The many faces of multivariate risk-taking

Risk apportionment for desirable and undesirable attributes

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Abstract

Many decisions under risk involve alternatives with multiple and possibly non-financial attributes. In this paper, we characterize risk apportionment preferences in a bivariate setting. We distinguish between desirable and undesirable attributes and show how to adapt the theory to obtain consistent results. We extend the definitions of correlation aversion, cross-prudence and cross-temperance in terms of simple lotteries to the case of undesirable attributes, provide a general characterization based on signs of cross-derivatives of the utility function, and discuss specific multivariate models for applications. Our results show how to unlock the powerful machinery of risk apportionment in the many situations in which decision-makers face undesirable attributes.

Keywords: Multivariate risk · risk apportionment · undesirable attributes · higher-order risk effects · correlation aversion · cross-prudence · cross-temperance

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1 Introduction

Risk attitudes are without doubt a crucial determinant of economic and financial decision-making. Many decisions under risk involve more than a single attribute (see Keeney et al., 1993). Treatment decisions have financial consequences but also affect people's health. After filing one's taxes, people may be uncertain about the size of the refund but also about how long they will have to wait to obtain it. In times of a public health crisis, policymakers have to weigh the economic costs of containment measures against the loss of life. These examples also illustrate that some attributes are desirable like money, consumption or health, while other attributes are undesirable like waiting time, costs or the number of fatalities. The distinction between desirable and undesirable attributes plays a key role in our paper.

While many economists have traditionally been thinking of risk attitude as risk-averse or risk-loving, so-called higher-order risk attitudes are receiving increased attention. In the early models of precautionary saving by Leland (1968), Sandmo (1970) and Drèze and Modigliani (1972), which were later revisited by Kimball (1990), a third-order attitude called prudence guarantees that income risk leads to precautionary saving. A fourth-order attitude called temperance ensures less risk-taking in the presence of greater background risk (Kimball, 1993). Although first received with some skepticism, the notions of prudence and temperance have now been widely accepted in the economic analysis of decision-making under risk.[1]

More generally, Ekern (1980) defines the notion of Kth-degree risk aversion as an aversion to Kth-degree risk increases where K is an integer. While general, his integral conditions lack intuition and it remains unclear how to test for higher-order risk attitudes in the data. A breakthrough came with the impactful works of Eeckhoudt and Schlesinger (2006), Eeckhoudt et al. (2007) and Eeckhoudt et al. (2009), who provide a simple and intuitive way of understanding higher-order risk preferences via risk apportionment. Two basic types of apportionment preferences arise from their analysis, "combining good with bad" and "combining good with good and bad with bad" (Deck and Schlesinger, 2014).

In this paper, we provide new results on multivariate risk-taking. We use the powerful tools of risk apportionment but explicitly distinguish between desirable and undesirable attributes in our analysis. Our first contribution is to revisit the concepts of correlation aversion, cross-prudence and cross-temperance (see Eeckhoudt et al., 2009). As Deck and Schlesinger (2014) say, "restricting any analyses within economic applications to only the first four orders seems a reasonable approximation." We start with such an approximation and provide definitions of correlation aversion, cross-prudence and cross-temperance in terms of simple lotteries. When one or both attributes are undesirable, some of these definitions need to be adjusted, and we explain how and why. In the expected utility model, these simple lottery preferences pin

[1] Noussair et al. (2014) summarize the many ways in which prudence and temperance affect economic behavior. Applications include auctions, bargaining, ecological discounting, precautionary saving, investment, rent-seeking, and prevention.

down the sign of specific cross-derivatives of the utility function. The approach with simple lotteries has the advantage that it remains valid even when expected utility falls short from a descriptive standpoint (see Starmer, 2000).[2]

Our second contribution is to provide a general characterization of risk apportionment preferences in the bivariate setting under expected utility. We revisit the univariate case and explain how to accommodate undesirable attributes. All we need to do is to adjust the "seed lotteries," and the rest of Eeckhoudt and Schlesinger's (2006) risk apportionment theory stays intact. For an undesirable attribute, decision-makers who prefer to combine good with bad have all subsequent derivatives of the utility function negative. Decision-makers who prefer to combine good with good and bad with bad have subsequent derivatives alternating in sign but starting with a negative instead of a positive (Ebert, 2020). For a desirable attribute, combining good with bad is characterized by alternating signs whereas combining good with good and bad with bad is characterized by a consistent positive sign. A reversal occurs when going from a desirable to an undesirable attribute.

