A Simple Approach for Measuring Higher-Order Risk Attitudes

Cary Deck* Rachel J. Huang†

Larry Y. Tzeng‡ Lin Zhao§

April 16, 2023

Abstract

While the importance of the Arrow-Pratt coefficients of second order risk aversion is well established in economics, the importance of higher-order risk attitudes has only recently begun to be recognized. In this paper, we provide the first approach to measure the higher-order Arrow-Pratt coefficients with choices between compound lotteries. Specifically, we provide a theoretical basis for using risk apportionment to reveal the intensity of higher-order risk attitudes, and then draw upon our theoretical results to develop a simple, systematic, and generalizable procedure for eliciting Arrow-Pratt coefficients of prudence, temperance, and other higher-order risk attitudes. We demonstrate our approach in a laboratory experiment and find that the modal subject exhibits mild prudence and mild temperancein addition to mild risk aversion. Further, we find that degrees of risk aversion are positively correlated across orders. Finally, while our approach is non-parametric, we note that behavior is broadly consistent with subjects having utility described by the exponential-power function.

Keywords: higher-order Arrow-Pratt risk aversion, risk apportionment, comparative risk attitude, measuring intensity of prudence and temperance, laboratory experiment.

*Corresponding Author, Department of Economics, University of Alabama, Tuscaloosa, AL 35487, United States. & Economic Science Institute, Chapman University. Email: cdeck@cba.ua.edu. phone: +1 205 348 8972. fax: +1 205 348 8983.

†Department of Finance, National Central University, Taoyuan 32001, Taiwan. Email: rachel@ncu.edu.tw.

‡Department of Finance, National Taiwan University, Taipei 10617, Taiwan. Email: tzeng@ntu.edu.tw.

§School of Business Administration, Faculty of Business Administration, Southwestern University of Finance and Economics, Chengdu 611130, China. Email: zhaoliniss@swufe.edu.cn.

1 Introduction

Risk attitudes of an economic agent have long been a fundamental issue in economics. Under expected utility, risk aversion equates to a negative second derivative of the utility function. The attitudes associated with higher order derivatives, referred to as higher-order risk attitudes, are now understood to be essential to economic decisions as well. For example, prudence (equated with a positive third derivative under expected utility) entails aversion to greater downside risks (Menezes, Geiss and Tressler 1980), and a stronger saving motive when future wealth becomes riskier (Kimball 1990). Temperance (equated with a negative fourth derivative under expected utility) implies less willingness to take on risk in the presence of greater background risk (Kimball 1993), and higher risk premium when the volatility of consumption growth increases (Gollier 2018).

While the direction of risk attitudes (equated with the sign of the appropriate derivative) serves as a primitive determinant for economic decisions, the intensity of risk attitudes is indispensable for quantitative analyses. For second-order risk attitude, Arrow (1971) and Pratt (1964) introduced the coefficients of absolute and relative risk aversion. For third order, Kimball (1990) introduced the coefficient of absolute prudence and linked it to the strength of the precautionary saving motives. For fourth order, Kimball (1992) introduced the coefficient of absolute temperance, which is useful for analyzing economic decisions involving two or more independent risks (Kimball 1993; Eeckhoudt, Gollier and Schlesinger 1996; Gollier and Pratt 1996). The above manner of quantifying the intensity of risk attitudes has been extended to all higher orders (Caballé and Pomansky 1996; Denuit and Eeckhoudt 2010). The resulting sequence of coefficients, which can be universally named as the coefficients of Arrow-Pratt absolute or relative risk aversion for a given order, play an important role in a wide range of economic applications including investment (Guiso, Jappelli and Terlizzese 1996), saving (Eeckhoudt and Schlesinger 2008), asset pricing (Gollier 2001), bargaining (White 2008), auctions (Esö and White 2004) and so on.

Despite the importance of higher-order risk attitudes, there have been relatively few empirical attempts to measure higher-order Arrow-Pratt coefficients of risk aversion. For third order, the coefficient of relative prudence has been estimated assuming a life-cycle consumption model and using savings data (Dynan 1993; Gourinchas and Parker 2002; Carroll and Kimball 2008). For the fourth order, we are not aware of any empirical study using naturally occurring data to estimate coefficients of absolute or relative temperance. In the laboratory, Ebert and Wiesen (2014) jointly measured the intensity of risk aversion, prudence, and temperance based on risk compensations while Noussair, Trautmann and van de Kuilen (2014) rely on the number of prudent choices as a measure of the degree of prudence, but neither of these papers are able to provide measures of the Arrow-Pratt coefficients.

In this paper, we provide the first approach to measure higher-order Arrow-Pratt risk aversion coefficients using choices between compound lotteries. In a seminal paper, Eeckhoudt and Schlesinger (2006) show apportioning zero mean risks, choosing between disaggregating and aggregating such risks in compound lotteries, can be used to identify the sign of the n^{th} derivative of utility.[1] We show that apportionment of non-zero-mean risks provides non-zero boundaries on the Arrow-Pratt coefficients of risk aversion. That is, while Eeckhoudt and Schlesinger (2006) provide theoretical justification for a method to determine the direction of higher-order risk attitudes, we provide a theoretical justification for a method to measure the Arrow-Pratt coefficients of risk aversion.[2]

Our paper is related to the literature on comparative risk aversion. Under a choice-based framework, Chiu (2005) and Denuit and Eeckhoudt (2010) construct lotteries such that the higher-order intensity of risk aversion between two individuals can be compared through their lottery choices.[3] The major difference between our paper and previous work is that we adopt the technique of risk apportionment. Furthermore, the approaches of both Chiu (2005) and Denuit and Eeckhoudt (2010) require mixed risk aversion, whereas our approach can be applied to decision makers who are risk seeking or imprudent or intemperate.

Our framework for identifying the intensity of higher-order risk preferences has several desirable properties. First, our method does not require any parametric assumption about the specific functional form of the decision makers' utility. While some empirical papers have tried to esti-

[1]This approach has served as the main method for studying higher-order risk preferences. Eeckhoudt, Rey and Schlesinger (2007) rely on the approach when considering bivariate utility functions; Eeckhoudt, Schlesinger and Tsetlin (2009) generalize risk apportionment to a broader class of lotteries; and Crainich, Eeckhoudt, and Trannoy (2013) apply the method to mixed risk lovers. Also, the approach of Eeckhoudt and Schlesinger (2006) has become a mainstream tool for experimental studies of higher-order risk preferences (e.g. Deck and Schlesinger 2010, 2014; Haering, Heinrich and Mayrhofer 2020).

[2]Recently, Jindapon, Liu and Neilson (2021) and Schneider and Sutter (2021) promote alternative measures to evaluate the strength of n^{th} degree risk apportionment. However, the measures proposed in those papers differ from the Arrow-Pratt measure of absolute and relative higher-order risk aversion. By contrast, our paper connects risk apportionment with the Arrow-Pratt coefficients.

[3]There have been other approaches to comparing the intensity of higher-order risk attitudes across individuals as well. For example, Jindapon and Neilson (2007) propose a comparative statics approach for n^{th}-degree risk aversion in an optimal effort decision. The approach of Liu and Meyer (2013) involves a comparison of matching probabilities.

mate the intensity of higher-order risk attitudes (e.g. Dynan 1993 and Noussair, Trautmann and van de Kuilen 2014), thus far such efforts have had to rely upon additional assumptions regarding preferences, which arbitrarily confines the degrees of freedom to describe an individual's behavior. Second, our approach is simple, systematic and generalizable. It is simple in that it only involves comparisons between two lotteries that are themselves composed of combinations of certain losses and fifty-fifty lotteries. It is systematic in that it involves a series of incremental comparisons, similar to the multiple price list approach popularized by Holt and Laury (2002) for measuring second-order risk attitudes. It is generalizable in that it can be readily adapted to risk preferences at any order, which stands in contrast to approaches such as Cohen and Einav (2007) whose approach is context-dependent and difficult to adapt to arbitrarily high orders of risk attitude. Finally, our framework provides bounds on intensity rather than a mean based estimation as developed by Noussair, Trautmann and van de Kuilen (2014).

As a demonstration of our approach, we implement it in a controlled laboratory experiment and measure the intensity of the higher-order risk attitudes of our subjects. In the experimental task, we ask subjects to apportion a sequence of nine risks including negative-mean, zero-mean, and positive-mean risks. The observed patterns for the direction of second-, third-, and fourth-order risk attitudes as identified by the apportionment of zero-mean risks are consistent with those reported previously by Deck and Schlesinger (2014) and Noussair, Trautmann, and van de Kuilen (2014). The modal behavior among our subjects is a mild degree of risk aversion consistent with a large experimental literature, a mild degree of prudence consistent with the relatively small literature attempting to measure the intensity of third order risk preferences (e.g. Noussair, Trautmann and van de Kuilen 2014 and Dynan 1993), and a mild degree of temperance which is a novel finding in the literature. However, we also observe a fraction of subjects who exhibit more extreme prudence and temperance as well as sizable fractions of subjects who exhibit moderate to extreme levels of imprudence or intemperance. Additionally, we extend Jindapon and Neilson (2007) to cases when decision makers are risk-loving or imprudent to demonstrate the implications of our laboratory findings.

Our approach also allows us to consider how degree of risk aversion are related across orders. Among our subjects, their degrees of second and third order risk aversion are positive and significant as are their degrees of third and fourth order risk aversion, but the greatest correlation is between second and fourth order degrees of risk aversion. Finally, we conduct a calibration exercise to determine how well common utility functions match observed behavior. Ultimately, we find that the behavior of most subjects can described with a exponential-power utility function.

2 Intensity of Higher-Order Risk Preferences

How to characterize the intensity of risk preferences within the expected utility framework is a fundamental question that has been extensively investigated in the economics literature. To briefly review the theoretical progress, let $u(x)$ be a von Neumann-Morgenstern utility function of wealth x that is defined on $(0, \infty)$ and continuously differentiable up to the desired order. For $n = 1, 2, ...,$ denote by $u^{(n)}(x)$ the n^{th} derivative of $u(x)$.

Arrow (1971) and Pratt (1964) introduce the coefficient of absolute risk aversion $-\frac{u^{(2)}(x)}{u^{(1)}(x)}$ as a measure of second-order risk attitude, which is well used in risk-taking decisions, e.g., investment and insurance choices. To examine the precautionary savings motive, Kimball (1990) introduces the coefficient of absolute prudence $-\frac{u^{(3)}(x)}{u^{(2)}(x)}$. The higher the coefficient of absolute prudence, the higher the strength of the precautionary saving motives. Kimball (1992) introduces the coefficient of absolute temperance $-\frac{u^{(4)}(x)}{u^{(3)}(x)}$ and shows that it is related to how strongly an individual is inclined to avoid binding one risk with another unavoidable independent risk. This pattern is extended to higher-order risk attitude by Caballé and Pomansky (1996) who define $-\frac{u^{(n)}(x)}{u^{(n-1)}(x)}$ as the coefficient of the n^{th}-order absolute risk aversion. Following Caballé and Pomansky's terminology, $-\frac{u^{(2)}(x)}{u^{(1)}(x)}$, $-\frac{u^{(3)}(x)}{u^{(2)}(x)}$ and $-\frac{u^{(4)}(x)}{u^{(3)}(x)}$ can be relabeled as absolute risk aversion of second order, third order and fourth order, respectively.[4] As shown by Jindapon and Neilson (2007), given a non-monetary cost of effort, the strength of the willingness to invest in effort to reduce risk depends on the coefficient of n^{th}-order absolute risk aversion.

In parallel, Pratt (1964) introduced the coefficients of relative risk aversion $-x\frac{u^{(2)}(x)}{u^{(1)}(x)}$, while relative prudence $-x\frac{u^{(3)}(x)}{u^{(2)}(x)}$ is proposed by Kimball (1990). Eeckhoudt and Schlesinger (2008) extend these coefficients to relative temperance $-x\frac{u^{(4)}(x)}{u^{(3)}(x)}$, as well as higher orders via defining $-x\frac{u^{(n)}(x)}{u^{(n-1)}(x)}$ as the coefficient of the n^{th}-order relative risk aversion. They show that the condition $-x\frac{u^{(n)}(x)}{u^{(n-1)}(x)} \geq n - 1$ is crucial to guarantee an increase in precautionary saving when there is an increase in risk

[4] Gollier (2018) labels absolute risk aversion of fifth order as absolute edginess when investigating aversion to risks on the variance of consumption.

in the return on saving.

