes n}, C_P \vec{u}_0^{\otimes n})$

But



But

Quantum computers do not give access to these values with perfect accuracy, but only to samples and, additionnally, they can be noisy

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- It can become easy for additive approximation for classes of functions that remain hard multiplicatively
- It can be easy when there is noise



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- I Useful QC algorithms need to be designed (or checked) on a case-by-case basis: no easy black-box approach
- **5** Keep in mind that we assumed perfect machines (without noise)



Current Impacts

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Examples of algorithms using coherent QC (large machines, error free)

- Discrete log (exponential)
- Linear algebra with quantum encoded data (possibly exponential, mostly polynomial)
- Search (quadratic)



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- Quantum Alternating Operator Ansatz (QAOA): combinatorial optimization
- Analog QC: physics simulations, optimization



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Quantum cryptography (QKD)

• Protecting information with statistical security (ie. without hardness asumptions)



Impacts

On cryptography

- 2016 NIST has started the process of changing the way public key crypto is done to become post-quantum (ie. quantum resistant)
- Calls issued, some protocols are being standardized
- Major impact on all industries (with increased operational risks)



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On computing

- A lot of work is being done to pinpoint possible use-cases
- Assessment of the current power of quantum machines
 - > Well chosen problem (hard for classical / easy for quantum): supremacy experiment
 - > Useful problem (but brute force classical simulation): latest IBM Nature paper
 - > Small scale proof of concept: hard to apprehend the scaling
- Trying to develop a GPU-like approach with HPC coupling



Looking into the future

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Impact your client's businesses

- Need to account for crypto uncertainty
 - > People store have long-term valuable documents
 - > Need to properly upgrade security of systems before it's too late
- Ensuring that some computations are correct / trusting computations

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Impact on your own business

- Dependent on applications
- Algebra + optim: Quite general



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- In a zone where there is some battle with classical computing (for well chosen problems)
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Bottlenecks

- Assessment of usefulness of QC requires reanalysing the full computational software stack
- Takes time and knowledge to know what you are trying to improve
- Improving over state of the art means you know what it is for your problem



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- Work with private companies (when getting inspiration from others / adapting something described elsewhere)
- Work with academic labs when you want to tackle something that (really) nobody has looked at before



Thank you! (time for questions)

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