Once we have the univariate apportionment lotteries in place, we characterize risk apportionment preferences across attributes. We use Eeckhoudt et al.'s (2009) approach of apportioning Ekern (1980) risk increases and determine the signs of successive cross-derivatives of the utility function in three cases. We consider two desirable attributes, one desirable and one undesirable attribute, and two undesirable attributes. The orders of the risk changes play different roles in the three cases, and these roles are determined by the apportionment preferences on the individual attributes. In the expected utility model, it is easy to show that all three cases can be reconciled with each other. Hence, our results are fully consistent.

Our third contribution is to relate our findings to popular multivariate models. We discuss multiplicatively separable utility and equivalent monetary utility. In the separable case, the apportionment preference across attributes is very easy to characterize. If the component utility functions have the same sign, the decision-maker prefers to combine good with good and bad with bad. If they have opposite signs, she prefers to combine good with bad. This insight allows us to construct any of the eight combinations of risk apportionment preferences studied in this paper for applications.

We proceed as follows. Section 2 outlines the model, defines risk apportionment, and revisits the single-attribute case. Section 3 defines correlation aversion, cross-prudence and cross-temperance while distinguishing between desirable and undesirable attributes. Section 4 provides the general theory. Section 5 connects the apportionment preference across attributes to signs of cross-derivatives of the utility function and reconciles all three cases. Section 6 relates our analysis to Gollier's (2021) generalized risk apportionment theory. Section 7 presents specific multivariate models and shows how to implement different combinations of apportionment preferences in applications. A final section concludes.

[2] One such example is the recent work by Eeckhoudt et al. (2020) who characterize risk apportionment over a single attribute in Yaari's (1987) dual theory of choice under risk.

2 The model

2.1 Preliminaries

We analyze bivariate preferences. Our analysis can be extended to higher dimensions by fixing all but two of the attribute levels. Let (x, y) denote a nonnegative vector of attributes with $x \in [0, \overline{x}]$ and $y \in [0, \overline{y}]$. The domain of attribute bundles is then given by $\mathcal{D} = [0, \overline{x}] \times [0, \overline{y}]$.[3] Previous literature has mainly focused on the case that x and y both represent desirable attributes, for example, if we interpret x as consumption or final wealth and y as health or quality of life. We refer to this case as **DD** where **D** is shorthand for "desirable." We revisit this case as a benchmark and provide some extensions. The decision-maker (DM) is better off when x increases, when y increases, or when both increase.

We can also consider situations in which x is desirable and y is undesirable. For example, x can be a monetary payoff and y the time it takes to receive it (waiting time). DMs prefer higher values of x but lower values of y. This setting is studied in Ebert (2020) under the assumption that only y is uncertain and x is deterministic. Households face uncertainty regarding the value of their investments, for which they prefer higher over lower outcomes, and at the same time uncertainty over potential losses arising from auto and home ownership or legal liability. For the second attribute, they clearly prefer lower over higher outcomes. We label this case as **DU** where **U** abbreviates "undesirable." The ordering assumption that the first attribute is desirable and the second one undesirable is without loss.

Finally we consider the case that both x and y are undesirable and label it as **UU**. For the sake of example, imagine a policymaker in times of a public health crisis who considers the stringency of lockdown measures. These measures affect the economy, potentially resulting in unemployment and loss of livelihood, but they curb the spread of infectious diseases, thus mitigating the number of hospitalizations and fatalities. If x denotes unemployment and y the number of fatalities, then both are undesirable because lower outcomes are preferred over higher ones for each of the two attributes.