In a seminal paper, Eeckhoudt and Schlesinger (2006) show that the sign of $u^{(n)}(x)$ can be revealed with choices between simple 50-50 lotteries composed of pure losses and zero-mean risks. However, their work remains silent on the intensity of n^{th}-order risk preferences. In this section we extend the approach of Eeckhoudt and Schlesinger (2006) for comparative higher-order risk aversion with choices between simple lotteries.

2.1 Intensity of Second-Order Risk Aversion

Let $w > 0$ denote an initial wealth level and $\tilde{\varepsilon}$ be a zero-mean risk. An individual is risk averse on a pre-specified interval $[a, b] \subset (0, \infty)$, if and only if for all lottery pairs supported on $[a, b]$ taking the form of $w + \tilde{\varepsilon}$ and w, w is always preferred to $w + \tilde{\varepsilon}$. Within the expected utility framework, risk aversion on $[a, b]$ is equivalent to $u^{(2)} \leq 0$ on $[a, b]$ (Rothschild and Stiglitz 1970).

Replacing the zero-mean risk $\tilde{\varepsilon}$ with a general non-zero-mean risk $\tilde{\delta}$, we can elicit a bound for $-\frac{u^{(2)}(x)}{u^{(1)}(x)}$ from a choice between

$$A_2 = w + \tilde{\delta} \quad \text{and} \quad B_2 = w. \tag{1}$$

Proposition 1. *Let A_2 and B_2 take the form of (1). For u and v that are twice continuously differentiable with $u^{(1)} > 0$ and $v^{(1)} > 0$, the following statements are equivalent:*

(i) For all $x \in [a, b]$, $-\frac{u^{(2)}(x)}{u^{(1)}(x)} \geq -\frac{v^{(2)}(x)}{v^{(1)}(x)}$;

(ii) For all A_2 and B_2 supported on $[a, b]$, $\mathbb{E}v(A_2) = \mathbb{E}v(B_2)$ always implies $\mathbb{E}u(A_2) \leq \mathbb{E}u(B_2)$.

All proofs are relegated to Appendix 7. Intuitively, Proposition 1 can be obtained from Pratt (1964). To see this, one can rewrite $\tilde{\delta} = \mathbb{E}\tilde{\delta} + (\tilde{\delta} - \mathbb{E}\tilde{\delta})$ where the first term is the mean and the second term is a zero-mean risk. If under v one is indifferent between A_2 and B_2, it means that the mean of the risk $\tilde{\delta}$ is exactly the compensating premium necessary to bear the zero-mean risk $\tilde{\delta} - \mathbb{E}\tilde{\delta}$. Since more risk aversion requires a greater compensating premium, B_2 would be more preferable than A_2 under u.

Proposition 1 demonstrates that comparative risk aversion can be revealed with simply lottery choices. In particular, Proposition 1 shows u is more risk averse than v, if and only if u always favors the risky lottery less than v. An analogous characterization of $-\frac{u^{(2)}(x)}{u^{(1)}(x)} \leq -\frac{v^{(2)}(x)}{v^{(1)}(x)}$ is available through reversing the inequality in statement (ii). In the special case where v is a linear (risk neutral) utility function, the equation $\mathbb{E}v(A_2) = \mathbb{E}v(B_2)$ amounts to requiring $\tilde{\delta}$ to have a zero mean, reproducing the equivalence of $u^{(2)} \leq 0$ with w always being preferred to $w + \tilde{\varepsilon}$.

The idea of comparing coefficients of risk aversion based on choice behavior was previously explored by Jewitt (1989) and Chiu (2005). However, in both of those papers the second derivative of the utility function is required to be negative. In contrast, our analysis imposes no condition on the second derivative. Thus, relative to Jewitt (1989) and Chiu (2005), we extend the comparison of risk attitudes to utility functions exhibiting risk seeking behavior while employing simpler lottery pairs.

2.2 Intensity of Third-Order Risk Aversion

Let $k > 0$ be a constant and recall that $\tilde{\varepsilon}$ denotes a zero-mean risk. Denote by $[x; y]$ a lottery with a 50-50 chance of receiving either outcome x or outcome y, where x and y themselves may be lotteries. According to Eeckhoudt and Schlesinger (2006), an individual is called prudent on $[a, b]$, if and only if for all lottery pairs taking the form of $[w; w - k + \tilde{\varepsilon}]$ and $[w + \tilde{\varepsilon}; w - k]$ supported on $[a, b]$, the latter is always preferred to the former. That is, a prudent individual prefers putting a zero-mean risk at the higher wealth level than at the lower wealth level. Prudence captures aversion to aggregating a loss with a zero mean risk. Within the expected utility framework, prudence on $[a, b]$ is equivalent to $u^{(3)} \geq 0$ on $[a, b]$.

As we show in Proposition 2, replacing the zero-mean risk $\tilde{\varepsilon}$ with a general non-zero-mean risk $\tilde{\delta}$, we can elicit a bound for $-\frac{u^{(3)}(x)}{u^{(2)}(x)}$ from choices between

$$A_3 = \left[w; w - k + \tilde{\delta}\right] \quad \text{and} \quad B_3 = \left[w + \tilde{\delta}; w - k\right]. \tag{2}$$

To do this, we need to distinguish between cases in which $u^{(2)} > 0$ and $u^{(2)} < 0$.

Proposition 2. *Let A_3 and B_3 take the form of (2). For u and v that are continuously differentiable up to the third order with $u^{(2)} \neq 0$ and $v^{(2)} \neq 0$, consider the following statements:*

(i) For all $x \in [a, b]$, $-\frac{u^{(3)}(x)}{u^{(2)}(x)} \geq -\frac{v^{(3)}(x)}{v^{(2)}(x)}$;

(ii) For all A_3 and B_3 supported on $[a,b]$, $\mathbb{E}v(A_3) = \mathbb{E}v(B_3)$ always implies $\mathbb{E}u(A_3) \leq \mathbb{E}u(B_3)$;

(iii) For all A_3 and B_3 supported on $[a,b]$, $\mathbb{E}v(A_3) = \mathbb{E}v(B_3)$ always implies $\mathbb{E}u(A_3) \geq \mathbb{E}u(B_3)$.

When $u^{(2)} < 0$ and $v^{(2)} < 0$, (i) and (ii) are equivalent; when $u^{(2)} > 0$ and $v^{(2)} > 0$, (i) and (iii) are equivalent.

Proposition 2 is analogous to Proposition 1, but for third-order risk attitude. Assuming risk aversion, u is more prudent than v, if and only if u exhibits a stronger propensity to disaggregate the loss and the risky lottery than v. An analogous characterization of $-\frac{u^{(3)}(x)}{u^{(2)}(x)} \leq -\frac{v^{(3)}(x)}{v^{(2)}(x)}$ is available through reversing the inequalities in statements (ii) and (iii). In the special case where v is a quadratic (prudence neutral) utility function, the equation $\mathbb{E}v(A_3) = \mathbb{E}v(B_3)$ amounts to requiring $\tilde{\delta}$ to have a zero mean.[5] This reproduces the equivalence of $u^{(3)} \geq 0$ with preferring $[w + \tilde{\varepsilon}; w - k]$ over $[w; w - k + \tilde{\varepsilon}]$ and the equivalence of $u^{(3)} \leq 0$ with preferring $[w; w - k + \tilde{\varepsilon}]$ over $[w + \tilde{\varepsilon}; w - k]$ as characterized by Eeckhoudt and Schlesinger (2006).

Chiu (2005) also uses choice behavior to compare coefficients of prudence. However, in Chiu's analysis the second and third derivatives of the utility function are required to be negative and positive, respectively. Our result shows that it is possible to identify the intensity of prudence without presupposing the signs of the second and third derivatives of the utility function although our lotteries are special cases of those in Chiu (2005). That is, our approach can accommodate utility functions exhibiting risk averse or seeking and prudent or imprudent behavior in the same manner. Moreover, the choices can be presented as simple lotteries with equiprobable outcomes.

2.3 Intensity of Higher-Order Risk Aversion

Comparative risk aversion can be identified for any order with choices between simple lotteries following the idea of risk apportionment. To do this, we first recall the lotteries introduced by Eeckhoudt and Schlesinger (2006), whose purpose is identifying the direction of preferences. Let $\{\tilde{\varepsilon}_i\}$ denote an indexed set of zero-mean risks that are mutually independent. For $w > 0$ and $k < 0$,

[5] To see this, we write $\mathbb{E}v(A_3) - \mathbb{E}v(B_3) = \frac{1}{2}[\mathbb{E}v_1(w + \tilde{\delta}) - v_1(w)]$, where $v_1(x) = v(x - k) - v(x)$ is linear in x when $v(x)$ is quadratic.

Eeckhoudt and Schlesinger define

$$\hat{A}_1 = w - k, \quad \hat{B}_1 = w,$$
$$\hat{A}_2 = w + \tilde{\varepsilon}_1, \quad \hat{B}_2 = w,$$

and

$$\hat{A}_n = \left[\hat{A}_{n-2} + \tilde{\varepsilon}_{\mathrm{Int}(n/2)}; \hat{B}_{n-2}\right], \quad \hat{B}_n = \left[\hat{A}_{n-2}; \hat{B}_{n-2} + \tilde{\varepsilon}_{\mathrm{Int}(n/2)}\right]$$

for $n \geq 3$ where $\mathrm{Int}(n/2)$ denotes the greatest integer not exceeding $n/2$. Based on this definition, we have

$$\hat{A}_3 = \left[\hat{A}_1 + \tilde{\varepsilon}_1; \hat{B}_1\right] = [w - k + \tilde{\varepsilon}_1; w], \quad \hat{B}_3 = \left[\hat{A}_1; \hat{B}_1 + \tilde{\varepsilon}_1\right] = [w - k; w + \tilde{\varepsilon}_1],$$
$$\hat{A}_4 = \left[\hat{A}_2 + \tilde{\varepsilon}_2; \hat{B}_2\right] = [w + \tilde{\varepsilon}_1 + \tilde{\varepsilon}_2; w], \quad \hat{B}_4 = \left[\hat{A}_2; \hat{B}_2 + \tilde{\varepsilon}_2\right] = [w + \tilde{\varepsilon}_1; w + \tilde{\varepsilon}_2].$$

In the above, $\hat{B}_1$ and $\hat{B}_2$ represent a fixed state, while $\hat{A}_1$ represents a certain loss relative to $\hat{B}_1$ and $\hat{A}_2$ represents a risky state relative to $\hat{B}_2$. For $n \geq 3$, $\hat{B}_n$ and $\hat{A}_n$ are lotteries involving $\hat{B}_{n-2}$ and $\hat{A}_{n-2}$ where $\hat{B}_n$ attaches an independent zero-mean risk to $\hat{B}_{n-2}$ and $\hat{A}_n$ attaches that zero-mean risk to $\hat{A}_{n-2}$. Eeckhoudt and Schlesinger prove that $(-1)^{n+1}u^{(n)} \geq 0$ if and only if for any zero-mean risks $\hat{B}_n$ is always preferred to $\hat{A}_n$. Such a preference is termed as "risk apportionment" of order n. Risk apportionment captures the aversion to combining "bad"—the additional pure risk $\tilde{\varepsilon}_{\mathrm{Int}(n/2)}$—with "bad"—the more risky state $\hat{A}_{n-2}$.