2.2 Risk apportionment

Eeckhoudt and Schlesinger (2006) develop the notion of risk apportionment and Eeckhoudt et al. (2009) apply it to stochastic dominance. Consider a DM who faces two independent stochastic changes that are unfavorable but unavoidable. If the DM would rather be exposed to the two changes in separate states, she exhibits a preference for combining good with bad. More formally, she prefers the 50-50 lottery that allocates one of the changes to one state and the other change to the other state over the 50-50 lottery that allocates both changes to the same state. The preferred lottery combines a relatively good outcome with a relatively bad outcome in each state whereas the dispreferred lottery has both good outcomes in the same

[3] A bounded domain avoids issues of sign permanence, see Scott and Horvath (1980) and Menegatti (2001).

state and both bad outcomes in the other state. We refer to this preference as combining good with bad or a preference for harms disaggregation, in short d for "disaggregate."

Some DMs may have the reverse preference and rather face the two unfavorable changes in the same state than in different states. They prefer the 50-50 lottery that allocates both changes to the same state over the 50-50 lottery that allocates one of the changes to one state and the other change to the other state. Consequently, these DMs prefer to combine the two relatively good outcomes in one state and the two relatively bad outcomes in the other state instead of combining relatively good with relatively bad outcomes in the same state. We refer to this preference as combining good with good and bad with bad or a preference for harms aggregation, and abbreviate it with a for "aggregate."[4]

In the univariate context, DMs who always prefer to disaggregate harms are called mixed risk-averse whereas DMs who always prefer to aggregate harms are called mixed risk-loving. Mixed risk aversion was first introduced by Caballé and Pomansky (1996) and Brockett and Golden (1987) whereas mixed risk lovers have not received much attention until recently, see Crainich et al. (2013) and Ebert (2013). In a laboratory experiment, Deck and Schlesinger (2014) provide evidence that the behavior of subjects classified as risk-averse is indeed consistent with mixed risk aversion while the behavior of subjects classified as risk-loving is consistent with mixed risk loving. Haering et al. (2020) confirm this dichotomy in different countries and with high stakes, and show that it is strengthened when lotteries are displayed in compound form instead of reduced form.[5]

In the bivariate context, DMs have an apportionment preference pertaining to each attribute individually and also an apportionment preference across attributes. Imagine a DM who always prefers to disaggregate harms. We label this preference as dd-d, where the first letter refers to the preference on the first attribute, the second letter to the preference on the second attribute, and the third letter to the preference across attributes. The example of wealth and health illustrates that a focus on dd-d is too narrow. A common combination of assumptions is risk aversion over wealth, risk aversion over health, and correlation loving over wealth and health. In our notation, this corresponds to dd-a because a DM with such a preference prefers to disaggregate harms pertaining to either wealth or either health but combines good with good and bad with bad when it comes to a sure reduction in wealth and a sure reduction in health. Following this reasoning, there is a total of eight possible combinations, dd-d, dd-a, da-d, da-a, ad-d, ad-a, aa-d and aa-a. We take these combinations as the primitive in our paper. We hope that our research will stimulate future empirical studies to investigate the relative prevalence of these preferences in the data.

[4] Of course, a DM's preference may not follow either of the two patterns, or one of the patterns in some cases and the other one in other cases. In this paper, we focus on preferences that are consistent.

[5] Bleichrodt and van Bruggen (2021) find a reflection effect for higher-order risk preferences similar to the reflection effect identified by Kahneman and Tversky (1979). Behavior in their experiment is, in general, not consistent with a preference for combining good with bad or good with good and bad with bad.

2.3 Relation to utility of univariate apportionment preferences

Before we consider multivariate risks, we recollect the relation to utility of attribute-specific risk apportionment preferences. To interpret the signs of successive derivatives of the utility function in terms of higher-order risk attitudes, we recall Ekern's (1980) definition of risk increases. Let $K \in \mathbb{N}$ be a whole number and let W_1 and W_2 be two random variables with values in $[0, \overline{w}]$. Denote by $F_1^{(1)}$ and $F_2^{(1)}$ their respective cumulative distribution functions. For $k \in \mathbb{N}$, define the functions $F_1^{(k)}$ on $[0, \overline{w}]$ recursively by setting $F_1^{(k+1)}(w) = \int_0^w F_1^{(k)}(t)\,\mathrm{d}t$ for $w \in [0, \overline{w}]$, and likewise for W_2. We state the following definition.