Our lotteries aimed at identifying the intensity of preferences are related to Eeckhoudt and Schlesinger's lotteries by replacing the zero-mean risk $\tilde{\varepsilon}_{\mathrm{Int}(n/2)}$ with a general non-zero-mean risk $\tilde{\delta}$. Assuming independence between $\tilde{\delta}$ and $\{\tilde{\varepsilon}_i\}$, we define

$$\begin{aligned} A_1 &= w - k, \quad B_1 = w, \\ A_2 &= w + \tilde{\delta}, \quad B_2 = w, \end{aligned} \tag{3}$$

and

$$A_n = \left[\hat{A}_{n-2} + \tilde{\delta}; \hat{B}_{n-2}\right], \quad B_n = \left[\hat{A}_{n-2}; \hat{B}_{n-2} + \tilde{\delta}\right], \tag{4}$$

for $n \geq 3$. Based on this definition, we have

$$A_3 = \left[\hat{A}_1 + \tilde{\delta}; \hat{B}_1\right] = \left[w - k + \tilde{\delta}; w\right], \quad B_3 = \left[\hat{A}_1; \hat{B}_1 + \tilde{\delta}\right] = \left[w - k; w + \tilde{\delta}\right],$$
$$A_4 = \left[\hat{A}_2 + \tilde{\delta}; \hat{B}_2\right] = \left[w + \tilde{\varepsilon}_1 + \tilde{\delta}; w\right], \quad B_4 = \left[\hat{A}_2; \hat{B}_2 + \tilde{\delta}\right] = \left[w + \tilde{\varepsilon}_1; w + \tilde{\delta}\right].$$

Allowing $\tilde{\delta}$ to have a non-zero mean and letting

$$\tilde{\varepsilon}_i = [-k_i; k_i] \text{ where } k_i > 0, \tag{5}$$

we can explore the intensity of the n^{th}-order risk aversion based on the choice between A_n and B_n.

Theorem 1. *For $n \geq 2$, let A_n and B_n be defined as in (3) and (4), with $\tilde{\varepsilon}_i$ specified in (5). For u and v that are continuously differentiable up to order n with $u^{(n-1)} \neq 0$ and $v^{(n-1)} \neq 0$, consider the following statements:*

(i) For all $x \in [a, b]$, $-\frac{u^{(n)}(x)}{u^{(n-1)}(x)} \geq -\frac{v^{(n)}(x)}{v^{(n-1)}(x)}$;

(ii) For all A_n and B_n supported on $[a, b]$, $\mathbb{E}v(A_n) = \mathbb{E}v(B_n)$ always implies $\mathbb{E}u(A_n) \leq \mathbb{E}u(B_n)$;

(iii) For all A_n and B_n supported on $[a, b]$, $\mathbb{E}v(A_n) = \mathbb{E}v(B_n)$ always implies $\mathbb{E}u(A_n) \geq \mathbb{E}u(B_n)$.

When $(-1)^n u^{(n-1)} > 0$ and $(-1)^n v^{(n-1)} > 0$, (i) and (ii) are equivalent; when $(-1)^n u^{(n-1)} < 0$ and $(-1)^n v^{(n-1)} < 0$, (i) and (iii) are equivalent.

Theorem 1 generalizes Propositions 1 to 2 to higher orders. For example, the bound for temperance measured by $-\frac{u^{(4)}(x)}{u^{(3)}(x)}$ can be elicited from the choice between A_4 and B_4. An analogous characterization of $-\frac{u^{(n)}(x)}{u^{(n-1)}(x)} \leq -\frac{v^{(n)}(x)}{v^{(n-1)}(x)}$ is available through reversing the inequalities in statements (ii) and (iii). In the special case where $v^{(n-1)} \equiv 0$, we can prove by induction that the equation $\mathbb{E}v(A_n) = \mathbb{E}v(B_n)$ amounts to requiring $\tilde{\delta}$ to have a zero mean, reproducing the characterization of $(-1)^{n+1}u^{(n)} \geq (\leq) 0$ that $\hat{B}_n$ is always more (less) preferable than $\hat{A}_n$.[6]

To our knowledge, Denuit and Eeckhoudt (2010) were the first to extend the equivalence between comparative risk aversion and a binary choice behavior to higher orders. Their lottery pairs,

[6]For example, on fourth order when v is a temperance neutral (cubic) utility function, we write $\mathbb{E}v(A_4) - \mathbb{E}v(B_4) = \frac{1}{2}[\mathbb{E}v_2(w + \tilde{\delta}) - v_2(w)]$, where $v_2(x) = \frac{1}{2}[v(x-k) + v(x+k)] - v(x)$ is linear in x when $v(x)$ is cubic. Thus, the equation $\mathbb{E}v(A_4) = \mathbb{E}v(B_4)$ amounts to requiring $\tilde{\delta}$ to have a zero mean, reproducing the characterization by Eeckhoudt and Schlesinger (2006) that $u^{(4)} \leq 0$ iff $[w + \tilde{\varepsilon}_1; w + \tilde{\varepsilon}_2]$ is always preferred to $[w; w + \tilde{\varepsilon}_1 + \tilde{\varepsilon}_2]$.

however, are designed only for utility functions that have positive odd numbered derivatives and negative even numbered derivatives up to the relevant order, which is a typical feature of the so-called "mixed risk averse" utility functions (Caballé and Pomansky 1996). In contrast, we construct lottery pairs by iteration of simple 50-50 lotteries, and hence impose no condition on the sign of derivatives of up to $n-1$. The iterative approach also allows the sign of the n^{th} derivative of utility functions to be either positive or negative, and simplifies the choices to involve only 50-50 lotteries.

3 An Implementable Procedure

Theorem 1 provides a means to compare the n^{th}-order absolute and relative risk aversion between two utility functions. We can bound the n^{th}-order absolute risk aversion of u by comparing u with v_1 and bound the n^{th}-order relative risk aversion of u by comparing it with v_2, where v_1 and v_2 satisfy

$$-\frac{v_1^{(n)}(x)}{v_1^{(n-1)}(x)} = \theta_1 \quad \text{and} \quad -x\frac{v_2^{(n)}(x)}{v_2^{(n-1)}(x)} = \theta_2, \quad \theta_1, \theta_2 \in \mathbb{R},$$

respectively. While Theorem 1 is stated in terms of comparing the intensities of absolute risk aversion, it works equally well for relative risk aversion when $x > 0$. Indeed, when comparing u with v_2 by Theorem 1, we get $-\frac{u^{(n)}(x)}{u^{(n-1)}(x)} \geq$ or $\leq -\frac{v_2^{(n)}(x)}{v_2^{(n-1)}(x)} = \frac{\theta_2}{x}$, which is equivalent to $-x\frac{u^{(n)}(x)}{u^{(n-1)}(x)} \geq$ or $\leq \theta_2$. The structural assumptions on v_1 and v_2 serve as bases for bounding a subject's risk attitude, which depends on u. Thus, we do not assume that one's utility function u exhibits constant n^{th}-order absolute or relative risk aversion and instead compare one's utility function to functions with those specific forms at a given level of wealth. In fact, our procedure proposes no assumption on the form of individual's utility function.

For a given order n, a subject faces a task that involves a series of risk apportionment choices between A_n and B_n that systematically varies $\tilde{\delta}$ holding other parameters fixed. Formally, for choice j in a Task of Order n, we construct A_n and B_n following (3) and (4) with task-specific values of $w > 0$, $k > 0$, $\tilde{\varepsilon}_i$ as in (5), and

$$\tilde{\delta}_j = [-h; h] + l_j = [-h + l_j; h + l_j], \tag{6}$$

where $h > 0$, $-h = l_1 < l_2 < ... < l_{J-1} < l_J = h$ and $J \geq 3$. As j increases from 1 to J, $\tilde{\delta}_j$ moves from $[-2h; 0]$ to $[0; 2h]$. We use $A_n(j)$ and $B_n(j)$ to indicate explicitly the dependence of A_n and B_n constructed in this way on j. Then, a Task of Order n is formulated as

$$\text{Task of Order } n = \{(A_n(j), B_n(j)) : j = 1, ..., J\}, \tag{7}$$

in which we present subjects a sequence of lottery pairs and ask them to select their preferred option in each pair.

To take a numerical third-order example, consider the series of 9 lottery pairs: $(A_3(j), B_3(j))$, $j = 1, 2, ..., 9$ with $A_1 = 13$, $B_1 = 23$, $h = 4$ and $l_{j+1} - l_j = 1$. Thus, for $j = 1$, we have $\delta_1 = [-8; 0]$, and Choice 1 is between the lottery pair

$$A_3(1) = [13 + [-8; 0]; 23] \quad \text{and} \quad B_3(1) = [13; 23 + [-8; 0]].$$

For $j = 2$, we have $\delta_2 = [-7; 1]$. Choice 2 is between the lottery pair

$$A_3(2) = [13 + [-7; 1]; 23] \quad \text{and} \quad B_3(2) = [13; 23 + [-7; 1]].$$

For $j = 9$, we have $\delta_9 = [0; 8]$ and Choice 9 is between the lottery pair

$$A_3(9) = [13 + [0; 8]; 23] \quad \text{and} \quad B_3(9) = [13; 23 + [0; 8]].$$

The lottery comparison of the form used by Eeckhoudt and Schlesinger (2006) occurs when $j = 5$, yielding $\delta_5 = [-4; 4]$ and a choice over the lottery pair

$$A_3(5) = [13 + [-4; 4]; 23] \quad \text{and} \quad B_3(5) = [13; 23 + [-4; 4]].$$

For any sequence of lotteries constructed through the process described above, for $j = 2, ..., J-1$, there exist unique n^{th}-order constant absolute and relative risk aversion coefficients, denoted by

$\Theta_1(n,j)$ and $\Theta_2(n,j)$ respectively, such that

$$\mathbb{E}v_1(A_n(j)) = \mathbb{E}v_1(B_n(j)) \text{ under } \theta_1 = \Theta_1(n,j), \text{ and}$$
$$\mathbb{E}v_2(A_n(j)) = \mathbb{E}v_2(B_n(j)) \text{ under } \theta_2 = \Theta_2(n,j).$$

These coefficients make individuals with utility functions v_1 or v_2 indifferent between $A_n(j)$ and $B_n(j)$.[7] It is shown in Lemma A3 in Appendix A that both $\Theta_1(n,j)$ and $\Theta_2(n,j)$ are strictly increasing in j. As a convention, we set $\Theta_1(n,1) = \Theta_2(n,1) = -\infty$ and $\Theta_1(n,J) = \Theta_2(n,J) = \infty$.

We can identify both upper and lower bounds on risk attitude at some wealth level from a single task. This approach to providing bounds on risk attitude is justified by the following corollary.

Corollary 1. *For $n \geq 2$, consider a Task of Order n as in (7) that is supported on $[a,b] \subset (0,\infty)$. Let u be a utility function that is continuously differentiable up to order n. When $(-1)^n u^{(n-1)} > 0$, there exists a unique $j^* \leq J-1$ such that the individual prefers $B_n(j)$ to $A_n(j)$ for $j \leq j^*$, but $A_n(j)$ to $B_n(j)$ for $j \geq j^*+1$. When $(-1)^n u^{(n-1)} < 0$, similar behavior holds with the individual preferring $A_n(j)$ to $B_n(j)$ for $j \leq j^*$, but $B_n(j)$ to $A_n(j)$ for $j \geq j^*+1$. For both cases, there exist $x_1, x_2 \in [a,b]$ such that*

$$\Theta_1(n,j^*) \leq -\frac{u^{(n)}(x_1)}{u^{(n-1)}(x_1)} \leq \Theta_1(n,j^*+1),$$
$$\Theta_2(n,j^*) \leq -x_2\frac{u^{(n)}(x_2)}{u^{(n-1)}(x_2)} \leq \Theta_2(n,j^*+1).$$

Corollary 1 demonstrates what we can infer from a single task. Let us recall the previous example to illustrate Corollary 1. Suppose that an individual prefers $B_3(j)$ to $A_3(j)$ for $j=1$ and $j=2$ and $A_3(j)$ to $B_3(j)$ for $j=3$ to $j=9$. The choices of $B_3(1)$ and $A_3(9)$ reveal the individual is risk-averse. An individual with a degree of absolute prudence equal to -0.69 and relative prudence equal to -14.26 would be indifferent between $A_3(2)$ and $B_3(2)$. That is $\Theta_1(3,2) = -0.69$ and $\Theta_2(3,2) = -14.26$. Furthermore, an individual with a degree of absolute prudence equal to -0.31 and relative prudence equal to -5.82 would be indifferent between $A_3(3)$ and $B_3(3)$. That is $\Theta_1(3,3) = -0.31$ and $\Theta_2(3,3) = -5.82$. Thus, from Corollary 1, we know that the individual who

[7] Here, v_1 and v_2 are unique up to scaling and addition of polynomials up to order $n-2$. Since the moments of A_n and B_n are the same up to order $n-2$, the scaling factor and the polynomial terms do not affect the subsequent analysis.

prefers $B_3(j)$ to $A_3(j)$ only for $j \leq 2$ exhibits

$$-0.69 \leq -\frac{u^{(3)}(x_1)}{u^{(2)}(x_1)} \leq -0.31 \text{ and } -14.26 \leq -x_2\frac{u^{(3)}(x_2)}{u^{(3)}(x_2)} \leq -5.82$$

for some x_1 and x_2 within $[5, 31]$.