Definition 1 (Ekern,1980)**.** W_2 has more Kth-degree risk than W_1 if:

(i) $F_1^{(k)}(\overline{w}) = F_2^{(k)}(\overline{w})$ for all $k = 1, \ldots, K$,

(ii) $F_1^{(K)}(w) \leq F_2^{(K)}(w)$ for all $w \in [0, \overline{w}]$.

Condition (i) ensures that W_1 and W_2 have the same first $(K-1)$ moments. Condition (ii) implies that the Kth moment is larger for W_2 than for W_1 when sign adjusted by $(-1)^K$. Well-known special cases include first-order stochastic dominance for $K = 1$, a mean-preserving increase in risk for $K = 2$ (see Rothschild and Stiglitz, 1970), a mean-variance-preserving increase in downside risk for $K = 3$ (see Menezes et al., 1980), and a mean-variance-skewness-preserving increase in outer risk for $K = 4$ (see Menezes and Wang, 2005).

Under expected utility a unambiguous preference over Kth-degree risk increases pins down the sign of the Kth derivative of the utility function. We formalize this in the next result.

Lemma 1. *Let $q : [0, \overline{w}] \to \mathbb{R}$ be a real-valued function that is K times continuously differentiable. The following two conditions are equivalent.*

(i) For all pairs (W_1, W_2) such that W_2 has more Kth-degree risk than W_1, we have $\mathbb{E}q(W_1) \geq \mathbb{E}q(W_2)$.

(ii) For all $w \in [0, \overline{w}]$, we have $(-1)^{K+1} q^{(K)}(w) \geq 0$.

Ekern (1980) showed that (ii) implies (i). Following the argument in Denuit et al. (1999), Jouini et al. (2013) also prove the reverse implication. Ekern (1980) calls DMs who dislike any increase in Kth-degree risk Kth-degree risk-averse. Analogously, we call DMs who like any increase in Kth-degree risk Kth-degree risk-loving. When preferences have an expected-utility representation with a smooth utility function, we can connect the DM's apportionment preference on the individual attributes to the notion of Kth-degree risk attitudes.

Let $u(x, y)$ represent the DM's preferences and consider the **DD** case, $u^{(1,0)} \geq 0$ and $u^{(0,1)} \geq 0$. As shown in Eeckhoudt and Schlesinger's (2006) main theorem, if the DM prefers to combine good with bad on x, then $(-1)^{M+1} u^{(M,0)} \geq 0$ for all $M \geq 1$. She is then Mth-degree risk-averse on the first attribute at all orders M. This holds for *dd-d*, *dd-a*, *da-d* and *da-a* DMs. If the DM prefers to combine good with good and bad with bad on x instead, then

$u^{(M,0)} \geq 0$ for all $M \geq 1$, see Deck and Schlesinger (2014). She is then Mth-degree risk-averse on the first attribute for all M that are odd, and Mth-degree risk-loving on the first attribute for all M that are even. This holds for *ad-d*, *ad-a*, *aa-d* and *aa-a* DMs. The same applies, *mutatis mutandis*, to the DM's apportionment preference over y.

Let us now move to the **DU** case, $u^{(1,0)} \geq 0$ and $u^{(0,1)} \leq 0$. The signs of the unidirectional derivatives of u regarding the first attribute are unaffected when going from **DD** to **DU**. For the second attribute, we follow in Eeckhoudt and Schlesinger's (2006) and Deck and Schlesinger's (2014) footsteps. If the DM prefers to combine good with bad on y, we now find $u^{(0,N)} \leq 0$ for all $N \geq 1$. She is then Nth-degree risk-loving on the second attribute when N is odd and Nth-degree risk-averse on the second attribute when N is even. This holds for *dd-d*, *dd-a*, *ad-d* and *ad-a* DMs. If the DM prefers to combine good with good and bad with bad on y instead, we have $(-1)^{N+1}u^{(0,N)} \leq 0$ for all $N \geq 1$. She is always Nth-degree risk-loving on the second attribute. This holds for *da-d*, *da-a*, *aa-d* and *aa-a* DMs.