Corollary 1 does not impose any assumption about the functional form of u, so that risk attitude elicited in a wealth range does not imply anything on risk attitude on other wealth ranges. To elicit the bound of absolute or relative risk aversion of order n, there is no need to rely on separate information about any risk attitude of a lower order.

A premise for this corollary to work is that $u^{(n-1)}$ does not vary in sign over the relevant domain. Since risk aversion coefficients, $-\frac{u^{(n)}(x)}{u^{(n-1)}(x)}$ or $-x\frac{u^{(n)}(x)}{u^{(n-1)}(x)}$, are definable only for segments with $u^{(n-1)} \neq 0$, this premise is not excessively demanding. Under this premise, the difference between the expected utility of $A_n(j)$ and that of $B_n(j)$ changes monotonically with j, yielding a single switch point from preferring $B_n(j)$ to preferring $A_n(j)$ under $(-1)^n u^{(n-1)} > 0$, or from preferring $A_n(j)$ to preferring $B_n(j)$ under $(-1)^n u^{(n-1)} < 0$. As with other techniques for measuring risk aversion, such as the multiple price list approach of Holt and Laury (2002), individuals whose preferences are captured by standard functional forms for utility, should exhibit a single switch point when going through the J choices of the task.

4 Experimental Design

To demonstrate the implementation of our procedure, we conducted a controlled laboratory experiment. For illustrative purposes, our experimental investigation will focus on the second, third and fourth order. For each order, we present subjects a task as described in (7), with each task consisting of 9 choices involving $J = 9$ pairs of lotteries. In total, subjects were asked to make 27 choices (9 choices / order $\times$ 3 orders). At the end of the experiment, one of these 27 choices was randomly selected and used to determine the subject's earnings.

Before continuing, we note that having 9 choices for a task means subjects can be placed into 10 continuous bins that only overlap at their endpoints.[8] The width of each bin depends on the specific lotteries that are used in the task and one could construct a finer or coarser set of bounds

[8] The bins overlap at the endpoints as a subject could be indifferent for a given choice.

Table 1: All Decision Tasks

Task of	Option $A_n(j)$	Option $B_n(j)$	$\tilde{\delta}_1$	$\tilde{\delta}_9$	$l_{j+1} - l_j$	Average Payoff
Order 2	$18 + \tilde{\delta}_j$	18	$[-8; 0]$	$[0; 8]$	1	18
Order 3	$[13 + \tilde{\delta}_j; 23]$	$[13; 23 + \tilde{\delta}_j]$	$[-8; 0]$	$[0; 8]$	1	18
Order 4	$[18 + [-5; 5] + \tilde{\delta}_j; 18]$	$[18 + [-5; 5]; 18 + \tilde{\delta}_j]$	$[-8; 0]$	$[0; 8]$	1	18

Note: This table reports the numerical payoffs used to construct tasks as formulated in (7). Recall that $[x; y]$ denotes a lottery where there is a 50% chance of receiving x and a 50% chance of receiving y. In $A_n(j)$ and $B_n(j)$, $\tilde{\delta}_j = [-h; h] + l_j$, with $\tilde{\delta}_1 = [-2h; 0]$, $\tilde{\delta}_9 = [0; 2h]$, and $l_{j+1} - l_j$ given in the second to the last column. For example, in our Task of Order 3, $\tilde{\delta}_1$ is $[-8; 0]$, $\tilde{\delta}_2$ is $[-7; 1]$, and $\tilde{\delta}_3$ is $[-6; 2]$, and so on.

by using more or fewer lotteries, respectively. This is also true for the multiple price list approach for measuring second-order risk aversion popularized by Holt and Laury (2002).[9]

4.1 Construction of Tasks

For all tasks, $\tilde{\delta}_1 = [-2h; 0]$ in Choice 1 only involves a loss, $\tilde{\delta}_5 = [-h; h]$ in Choice 5 involves 50-50 of equal sized gain and loss, and $\tilde{\delta}_9 = [0; 2h]$ in Choice 9 only involves a gain. Thus, for a Task of Order n Choice 1 and Choice 9 reveal the direction of the $(n-1)^{th}$-order risk attitude, while Choice 5 is consistent with Eeckhoudt and Schlesinger (2006) and reveals the direction of the n^{th}-order risk attitude.

Table 1 provides all 3 tasks used in the experiment, where payoffs are in US dollars. Our Task of Order 3 is the basis for the numerical example used in the previous section. The numerical payoffs in the tasks are designed such that risk attitudes associated with indifference between choices are in the neighborhood of the risk attitudes that have been reported previously in the literature (e.g, Holt and Laury 2002; Bliss and Panigirtzoglou 2004; Noussair, Trautmann and van de Kuilen 2014). Thus, our specific tasks are not calibrated to identify particularly extreme levels of risk attitude, although one could design tasks to partition more extreme risk attitudes using the same technique.

4.2 Steps for Making Choices

Our subjects face multiple choices in each task. To help subjects understand the relationship between the choices on a task, all nine lotteries for a given order are displayed on the screen at the

[9] We also note that one could attempt to identify the $\tilde{\delta}$ that makes a respondent indifferent between A_n versus B_n, using a method similar to that of Becker, Degroot, and Marschak (1964). However, our approach only requires subjects to make binary comparisons.

Table 2: The Degree of Risk Aversion Making $A_n(j)$ and $B_n(j)$ Indifferent

Task of	Choice j						
	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$	$j=7$	$j=8$
Panel A: Constant Absolute Degrees $\Theta_1(n,j)$							
Order 2	-0.69	-0.31	-0.14	0.00	0.14	0.31	0.69
Order 3	-0.69	-0.31	-0.14	0.00	0.14	0.31	0.69
Order 4	-0.69	-0.31	-0.14	0.00	0.14	0.31	0.69
Panel B: Constant Relative Degrees $\Theta_2(n,j)$							
Order 2	-11.81	-5.19	-2.27	0.00	2.40	5.72	12.96
Order 3	-14.26	-5.82	-2.36	0.00	2.21	4.97	10.48
Order 4	-13.44	-5.51	-2.31	0.00	2.30	5.28	11.21

Note: In this table, Panel A and Panel B report the constant degrees of n^{th}-order absolute and relative risk aversion that make $A_n(j)$ and $B_n(j)$ in Choice j within the Task of Order n indifferent. Here, $n = 2, 3, 4$ and $j = 2, ..., 8$.

same time. The subject then makes choices about apportioning the lotteries in order from $\tilde{\delta}_1$ to $\tilde{\delta}_9$. Figure 1 provides an example of a subject facing Choice 5 in our Task of Order 3 where they are asked to apportion $\tilde{\delta}_5$. Once the subject has made all 9 choices for a task, a button appears on the screen enabling the subject to submit all 9 responses simultaneously. A subject is free to change their decision regarding the apportionment of a lottery at any point prior to pressing the submit button.[10]

The presentation of each choice follows Deck and Schlesinger (2014) and is meant to facilitate a choice as deciding to combine "good" with "good" or to combine "good" with "bad." For example, in the fifth lottery shown in Figure 1 the decision is if one wants to combine a 50-50 lottery that pays either -\$4 or \$4 with the bad \$13 outcome or the good \$23 outcome of an independent 50-50 lottery. A risk averse person would view the -\$4 or \$4 lottery as a bad while a risk-loving person would view it as a good. A prudent person would opt to combine the -\$4 or \$4 lottery with the \$23 outcome. For a person who is risk averse this is combining a good and a bad, whereas for a risk-loving person this is combining a good with a good. The top lottery shown in Figure 1 also demonstrates how choice 1 an n^{th}-order task each identify the subject's $(n-1)^{th}$-order preferences. The example task measures third-order risk attitude, but for the first choice the preferred option

[10]If a subject changes their decision for Choice i with $i \leq 8$ the software proceeds to Choice $i+1$ and continues sequentially from that point. Additionally, the software imposes that a subject make a single switch on each task; however, the subjects are not informed of this in the instructions. Rather, those that attempt to provide responses that do not conform to a single switch rule are notified of this requirement. This allows us to identify how many subjects naturally follow a single switch rule. 30% of subjects never exhibited behavior inconsistent with a single switch. 47% and 8% of the subjects exhibited single switch only for the second and only for the second and third order task, respectively. The other 15% of subjects did not exhibit single switch on the second order task.

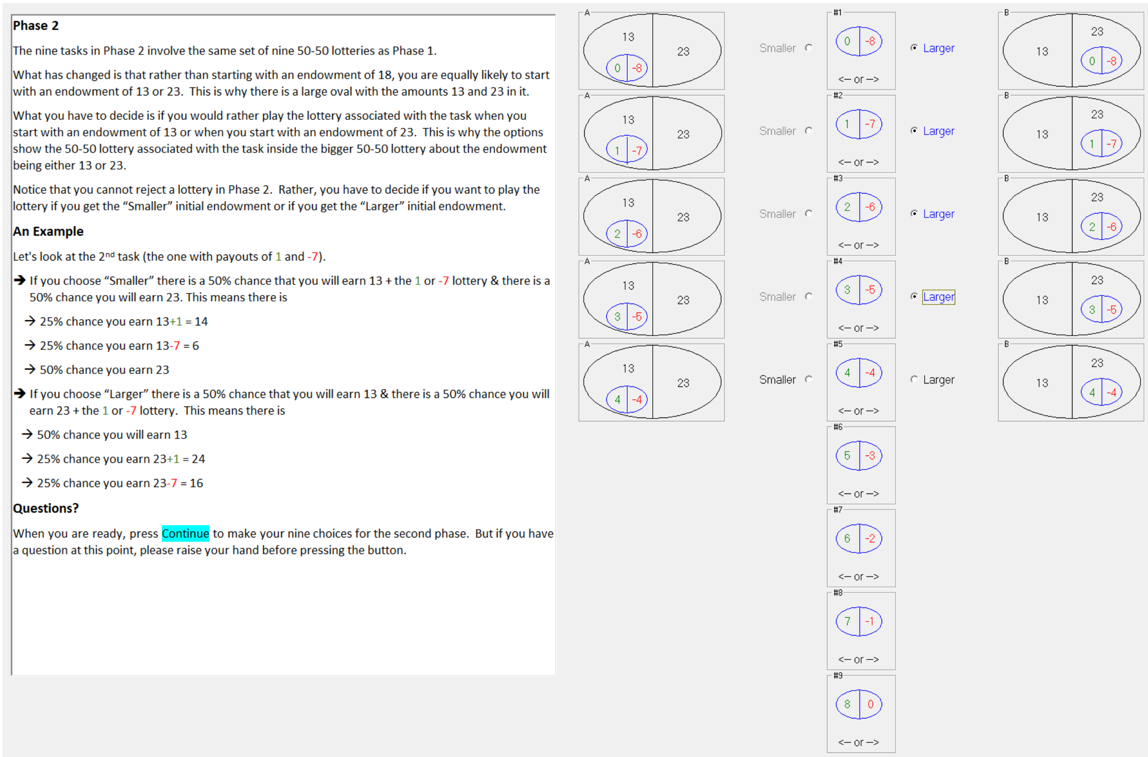

Figure 1: Subject Interface.

only depends on one's second-order risk attitude since the 50-50 lottery with -\$8 and \$0 is a pure loss. Similarly, the ninth choice, which is not visible in Figure 1 apportions the 50-50 lottery with \$0 and \$8, which is a pure gain and thus depends only one's second-order risk attitude.