The signs of $u^{(0,N)}$ coincide with Ebert's (2020) results who studied the first four orders in the context of discounting. We show in Appendix A.1 how to obtain all signs via Eeckhoudt et al.'s (2009) approach of apportioning risk increases. As in the **DD** case, DMs agree on odd-order risk preferences but disagree on even-order risk preferences. However, odd-order risk increases on y switch from being favorable to unfavorable and from unfavorable to favorable when going from the **DD** case to the **DU** case. The reason is that lower values of y are preferred over higher ones when y is undesirable. When combined with the DM's apportionment preference, this affects her higher-order risk attitude at all odd orders.[6]

3 Lottery preference and relation to utility: Correlation aversion, cross-prudence and cross-temperance

3.1 Two desirable attributes (case DD)

We begin with the **DD** case so that both x and y are desirable. For each attribute, an unfavorable change is then a reduction or a sure loss. We have seen in Section 2.3 that a DM is averse to mean-preserving spreads on a particular attribute if her apportionment preference on that attribute is combining good with bad. If it is combining good with good and bad with bad instead, the DM likes mean-preserving spreads on that attribute. Zero-mean risks can thus be unfavorable or favorable changes compared to the status quo depending on the DM's apportionment preference.

We now characterize the DM's apportionment preference across attributes with the help of simple lotteries as in Eeckhoudt et al. (2007). For positive constants $k > 0$ and $\ell > 0$,

[6] Menegatti and Peter (2021) observe a similar reversal when comparing the comparative statics of a risky benefit with that of a risky cost. Courbage and Rey (2016) notice a reversal as well when looking at the effect of changes in risky health losses on decision thresholds for preventive treatment.

a DM is called *correlation averse* if she prefers the lottery $[(x-k,y);(x,y-\ell)]$ over the lottery $[(x-k,y-\ell);(x,y)]$ for all $(x,y)\in\mathcal{D}$ such that $x-k\geq 0$ and $y-\ell\geq 0$, and *correlation loving* if she always has the reverse preference. When both attributes are desirable, correlation aversion is consistent with a preference to disaggregate harms across attributes whereas correlation loving represents a desire to aggregate harms across attributes. In terms of our risk apportionment taxonomy, *dd-d*, *da-d*, *ad-d* and *aa-d* DMs are correlation averters whereas *dd-a*, *da-a*, *ad-a* and *aa-a* DMs are correlation lovers. The apportionment preference on the individual attributes plays no role for this classification.

Let $\widetilde{\varepsilon}$ be an arbitrary zero-mean risk on x. Eeckhoudt et al. (2007) call a DM *cross-prudent in y* if she prefers the lottery $[(x+\widetilde{\varepsilon},y);(x,y-\ell)]$ over the lottery $[(x+\widetilde{\varepsilon},y-\ell);(x,y)]$ for all $(x,y)\in\mathcal{D}$ such that $\text{Supp}[x+\widetilde{\varepsilon}]\subseteq[0,\overline{x}]$ and $y-\ell\geq 0$, and *cross-imprudent in y* if she always has the reverse lottery preference.[7] There are now two ways to interpret this lottery preference. If the DM prefers to disaggregate harms on x, then the zero-mean risk $\widetilde{\varepsilon}$ is a harm relative to zero and cross-prudence in y represents a preference to disaggregate harms across attributes. This is Eeckhoudt et al.'s (2007) interpretation. If, however, the DM prefers to aggregate harms on x, then $\widetilde{\varepsilon}$ is preferred over zero and cross-prudence in y is a preference to aggregate harms across attributes. So *dd-d*, *da-d*, *ad-a* and *aa-a* DMs are cross-prudent in y whereas *dd-a*, *da-a*, *ad-d* and *aa-d* DMs are cross-imprudent in y. What matters is whether the apportionment preference on x is aligned with the apportionment preference across attributes or not. The apportionment preference on the second attribute y is irrelevant because a sure loss of ℓ on the second attribute is always unfavorable in the **DD** case.