4.3 Procedures

The study was conducted at The University of Alabama's TIDE Lab. One hundred subjects were recruited from the lab's standing pool of volunteers.[11] While many of the subjects had participated in other unrelated studies, none had participated in a study about risk. The average salient earnings were \$18.74 (with a minimum of \$7 and a maximum of \$31). Subjects also received a \$5 payment for participating in the study.

Data were collected during fourteen sessions with subjects being recruited for 30 minutes. Each session involved between 4 and 14 subjects; however, subjects did not interact with each other during a session. At the start of a session, subjects were seated at individual computer stations

[11] Because the experiment is meant to demonstrate the implementation of the procedure laid out in the previous section and to provide a general sense of the observed degrees of relative and absolute prudence and temperance rather than testing specific hypotheses, the sample size is arbitrary and not based on statistical power.

separated by privacy dividers. Subjects read general computerized instructions.[12] The paid portion of the study in which they completed the 3 tasks shown in Table 1 was self-paced. To facilitate subject understanding, the tasks were presented in order. Task specific instructions were presented just prior to the subjects making their decisions for that task. These instructions remained visible on the left portion of the screen throughout the time the subject was making decisions.

One the subject completed all 27 choices over the 3 tasks, the computer randomly selected one choice from one task to be used in determining the subject's payment. Any 50-50 lottery required to determine the outcome of the selected option was resolved through the use of a physical spinner as had been explained previously to the subjects.[13] Each subject complete a survey that consisted of a single question about gender and was then paid in private and dismissed from the study.

5 Experimental Results

The main results of the experiment are captured in Figure 2, which presents histograms for each order of the choice at which subjects changed their apportionment decisions. The relevant risk attitudes associated with a particular switch point can be found in Table 2. But as preliminary point, we note that the behavior we observe is consistent with the directional results of Deck and Schlesinger (2014) and Noussair, Trautmann, and van de Kuilen (2014). Effectively, those experiments only considered choices with zero mean lotteries (i.e. apportionment decisions of the form in Choice 5 of each of our tasks) and we find that a majority of our subjects indicate a preferences for $B_n(5)$ over $A_n(5)$ for all n = 2, 3, and 4.

Overall, 60% of our subjects made choices indicating some degree of 2^{nd} order risk aversion. For the second order task, the most common switching point was at Choice 6 indicating that 31% of the subjects are slightly risk averse exhibiting 2^{nd} order absolute risk aversion between 0.00 and 0.14 and 2^{nd} order relative risk aversion between 0.00 and 2.40. Further, 75% of the subjects exhibiting 2^{nd} order absolute risk aversion between -0.14 and 0.31 and 2^{nd} order relative risk aversion between -2.27 and 5.72 indicating that few subjects have extreme second order risk attitudes. As an aside, we also report that 2^{nd} order behavior did no differ by gender (p-value for χ^2 test = 0.598).

[12]The instructions are available in the supplementary materials.

[13]The subjects were invited to inspect the spinner before beginning the paid portion of the experiment and again before their final payoff was determined.

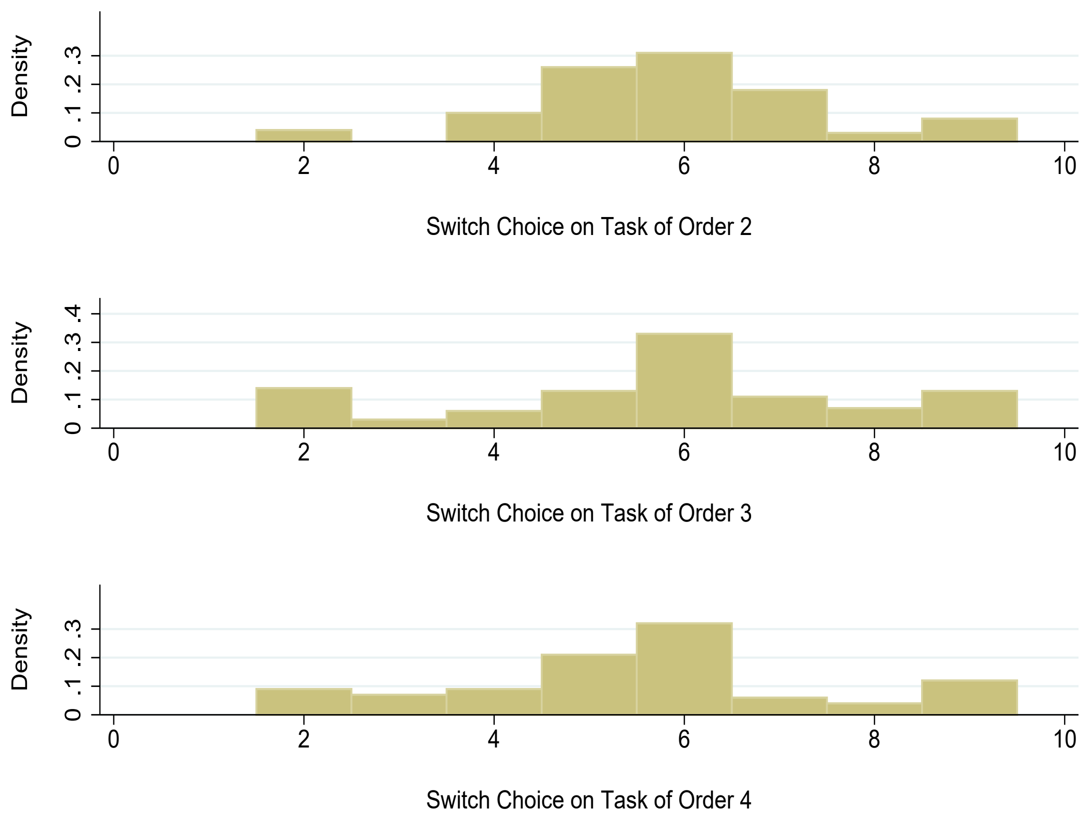

Figure 2: Histogram of Switching in Task of Order n

An approximately two-thirds majority of the subjects exhibited some degree of prudence. As with the second order task, for the third order task the most common switching point was at Choice 6 indicating that 33% of the subjects are slightly prudent exhibiting 3^{rd} order absolute risk aversion between 0.00 and 0.14 and 3^{rd} order relative risk aversion between 0.00 and 2.21. Unlike what was observed for second order risk, on the third order tasks a sizeable fraction of the subjects exhibit extreme attitudes. Only 57% of the subjects exhibiting 3^{rd} order absolute risk aversion between -0.14 and 0.31 and 3^{rd} order relative risk aversion between -2.36 and 4.97, while 14% (13%) of the subjects exhibit 3^{rd} order absolute risk aversion below -0.69 (above 0.69) and 3^{rd} order relative risk aversion below -14.26 (above 10.48). We also note that there is no gender difference in third order behavior (p-value for χ^2 test = 0.266).

For the fourth order task, only a slight majority of 54% exhibited temperance. The modal response was again to switch at choice 6 indicating that this 32% of the subjects exhibited 4^{th} order absolute risk aversion between 0.00 and 0.14 and 4^{th} order relative risk aversion between 0.00 and 2.30. As with the prudence, the degree of temperance exhibited by the subjects is more extreme than the degrees of second order risk aversion that were exhibited. Only 59% of the subjects exhibited 4^{th} order absolute risk aversion between -0.14 and 0.31 and 4^{th} order relative risk aversion between -2.31 and 5.28. Finally, as with second and third order behavior, there is no difference in 4^{th} order behavior by gender (p-value for χ^2 test = 0.582).

As detailed in Section 2, the degree of n^{th} order risk aversion depends on the sign of the $(n-1)^{th}$ derivative of the utility function. The structure of the tasks we implement is such that Choice 1 (or Choice 9) of a Task of Order n is sufficient to determine the sign of the $(n-1)^{th}$. Hence, in Figure 3 we separate the behavior of the subjects based on their selection of $A_n(1)$ or $B_n(1)$. Because every subject selected $B_2(1)$ rather than $A_2(1)$, indicating that all of our subjects exhibit behavior consistent with a monotonically increasing utility function, the figure only shows behavior for the Tasks of Orders 3 and 4. The top-right portion of Figure 3 shows that risk-averse subjects most commonly switch at Choice 6. It also indicates that far more subjects switch at Choices 6 through 9 than at Choices 2 through 5. The top-left portion of Figure 3 indicates that risk-loving subjects most commonly switch at Choice 6 as well. However, by contrast with what is observed for risk-averse subjects, the numbers of risk-loving subjects who switch at Choices 6 through 9 is similar to the number who switch at Choices 2 through 5.

What is clear from the top portion of the figure and supported statistically is that those who are risk averse are more likely to have greater degrees of absolute and relative prudence than are those who are risk seeking (p-value for χ^2 test $= 0.027$). However, as suggested by the lower portion of the figure prudent and imprudent people do not exhibit substantially different degrees of absolute and relative temperance (p-value for χ^2 test $= 0.596$).

One implication of our results draws upon Jindapon and Neilson (2007) who examine comparative risk aversion in a model where decision makers can exert effort to shift an initial wealth distribution to a preferred distribution. Specifically, Jindapon and Neilson (2007) showed that if the initial distribution differs from the preferred distribution by a simple increase in 3^{rd} degree risk, then a risk-averse agent with preferences captured by the utility function u would invest more effort than another risk-averse agent whose preferences are captured by the utility function v if and only if u has a higher degree of absolute prudence than v.[14] Appendix 7 extends Jindapon and Neilson (2007) by assuming that agents are risk-loving instead of risk averse and showing that when the initial distribution differs from the preferred distribution by a simple increase in 3^{rd} degree risk, then a risk-loving agent with utility function u would invest more effort than another risk-loving agent with utility v if and only if u has a lower degree of absolute prudence than v. Note that subjects who switch at Choice $j+1$ has a higher degree of absolute prudence than subjects who switch at Choice j regardless of second order risk preferences. Thus, in the top-right portion of Figure 3 subjects who switch at Choice $j+1$ would exert more efforts than the subjects who switch at Choice j or earlier whereas in the top-left portion of Figure 3 subjects who switch at Choice $j+1$ would exert less efforts than subjects who switch at Choice j or earlier.

Jindapon and Neilson (2007) and our Appendix 7 also provide a basis for understanding the implications of observed 4^{th} order behavior. Jindapon and Neilson (2007) showed that if the initial distribution differs from the preferred distribution by a simple increase in 4^{th} degree risk, then a prudent agent with higher degree of absolute temperance is willing to exert more effort to pursue the preferred distribution than is a prudent agent with a lower degree of absolute temperance. Thus, in the bottom-right portion of Figure 3 subjects who switch at Choice $j+1$ would exert more effort than than subjects who switch at Choice j or earlier. Our Appendix 7 shows that an imprudent agent with a lower degree of absolute temperance is willing to exert more effort to pursue

[14] See Appendix 7 for the definition of a simple increase (decrease) in 3^{rd} degree risk.