Now let $\widetilde{\delta}$ be an arbitrary zero-mean risk on y. Eeckhoudt et al. (2007) call a DM *cross-prudent in x* if she prefers the lottery $[(x,y+\widetilde{\delta});(x-k,y)]$ over the lottery $[(x-k,y+\widetilde{\delta});(x,y)]$ for all $(x,y)\in\mathcal{D}$ such that $\text{Supp}[y+\widetilde{\delta}]\subseteq[0,\overline{y}]$ and $x-k\geq 0$, and *cross-imprudent in x* if she always has the reverse lottery preference. There are again two ways to interpret this lottery preference. The first one by Eeckhoudt et al. (2007) relies on the zero-mean risk $\widetilde{\delta}$ being a harm relative to zero. The second one is for the case that $\widetilde{\delta}$ is preferred over zero. For cross-prudence in x, what matters is the apportionment preference on y relative to the apportionment preference across attributes. If they are aligned, we obtain cross-prudence in x, which is the case for *dd-d*, *da-a*, *ad-d* and *aa-a* DMs. If they are not aligned, we obtain cross-imprudence in x, which is the case for *dd-a*, *da-d*, *ad-a* and *aa-d* DMs. For cross-prudence in x, the apportionment preference on x does not play a role because a sure loss of k on the first attribute is always bad in the **DD** case.

Let $\widetilde{\varepsilon}$ be an arbitrary zero-mean risk on x, let $\widetilde{\delta}$ be an arbitrary zero-mean risk on y, and let $\widetilde{\varepsilon}$ and $\widetilde{\delta}$ be independent. Eeckhoudt et al. (2007) call a DM *cross-temperate* if she prefers the lottery $[(x+\widetilde{\varepsilon},y);(x,y+\widetilde{\delta})]$ over the lottery $[(x+\widetilde{\varepsilon},y+\widetilde{\delta});(x,y)]$ for all $(x,y)\in\mathcal{D}$ such

[7] We adopt Eeckhoudt et al.'s (2007) notation and let $\text{Supp}[x+\widetilde{\varepsilon}]$ denote the support of the probability distribution function associated with the random variable $x+\widetilde{\varepsilon}$. We assume that the realizations of $\widetilde{\varepsilon}$ are between $-x$ and $\overline{x}-x$ almost surely to remain in the domain of the first attribute.

that $\mathrm{Supp}[x+\widetilde{\varepsilon}] \subseteq [0,\overline{x}]$ and $\mathrm{Supp}[y+\widetilde{\delta}] \subseteq [0,\overline{y}]$, and *cross-intemperate* if she always has the reverse lottery preference. The interpretation of this lottery preference now depends on all three apportionment preferences. If the DM prefers to disaggregate harms on x and y individually, the lottery preference is consistent with a desire to disaggregate harms across attributes. This is Eeckhoudt et al.'s (2007) interpretation. However, this lottery preference is also consistent with disaggregating harms across attributes when the DM prefers to aggregate harms on x and y individually. The difference is that the harm is now not to receive the zero-mean risk. Let the individual apportionment preferences on x and y not be aligned, and assume the DM prefers to disaggregate harms on x but prefers to aggregate harms on y. Then, $\widetilde{\varepsilon}$ is a harm relative to zero but $\widetilde{\delta}$ is preferred over zero. The lottery preference for cross-temperance can now be understood as a preference to aggregate harms across attributes. In summary, when the individual apportionment preferences are aligned, cross-temperance represents a desire to disaggregate harms across attributes. When, however, the individual apportionment preferences are not aligned, it represents a desire to aggregate harms across attributes. So *dd-d*, *da-a*, *ad-a* and *aa-d* DMs are cross-temperate whereas *dd-a*, *da-d*, *ad-d* and *aa-a* DMs are cross-intemperate.

When the DM's preferences can be represented with a bivariate utility function $u(x,y)$, Eeckhoudt et al. (2007) show that correlation aversion, cross-prudence in x and y, and cross-temperance can be characterized via the sign of specific cross-derivatives of the utility function. We use $u^{(M,N)}(x,y)$ to denote $\partial^{M+N}u(x,y)/\partial^M x \partial^N y$ for $M \geq 0$ and $N \geq 0$ with $M+N \geq 1$. In the **DD** case, we have $u^{(1,0)} \geq 0$ and $u^{(0,1)} \geq 0$ because both x and y are desirable. Based on the above discussion and with Eeckhoudt et al.'s (2007) Proposition 1, we can then sign specific cross-derivatives of the utility function for the various underlying apportionment preferences. We summarize our findings in the following proposition.