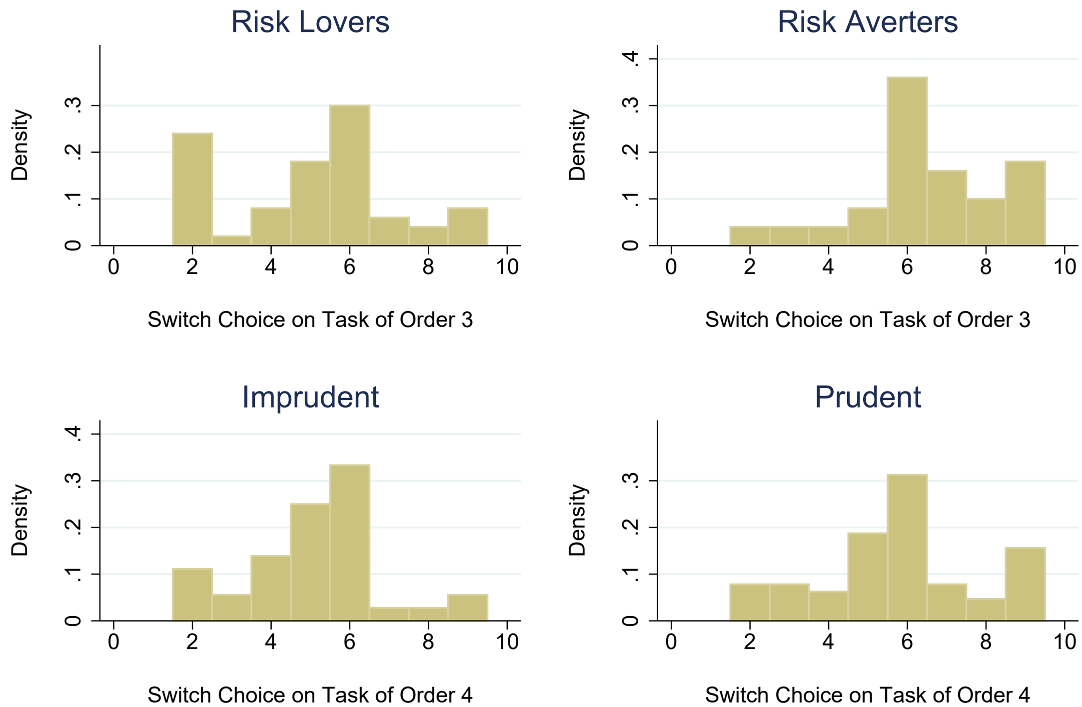

Figure 3: Histogram of Switching in Task of Order n Given Sign of $(n-1)^{th}$ Order Risk Attitude

the preferred distribution than is an imprudent agent with a higher degree of absolute temperance. Thus, in bottom-left portion of Figure 3 subjects who switch at Choice $j+1$ would exert less efforts than subjects who switch at Choice j or earlier.

While not the main focus of our study, the within-subject nature of our data allows us to examine how degrees of risk aversion are related across orders. Specifically, the correlation between switching points on the Tasks of Orders 2 and 3 is 0.209 (p-value = 0.037) indicating that a greater degree of absolute (relative) second order risk aversion is associated with a greater degree of absolute (relative) prudence. Similarly, the degrees of absolute (relative) prudence and absolute (relative) temperance are positively correlated among our subjects; the correlation between switching points on the Tasks of Orders 3 and 4 is 0.258 (p-value = 0.010). But the strongest relationship that we observer is between the degree of second order risk aversion and the degree of temperance; the correlation between switching points on the Tasks of Orders 2 and 4 is 0.422 (p-value < 0.001). That the connection between even order risk attitudes is stronger than the relationship between even and odd orders is consistent with the notion of people being mixed risk averters and mixed risk seekers.

Finally, while our approach is non-parametric, we report a calibration exercise to determine how well different utility functions describe observed behavior. For each subject for each utility function, we identify the parameter values that best fit the observed behavior of the subject across all three orders.[15] Table **??** reports the mean and standard deviation of the subject specific model parameters. For example, when considering the exponential utility function, the average value of γ across the subjects is 0.15.[16] The table also reports the mean and standard deviation of the accuracy rate for a function when parameter values are subject specific. That is, allowing for each subject to have a unique value of γ, on average 80% of a subject's choices are consistent with the exponential utility function. Among the utility functions listed in Table **??**, the exponential-power utility function has the greatest average accuracy at 91%.[17]

For behavior to be consistent with a well behaved utility function, then a subject should prefer $A_1(5)$ to $B_1(5)$ iff they prefer $A_2(1)$ to $B_2(1)$ as both choices depend only on the sign of $u^{(1)}$. Similarly, the choices between $A_2(5)$ and $B_2(5)$ and $A_3(1)$ and $B_3(1)$ both identify the sign of $u^{(3)}$. If attention is restricted to those subjects who make consistent decisions over these two pairs of choices, then exponential-power utility function's average accuracy rate increases to 98% although this performance is not substantially better than that of the power utility function that has an accuracy rate of 95% among these subjects as shown in the top portion of Table **??**. Interestingly, as shown in the lower portion of Table **??**, the subjects who are not consistent on either pair of choices behave as if they are close to risk neutral on average, which imply they were indifferent for all third and fourth order choices.

6 Conclusions

This paper introduces a simple and systematic procedure for identifying the intensity of risk attitudes using the notion of risk apportionment. Our process is systematic in that it involves a series of

[15] Matlab code for the identification is available in the supplementary materials.

[16] This does not imply that 0.15 is the value of γ that best fits all of the data for the exponential utility function.

[17] Because options in the third order task have equal means and options in the fourth order task have equal means and variances, we do not consider linear or quadratic utility functions as these wold imply indifference over all third and fourth order choices and all fourth order choices, respectively.

binary comparisons where each comparisons differs from the others in the same incremental manner. The process is simple in that it only involves comparisons between two lotteries that are themselves composed of combinations of certain losses and fifty-fifty lotteries. Further, our approach can be used to identify both relative and absolute risk aversion of any arbitrary degree without relying upon assumptions regarding the respondent's underlying preference structure.

We also demonstrate the implementation of our approach in a laboratory setting. Consistent with previous work, we find that a majority of our subjects are non-satiated, risk averse, prudent, and temperate. Our approach allows us to go further and identify that the typical behavior of our subjects is modest relative and absolute risk aversion, modest relative and absolute prudence, and modest relative and absolute temperance. Further, we find that higher order degrees of risk aversion are positively correlated, although the strongest relationship that we observe is between second order risk aversion and prudence. Finally, our calibration exercise suggests that behavior is generally consistent with the exponential-power utility function.

7 Acknowledgments

The authors are grateful to Manel Baucells, Henry Chiu, David Crainich, Paan Jindapon, Liqun Liu, Diego Nocetti, Richard Peter and Ilia Tsetlin for helpful comments. All authors share first authorship, but are listed alphabetically by convention.

References

Arrow, K.J., 1971. *Essays in the theory of risk-bearing.* Markham Publishing, Chicago.

Becker, G.M., DeGroot, M.H. and Marschak, J., 1964. Measuring utility by a single-response sequential method. *Behavioral Science*, 9(3), 226-232.

Bliss, R.R. and Panigirtzoglou, N., 2004. Option-implied risk aversion estimates. *Journal of Finance*, 59(1), 407-446.

Caballé, J. and Pomansky, A., 1996. Mixed risk aversion, *Journal of Economic Theory* 71(2), 485-513.

Carroll, C.D. and Kimball, M.S., 2008. Precautionary Saving and Precautionary Wealth. in Durlauf, N.S. and Blume, L.E. (eds), *The New Palgrave Dictionary of Economics*, 2nd edn. (London: MacMillan).

Chiu, W.H., 2005. Skewness preference, risk aversion, and the precedence relations on stochastic changes. *Management Science*, 51(12), 1816-1828.

Cohen, A. and Einav, L., 2007. Estimating risk preferences from deductible choice. *American Economic Review*, 97(3), 745-788.

Crainich, D., Eeckhoudt, L. and Trannoy, A., 2013. Even (mixed) risk lovers are prudent. *American Economic Review*, 103(4), 1529-1535.

Deck, C. and Schlesinger, H., 2010. Exploring higher order risk effects. *Review of Economic Studies*, 77(4), 1403-1420.

Deck, C. and Schlesinger, H., 2014. Consistency of higher order risk preferences. *Econometrica*, 82(5), 1913-1943.

Denuit, M.M. and Eeckhoudt L., 2010. A general index of absolute risk attitude. *Management Science*, 56, 712-715.

Dynan, K.E., 1993. How prudent are consumers?. *Journal of Political Economy*, 101(6), 1104-1113.

Ebert, S. and Wiesen, D., 2014. Joint measurement of risk aversion, prudence, and temperance. *Journal of Risk and Uncertainty*, 48(3), 231-252.

Eeckhoudt, L., Gollier, C. and Schlesinger, H., 1996. Changes in background risk and risk taking behavior. *Econometrica*, 64(3), 683-689.

Eeckhoudt, L., Rey, B. and Schlesinger, H., 2007. A good sign for multivariate risk taking. *Management Science*, 53(1), 117-124.

Eeckhoudt, L. and Schlesinger, H., 2006. Putting risk in its proper place. *American Economic Review*, 96, 280-289.

Eeckhoudt, L. and Schlesinger, H., 2008. Changes in risk and the demand for saving. *Journal of Monetary Economics*, 55(7), 1329-1336.

Eeckhoudt, L., Schlesinger H. and Tsetlin I., 2009. Apportioning of risks via stochastic dominance. *Journal of Economic Theory*, 144, 994-1003.

Esö, P. and White, L., 2004. Precautionary bidding in auctions. *Econometrica*, 72(1), 77-92.

Gollier, C., 2001. Wealth inequality and asset pricing. *The Review of Economic Studies*, 68(1), 181-203.

Gollier, C., 2018. Stochastic volatility implies fourth-degree risk dominance: Applications to asset pricing. *Journal of Economic Dynamics and Control*, 95, 155-171.

Gollier, C. and Pratt, J.W., 1996. Risk vulnerability and the tempering effect of background risk. *Econometrica*, 64(5), 1109-1123.

Gourinchas, P.O. and Parker, J., 2002. Consumption over the life cycle. *Econometrica*, 70(1), 47-89.

Guiso, L., Jappelli, T. and Terlizzese, D., 1996. Income risk, borrowing constraints, and portfolio choice. *American Economic Review*, 86(1), 158-172.

Haering, A., Heinrich, T. and Mayrhofer, T., 2020. Exploring the consistency of higher order risk preferences. *International Economic Review*, 61(1), 283-320.

Holt, C.A. and Laury, S.K., 2002. Risk aversion and incentive effects. *American Economic Review*, 92(5), 1644-1655.

Jewitt, I., 1989. Choosing between risky prospects: The characterization of comparative statics results, and location independent risk. *Management Science*, 35(1), 60-70.

Jindapon, P. and Neilson, W.S., 2007. Higher-order generalizations of Arrow-Pratt and Ross risk aversion: A comparative statics approach. *Journal of Economic Theory*, 136(1), 719-728.

Jindapon, P., Liu, L. and Neilson, W. S., 2021. Comparative risk apportionment. *Economic Theory Bulletin*, 9(1), 91-112.

Kimball, M.S., 1990. Precautionary saving in the small and in the large. *Econometrica*, 58(1), 53-73.

Kimball, M.S., 1992. Precautionary motives for holding assets. In P. Newman, M. Milgate and J. Eatwell (eds.) *The New Palgrave Dictionary of Money and Finance*. Vol. 3, London: The Macmillan Press Limited, pp. 158-161.

Kimball, M.S., 1993. Standard risk aversion. *Econometrica*, 61(3), 589-611.

Liu, L. and Meyer, J., 2013. Substituting one risk increase for another: A method for measuring risk aversion. *Journal of Economic Theory*, 148(6), 2706-2718.

Menezes, C., Geiss, C. and Tressler, J., 1980. Increasing downside risk. *American Economic Review*, 70(5), 921-932.

Noussair, C.N., Trautmann, S.T. and van de Kuilen, G., 2014. Higher order risk attitudes, demographics, and financial decisions. *Review of Economic Studies*, 81(1), 325-355.

Pratt, J.W., 1964. Risk aversion in the small and in the large. *Econometrica*, 32(1), 122-136

Rothschild, M. and Stiglitz, J.E., 1970. Increasing risk: I. A definition. *Journal of Economic Theory*, 2(3), 225-243.

Schneider, S. O. and Sutter, M., 2021. Higher order risk preferences: New measures, determinants and field behavior. Working Paper, MPI Collective Goods Discussion Paper, (2020/22).

White L., 2008. Prudence in bargaining: The effect of uncertainty on bargaining outcomes. *Games and Economic Behavior*, 62(1), 211-231.

Appendix A Mathematical Proofs

To prove our formal results, we first establish three technical lemmas.

Lemma A1. *For u and v that are twice continuously differentiable with $u^{(1)}$ and $v^{(1)}$ having the same sign, the following statements are equivalent:*

(i) $-\frac{u^{(2)}(x)}{u^{(1)}(x)} \geq -\frac{v^{(2)}(x)}{v^{(1)}(x)}$ *for all* $x \in [a, b]$*;*

(ii) $-\frac{u^{(1)}(x)-u^{(1)}(x-k)}{u(x)-u(x-k)} \geq -\frac{v^{(1)}(x)-v^{(1)}(x-k)}{v(x)-v(x-k)}$ *for all* $x, x-k \in [a, b]$ *with* $k > 0$*.*

Proof. Assume first that $u^{(1)} > 0$ and $v^{(1)} > 0$. Since (i) is a direct consequence of (ii) after letting $k \to 0$, we concentrate on the proof that (i) implies (ii). Statement (i) is equivalent to $u(x) = \varphi(v(x))$, where φ is twice differentiable with $\varphi^{(1)} > 0$ and $\varphi^{(2)} \leq 0$ (Pratt 1964). By the mean value theorem, $u(x) - u(x-k) = \varphi(v(x)) - \varphi(v(x-k)) = \varphi^{(1)}(v(x-\theta k))[v(x) - v(x-k)]$ with some $\theta \in (0, 1)$, and accordingly,

$$
\begin{aligned}
-\frac{u^{(1)}(x) - u^{(1)}(x-k)}{u(x) - u(x-k)} &= -\frac{\varphi^{(1)}(v(x))v^{(1)}(x) - \varphi^{(1)}(v(x-k))v^{(1)}(x-k)}{\varphi^{(1)}(v(x-\theta k))[v(x) - v(x-k)]} \\
&\geq -\frac{v^{(1)}(x) - v^{(1)}(x-k)}{v(x) - v(x-k)},
\end{aligned}
$$

which follows from $\varphi^{(1)}(v(x)) \leq \varphi^{(1)}(v(x-\theta k)) \leq \varphi^{(1)}(v(x-k))$.

To address the alternative case with $u^{(1)} < 0$ and $v^{(1)} < 0$, we introduce $\hat{u} = -u$ and $\hat{v} = -v$, which satisfy $\hat{u}^{(1)} > 0$ and $\hat{v}^{(1)} > 0$. Because $-\frac{\hat{u}^{(2)}(x)}{\hat{u}^{(1)}(x)} = -\frac{u^{(2)}(x)}{u^{(1)}(x)}$, $-\frac{\hat{u}^{(1)}(x)-\hat{u}^{(1)}(x-k)}{\hat{u}(x)-\hat{u}(x-k)} = -\frac{u^{(1)}(x)-u^{(1)}(x-k)}{u(x)-u(x-k)}$ and similar equations also hold for $\hat{v}$ and v, the result follows straightforwardly by adapting the former analysis to $\hat{u}$ and $\hat{v}$. Q.E.D.

Lemma A2 below extends Lemma A1 to higher orders.

Lemma A2. *For $n \geq 3$, let $\hat{A}_n$ and $\hat{B}_n$ be the lotteries introduced by Eeckhoudt and Schlesinger (2006) with $w = 0$ and $\tilde{\varepsilon}_i$ as in (5). For u and v that are continuously differentiable up to order n with $u^{(n-1)}$ and $v^{(n-1)}$ having the same sign, define*

$$
\begin{aligned}
u_{n-2}(x) &\equiv \mathbb{E}u(x + \hat{A}_{n-2}) - \mathbb{E}u(x + \hat{B}_{n-2}), \\
v_{n-2}(x) &\equiv \mathbb{E}v(x + \hat{A}_{n-2}) - \mathbb{E}v(x + \hat{B}_{n-2}).
\end{aligned}
$$

The following statements are equivalent:

(i) $-\frac{u^{(n)}(x)}{u^{(n-1)}(x)} \geq -\frac{v^{(n)}(x)}{v^{(n-1)}(x)}$ *for all* $x \in [a, b]$*;*

(ii) $-\frac{u_{n-2}^{(2)}(x)}{u_{n-2}^{(1)}(x)} \geq -\frac{v_{n-2}^{(2)}(x)}{v_{n-2}^{(1)}(x)}$ *for all* $x + \hat{A}_{n-2}, x + \hat{B}_{n-2} \in [a, b]$*.*

Proof. For $n = 3$, $u_1(x) = u(x - k) - u(x)$ and $v_1(x) = v(x - k) - v(x)$. We apply Lemma A1 with $u^{(1)}$ and $v^{(1)}$ to get the equivalence between (i) and (ii).

For $n = 4$, $u_2(x) = \frac{1}{2}[u(x - k_1) + u(x + k_1)] - u(x)$ and $v_2(x) = \frac{1}{2}[v(x - k_1) + v(x + k_1)] - v(x)$. Accordingly, (ii) implies (i) after letting $k_1 \to 0$. To prove that (i) implies (ii), we apply Lemma A1 to $u^{(2)}$ and $v^{(2)}$, and obtain that (i) implies

$$-\frac{u^{(3)}(x) - u^{(3)}(x - k_1)}{u^{(2)}(x) - u^{(2)}(x - k_1)} \geq -\frac{v^{(3)}(x) - v^{(3)}(x - k_1)}{v^{(2)}(x) - v^{(2)}(x - k_1)}.$$

Similarly, applying Lemma A1 to $\hat{u}(x) \equiv u^{(1)}(x) - u^{(1)}(x - k_1)$ and $\hat{v}(x) \equiv v^{(1)}(x) - v^{(1)}(x - k_1)$ leads to

$$-\frac{\hat{u}^{(1)}(x + k_1) - \hat{u}^{(1)}(x)}{\hat{u}(x + k_1) - \hat{u}(x)} \geq -\frac{\hat{v}^{(1)}(x + k_1) - \hat{v}^{(1)}(x)}{\hat{v}(x + k_1) - \hat{v}(x)},$$

which yields (ii) as $\hat{u}(x + k_1) - \hat{u}(x) = 2u_2^{(1)}(x)$ and $\hat{v}(x + k_1) - \hat{v}(x) = 2v_2^{(1)}(x)$.

For $n \geq 5$ the proof is by induction. Suppose that the equivalence between (i) and (ii) holds true for all orders up to $n - 1$. For order n, recall that $\tilde{\varepsilon}_{\text{Int}(n/2)-1} = [-k_{\text{Int}(n/2)-1}; k_{\text{Int}(n/2)-1}]$ and

$$\begin{aligned} u_{n-2}(x) &= \frac{1}{4}\left[u_{n-4}\left(x - k_{\text{Int}(n/2)-1}\right) + u_{n-4}\left(x + k_{\text{Int}(n/2)-1}\right)\right] - \frac{1}{2}u_{n-4}(x), \\ v_{n-2}(x) &= \frac{1}{4}\left[v_{n-4}\left(x - k_{\text{Int}(n/2)-1}\right) + v_{n-4}\left(x + k_{\text{Int}(n/2)-1}\right)\right] - \frac{1}{2}v_{n-4}(x). \end{aligned}$$

Applying the equivalence for the fourth order to u_{n-4} and v_{n-4}, we obtain that (ii) is equivalent to

$$-\frac{u_{n-4}^{(4)}(x)}{u_{n-4}^{(3)}(x)} \geq -\frac{v_{n-4}^{(4)}(x)}{v_{n-4}^{(3)}(x)} \quad \text{for all } x + \hat{A}_{n-4}, x + \hat{B}_{n-4} \in [a, b].$$

We further apply the equivalence for the $(n - 4)^{th}$ order to $u^{(2)}$ and $v^{(2)}$ to obtain that (ii) is equivalent to (i) for order n. Q.E.D.

Lemma A3. *For $n \geq 2$, consider a Task of Order n as specified in (7) that is supported on $[a,b] \subset (0,\infty)$. For each $j = 2,...,J-1$, there exists a unique constant absolute risk aversion coefficient $\Theta_1(n,j)$ such that $\mathbb{E}v_1(A_n(j)) = \mathbb{E}v_1(B_n(j))$ under $\theta_1 = \Theta_1(n,j)$. Moreover, $\Theta_1(n,j)$ increases strictly in j. The same statement holds true for $\Theta_2(n,j)$, the constant relative risk aversion coefficient such that $\mathbb{E}v_2(A_n(j)) = \mathbb{E}v_2(B_n(j))$.*

Proof. We proceed under the assumption that $(-1)^n v_1^{(n-1)} > 0$ as the alternative case $(-1)^n v_1^{(n-1)} < 0$ can be addressed by adapting the analysis to $-v_1$. By induction, we have $v_{1(n-2)}^{(1)} > 0$ and $\mathbb{E}v_1(B_n(j)) - \mathbb{E}v_1(A_n(j)) = v_{1(n-2)}(w) - \mathbb{E}v_{1(n-2)}(w + \tilde{\delta}_j)$.

For $j = 2,...,J-1$, the existence and uniqueness of $\Theta_1(n,j)$ follow from the monotonicity of $v_{1(n-2)}^{-1}(\mathbb{E}v_{1(n-2)}(w + \tilde{\delta}_j))$ with respect to θ_1 (Pratt 1964), together with the facts that $\lim_{\theta_1 \to -\infty} v_{1(n-2)}^{-1}(\mathbb{E}v_{1(n-2)}(w + \tilde{\delta}_j)) = w + \text{ess sup}(\tilde{\delta}_j)$ and $\lim_{\theta_1 \to \infty} v_{1(n-2)}^{-1}(\mathbb{E}v_{1(n-2)}(w + \tilde{\delta}_j)) = w + \text{ess inf}(\tilde{\delta}_j)$.

As j increases, $\mathbb{E}v_{1(n-2)}(w + \tilde{\delta}_j)$ increases strictly, but $\mathbb{E}v_{1(n-2)}(w)$ does not change. Thus, for any $j_1 < j_2$, $\mathbb{E}v_{1(n-2)}(w) = \mathbb{E}v_{1(n-2)}(w + \tilde{\delta}_{j_1})$ under $\theta_1 = \Theta_1(n,j_1)$ implies $\mathbb{E}v_{1(n-2)}(w) < \mathbb{E}v_{1(n-2)}(w + \tilde{\delta}_{j_2})$ under the same θ_1, which further implies $\mathbb{E}v_{1(n-2)}(w) < \mathbb{E}v_{1(n-2)}(w + \tilde{\delta}_{j_2})$ under all $\theta_1 \leq \Theta_1(n,j_1)$ by virtue of Proposition 1. Accordingly, to achieve $\mathbb{E}v_{1(n-2)}(w) = \mathbb{E}v_{1(n-2)}(w+\tilde{\delta}_{j_2})$ under $\theta_1 = \Theta_1(n,j_2)$, it must hold that $\Theta_1(n,j_2) > \Theta_1(n,j_1)$, which proves the monotonicity of $\Theta_1(n,j)$ with respect to j.

When $j = 1$, $\mathbb{E}v_{1(n-2)}(w) > \mathbb{E}v_{1(n-2)}(w + \tilde{\delta}_1)$ for any finite $\Theta_1 \in \mathbb{R}$ yielding the convention $\Theta_1(n,1) = -\infty$. When $j = J$, $\mathbb{E}v_{1(n-2)}(w) < \mathbb{E}v_{1(n-2)}(w + \tilde{\delta}_J)$ for any finite $\Theta_1 \in \mathbb{R}$ yielding the convention $\Theta_1(n,j) = \infty$. Q.E.D.

In what follows, we prove our main results.