Proposition 1. *Consider the case of two desirable attributes (case **DD**).*

(i) DMs with apportionment preferences dd-d, da-d, ad-d or aa-d have $u^{(1,1)} \leq 0$ *(correlation aversion), DMs with apportionment preferences dd-a, da-a, ad-a or aa-a have* $u^{(1,1)} \geq 0$ *(correlation loving).*

(ii) DMs with apportionment preferences dd-d, da-d, ad-a or aa-a have $u^{(2,1)} \geq 0$ *(cross-prudence in y), DMs with apportionment preferences dd-a, da-a, ad-d or aa-d have* $u^{(2,1)} \leq 0$ *(cross-imprudence in y).*

(iii) DMs with apportionment preferences dd-d, da-a, ad-d or aa-a have $u^{(1,2)} \geq 0$ *(cross-prudence in x), DMs with apportionment preferences dd-a, da-d, ad-a or aa-d have* $u^{(1,2)} \leq 0$ *(cross-imprudence in x).*

(iv) DMs with apportionment preferences dd-d, da-a, ad-a or aa-d have $u^{(2,2)} \leq 0$ *(cross-temperance), DMs with apportionment preferences dd-a, da-d, ad-d or aa-a have* $u^{(2,2)} \geq 0$ *(cross-intemperance).*

Table 1 in the appendix collects these signs and organizes them according to the DM's apportionment preference. Only DMs who prefer to disaggregate harms, on the individual attributes as well as across attributes, are correlation averse, cross-prudent in x and y, and cross-temperate. As soon as one of the apportionment preferences changes, at least some of the signs flip. For example, we obtain correlation loving, cross-imprudence in x and y, and cross-intemperance when the DM prefers to combine good with bad on the individual attributes but prefers to aggregate harms across attributes.

3.2 One desirable and one undesirable attribute (case DU)

We now turn to the **DU** case so that x is desirable and y is undesirable. A sure reduction is still an unfavorable change when applied to the first attribute but it is now a favorable change when applied to the second attribute. In fact, a sure *increase* is now an unfavorable change of the second attribute. As explained in Section 2.3, if the DM prefers to combine good with bad on the second attribute, she is averse to mean-preserving spreads on the second attribute and zero-mean risks are unfavorable compared to the status-quo. Conversely, if the DM prefers to combine good with good and bad with bad on y, she likes mean-preserving spreads on y and zero-mean risks are favorable changes relative to the satus-quo.

Taking into account that y is undesirable, we now define a DM to be *correlation averse* if she prefers the lottery $[(x-k, y);(x, y+\ell)]$ over the lottery $[(x-k, y+\ell);(x, y)]$ for all $(x, y) \in \mathcal{D}$ such that $x-k \geq 0$ and $y+\ell \leq \overline{y}$, and *correlation loving* if she always has the reverse preference. As in the **DD** case, correlation aversion represents a preference to disaggregate harms across attributes wheres correlation loving represents a desire to aggregate harms across attributes. What has changed is the definition of a harm on the second attribute because y is now undesirable. In terms of our classification *dd-d*, *da-d*, *ad-d* and *aa-d* DMs are correlation averters whereas *dd-a*, *da-a*, *ad-a* and *aa-a* DMs are correlation lovers.