Proof of Proposition 1. To prove that (i) implies (ii), let F and G be the cumulative distribution functions of B_2 and A_2, respectively. Then, integration-by-parts yields $\mathbb{E}v(B_2) - \mathbb{E}v(A_2) =$

$\int_a^b v^{(1)}(x)[G(x) - F(x)]dx$ and

$$\begin{aligned}
&\mathbb{E}u(B_2) - \mathbb{E}u(A_2) \\
&= \int_a^b \frac{u^{(1)}(x)}{v^{(1)}(x)} \left(-\frac{u^{(2)}(x)}{u^{(1)}(x)} + \frac{v^{(2)}(x)}{v^{(1)}(x)} \right) \int_a^x v^{(1)}(y)[G(y) - F(y)]dydx \\
&\quad + \frac{u^{(1)}(b)}{v^{(1)}(b)} \int_a^b v^{(1)}(x)[G(x) - F(x)]dx.
\end{aligned}$$

Since F intersects G from below once, $\mathbb{E}v(B_2) = \mathbb{E}v(A_2)$ implies $\int_a^b v^{(1)}(y)[G(y) - F(y)]dy = 0$ but $\int_a^x v^{(1)}(y)[G(y) - F(y)]dy \geq 0$ for all $x \in [a, b]$, which in turn implies $\mathbb{E}u(B_2) \geq \mathbb{E}u(A_2)$.

To show that (ii) implies (i), we argue by contradiction. If $-\frac{u^{(2)}(x_0)}{u^{(1)}(x_0)} < -\frac{v^{(2)}(x_0)}{v^{(1)}(x_0)}$ for some $x_0 \in [a, b]$, then continuity implies $-\frac{u^{(2)}(x)}{u^{(1)}(x)} < -\frac{v^{(2)}(x)}{v^{(1)}(x)}$ on $(x_0 - \varepsilon, x_0 + \varepsilon) \cap [a, b]$ for some $\varepsilon > 0$. For $\tilde{\delta}$ supported on $(x_0 - \varepsilon, x_0 + \varepsilon) \cap [a, b]$, we can follow the proof for (i) implying (ii) to obtain that $\mathbb{E}v(B_2) = \mathbb{E}v(A_2)$ always implies $\mathbb{E}u(B_2) \leq \mathbb{E}u(A_2)$ and a strict inequality $\mathbb{E}u(B_2) < \mathbb{E}u(A_2)$ holds when $\tilde{\delta}$ is not constant. This a contradiction to (ii). Q.E.D.

Proof of Proposition 2. For $k > 0$, recall $u_1(x) = u(x-k) - u(x)$ and $v_1(x) = v(x-k) - v(x)$. For individuals with utility function u, the choice between A_3 and B_3 is based on comparing $\mathbb{E}u_1(A_2)$ and $\mathbb{E}u_1(B_2)$ with a similar result holding for v. We apply Proposition 1 to obtain that (ii) is equivalent to $-\frac{u_1^{(2)}(x)}{u_1^{(1)}(x)} \geq -\frac{v_1^{(2)}(x)}{v_1^{(1)}(x)}$ for all $x, x - k \in [a, b]$, and further apply Lemma A2 at the third order to obtain that (ii) is equivalent to (i). Q.E.D.

Proof of Theorem 1. For $n \geq 3$, let u_{n-2} and v_{n-2} be defined as in Lemma A2. For an individual with utility function u, the choice between A_n and B_n is based on comparing $\mathbb{E}u_{n-2}(A_2)$ and $\mathbb{E}u_{n-2}(B_2)$. A similar statement holds for v. We apply Proposition 1 to obtain that (ii) is equivalent to $-\frac{u_{n-2}^{(2)}(x)}{u_{n-2}^{(1)}(x)} \geq -\frac{v_{n-2}^{(2)}(x)}{v_{n-2}^{(1)}(x)}$ for all $x + A_{n-2}, x + B_{n-2} \in [a, b]$, and further apply Lemma A2 to obtain that (ii) is equivalent to (i). Q.E.D.

Proof of Corollary 1. We assume $(-1)^n u^{(n-1)} > 0$ as the case $(-1)^n u^{(n-1)} < 0$ can be addressed by adapting the above analysis to $-u$. Recall that for an individual with utility function u, the choice between $A_n(j)$ and $B_n(j)$ is based on comparing $\mathbb{E}u_{n-2}(w + \tilde{\delta}_j)$ and $\mathbb{E}u_{n-2}(w)$. We prove by induction that $u_{n-2}^{(1)} > 0$. When $j = 1$, $\tilde{\delta}_1 = [-2h; 0]$ is a first-degree deterioration relative to 0 and thus w is preferred to $w + \tilde{\delta}_1$ by u_{n-2}; when $j = J$, $\tilde{\delta}_J = [0; 2h]$ is a first-degree improvement relative to 0 and thus $w + \tilde{\delta}_J$ is preferred to w by u_{n-2}. As j increases from 1 to J, $\mathbb{E}u_{n-2}(w + \tilde{\delta}_j)$

increases but $\mathbb{E}u_{n-2}(w)$ does not change, yielding a single point at which the individual switches from preferring w to preferring $w+\tilde{\delta}_j$ under u_{n-2}, or equivalently, from preferring $B_n(j)$ to preferring $A_n(j)$ under u.

When the individual prefers $B_n(j)$ to $A_n(j)$ for $j \leq j^*$ and prefers $A_n(j)$ to $B_n(j)$ for $j \geq j^*+1$, we have $\mathbb{E}u(B_n(j^*)) \geq \mathbb{E}u(A_n(j^*))$ and $\mathbb{E}u(B_n(j^*+1)) \leq \mathbb{E}u(A_n(j^*+1))$. If otherwise $-\frac{u^{(n)}(x)}{u^{(n-1)}(x)} < \Theta_1(j^*)$ or $-\frac{u^{(n)}(x)}{u^{(n-1)}(x)} > \Theta_1(j^*+1)$ for all $x \in [a,b]$, then Theorem 1 would imply $\mathbb{E}u(A_n(j^*)) > \mathbb{E}u(B_n(j^*))$ or $\mathbb{E}u(A_n(j^*+1)) < \mathbb{E}u(B_n(j^*+1))$, which is a contradiction. Q.E.D.

Appendix B Effort-Making Problem

Jindapon and Neilson (2007) examined comparative risk aversion in a model where decision makers can exert effort to shift an initial wealth distribution (G) to a preferred distribution (F). Given that u and v are utility functions that exhibit risk-aversion and/or prudence, their paper provides conditions on u and v for an individual whose preference is captured by u to exert more effort than an individual whose preference is captured by v. In this appendix, we extend their model to consider individuals who are risk loving and/or imprudent.

Assume that an individual with utility function u can invest in an effort $e \in [0,1]$ with a non-monetary cost of effort $c(e)$ where $c'(e) > 0$ and $c''(e) > 0$ such that her wealth distribution will become $eF(x) + (1-e)G(x)$, where $x \in [a,b]$. Thus, the objective of the individual is as follows:

$$\max_{e\in[0,1]} \int_a^b u(x)[edF(x) + (1-e)dG(x)] - c(e).$$

The first-order condition is

$$\int_a^b u(x)d[F(x) - G(x)] - c'(e) = 0,$$

and the second-order condition holds automatically due to $-c''(e) < 0$. Since the second-order condition does not depend on the sign of $u^{(2)}$, there always exists a unique interior solution if $\int_a^b u(x)d[F(x) - G(x)] > 0$, regardless of whether or not the individual is risk averse or risk loving.

Define $F^{(2)}(x) = \int_a^x F(t)dt$ and $G^{(2)}(x) = \int_a^x G(t)dt$. Following Jindapon and Neilson (2007), we assume that F differs from G by a simple decrease in third order risk, i.e., $F^{(2)}(x)$ crosses $G^{(2)}(x)$

only once from below and $E_F(x) = E_G(x)$. Taking integration by parts, the first-order condition becomes

$$\int_a^b [-u^{(2)}(x)][G^{(2)}(x) - F^{(2)}(x)]dx - c'(e) = 0.$$

To compare two individuals with utility functions u and v, denote the optimal effort levels corresponding to u and v by e_u^* and e_v^*, respectively.

Proposition B1. *Regarding the effort-making problem, we have:*

(i) Under $u^{(2)} < 0$ and $v^{(2)} < 0$, $e_u^* \geq e_v^*$ for any F and G such that F differs from G by a simple decrease in third degree risk, if and only if $-\frac{u^{(3)}(x)}{u^{(2)}(x)} \geq -\frac{v^{(3)}(x)}{v^{(2)}(x)}$ for all $x \in [a,b]$;

(ii) Under $u^{(2)} > 0$ and $v^{(2)} > 0$, $e_u^* \geq e_v^*$ for any F and G such that F differs from G by a simple decrease in third degree risk, if and only if $-\frac{u^{(3)}(x)}{u^{(2)}(x)} \leq -\frac{v^{(3)}(x)}{v^{(2)}(x)}$ for all $x \in [a,b]$.

Statement (i) in the above proposition is owing to Jindapon and Neilson (2007). Here, we examine the "if" part of statement (ii). Assume $F^{(2)}(x) \leq G^{(2)}(x)$ for $x \leq x_0$ and $F^{(2)}(x) \geq G^{(2)}(x)$ for $x \geq x_0$, and scale u and v so that $u^{(2)}(x_0) = v^{(2)}(x_0)$. In light of the first-order condition, $e_u^* \geq e_v^*$ if and only if

$$\int_a^b \left[-\frac{u^{(2)}(x)}{u^{(2)}(x_0)} + \frac{v^{(2)}(x)}{v^{(2)}(x_0)}\right][G^{(2)}(x) - F^{(2)}(x)]dx \geq 0.$$

If $-\frac{u^{(3)}(x)}{u^{(2)}(x)} \leq -\frac{v^{(3)}(x)}{v^{(2)}(x)}$ for all $x \in [a,b]$, then it holds $\frac{u^{(2)}(y)}{u^{(2)}(z)} \leq \frac{v^{(2)}(y)}{v^{(2)}(z)}$ for all $z \geq y$. Accordingly, we have $F^{(2)}(x) \leq G^{(2)}(x)$ and $\frac{u^{(2)}(x)}{u^{(2)}(x_0)} \leq \frac{v^{(2)}(x)}{v^{(2)}(x_0)}$ for $x \leq x_0$ and $F^{(2)}(x) \geq G^{(2)}(x)$ and $\frac{u^{(2)}(x)}{u^{(2)}(x_0)} \geq \frac{v^{(2)}(x)}{v^{(2)}(x_0)}$ for $x \geq x_0$, which in turn implies the desired inequality. The "only if" portion of the statement can be proved using the same approach as in the proof of Theorem 3 in Jindapon and Neilson (2007).

By the same token, we can extend the above analysis to the fourth order. We say F differs from G by a simple decrease in fourth degree risk, if $F^{(3)}(x) = \int_a^x F^{(2)}(t)dt$ crosses $G^{(3)}(x) = \int_a^x G^{(2)}(t)dt$ only once from below, and moreover, $E_F(x) = E_G(x)$ and $E_F(x^2) = E_G(x^2)$ hold. The result is formally stated as follows, and the proof is omitted.

Proposition B2. *Regarding the effort-making problem, we have:*

(i) Under $u^{(3)} > 0$ *and* $v^{(3)} > 0$, $e_u^* \geq e_v^*$ *for any* F *and* G *such that* F *differs from* G *by a simple decrease in fourth degree risk, if and only if* $-\frac{u^{(4)}(x)}{u^{(3)}(x)} \geq -\frac{v^{(4)}(x)}{v^{(3)}(x)}$ *for all* $x \in [a, b]$*;*

(ii) Under $u^{(3)} < 0$ *and* $v^{(3)} < 0$, $e_u^* \geq e_v^*$ *for any* F *and* G *such that* F *differs from* G *by a simple decrease in fourth degree risk, if and only if* $-\frac{u^{(4)}(x)}{u^{(3)}(x)} \leq -\frac{v^{(4)}(x)}{v^{(3)}(x)}$ *for all* $x \in [a, b]$.