Let $\widetilde{\varepsilon}$ be a zero-mean risk on the first attribute and let ℓ be a sure increase of the second attribute. We now call a DM *cross-prudent in y* if she prefers the lottery $[(x+\widetilde{\varepsilon}, y);(x, y+\ell)]$ over the lottery $[(x+\widetilde{\varepsilon}, y+\ell);(x, y)]$ for all $(x, y) \in \mathcal{D}$ such that $\operatorname{Supp}[x+\widetilde{\varepsilon}] \subseteq [0, \overline{x}]$ and $y+\ell \leq \overline{y}$, and *cross-imprudent in y* if she always has the reverse lottery preference. We can interpret this lottery preference in two ways. If the DM prefers to disaggregate harms on x, the zero-mean risk is bad relative to zero and cross-prudence in y represents a preference to disaggregate harms across attributes. If the DM prefers to combine good with good and bad with bad on x instead, the zero-mean risk is preferred over zero and cross-prudence in y is consistent with a preference to aggregate harms across attributes. So *dd-d*, *da-d*, *ad-a* and *aa-a* DMs are cross-prudent in y whereas *dd-a*, *da-a*, *ad-d* and *aa-d* DMs are cross-imprudent in y. As in the **DD** case, the alignment between the apportionment preference on x and the apportionment preference across attributes matters. The apportionment preference on y does not matter for cross-prudence in y because a sure increase of ℓ is always unfavorable.

Now let $\widetilde{\delta}$ be a zero-mean risk on y. We call a DM *cross-prudent in x* if she prefers the lottery $[(x-k,y);(x,y+\widetilde{\delta})]$ over the lottery $[(x-k,y+\widetilde{\delta});(x,y)]$ for all $(x,y)\in\mathcal{D}$ such that $\mathrm{Supp}[y+\widetilde{\delta}]\subseteq[0,\overline{y}]$ and $x-k\geq 0$, and *cross-imprudent in x* if she always has the reverse lottery preference. It is now the apportionment preference on y and the apportionment preference across attributes that matter. If they are aligned, we find cross-prudence in x, if they are not, we obtain cross-imprudence in x. So *dd-d*, *da-a*, *ad-d* and *aa-a* DMs are cross-prudent in x whereas *dd-a*, *da-d*, *ad-a* and *aa-d* DMs are cross-imprudent in x. The apportionment preference on x does not play a role because a sure loss of k is always unfavorable.

Let $\widetilde{\varepsilon}$ and $\widetilde{\delta}$ be two independent zero-mean risks on x and y. We call a DM *cross-temperate* if she prefers the lottery $[(x+\widetilde{\varepsilon},y);(x,y+\widetilde{\delta})]$ over $[(x+\widetilde{\varepsilon},y+\widetilde{\delta});(x,y)]$ for all $(x,y)\in\mathcal{D}$ such that $\mathrm{Supp}[x+\widetilde{\varepsilon}]\subseteq[0,\overline{x}]$ and $\mathrm{Supp}[y+\widetilde{\delta}]\subseteq[0,\overline{y}]$, and *cross-intemperate* if she always has the reverse lottery preference. If the DM prefers to combine good with bad on x and y individually, this lottery preference is consistent with combining good with bad across attributes. The same holds if the DM prefers to combine good with good and bad with bad on x and y individually. The stated lottery preference is to rather face the undesirable changes in separate states (i.e., not getting the zero-mean risks) instead of taking the chance to face them together. Let the apportionment preferences on x and y not be aligned, and say the DM prefers to disaggregate harms on x but prefers to aggregate harms on y. In this case, $\widetilde{\varepsilon}$ is a harm relative to zero but $\widetilde{\delta}$ is preferred over zero. The lottery preference for cross-temperance can now be interpreted as a preference to aggregate harms across attributes. In summary, *dd-d*, *da-a*, *ad-a* and *aa-d* DMs are cross-temperate whereas *dd-a*, *da-d*, *ad-d* and *aa-a* DMs are cross-intemperate.

Our classification in the **DU** case is thus identical to the one in the **DD** case. We achieved this by adjusting the defining lottery preferences. When we represent preferences with a bivariate utility function, this affects some of the signs of the cross-derivatives as follows.

Proposition 2. *Consider the case in which the first attribute is desirable and the second attribute is undesirable (case **DU**).*

(i) DMs with apportionment preferences dd-d, da-d, ad-d or aa-d have $u^{(1,1)}\geq 0$ (correlation aversion), DMs with apportionment preferences dd-a, da-a, ad-a or aa-a have $u^{(1,1)}\leq 0$ (correlation loving).

(ii) DMs with apportionment preferences dd-d, da-d, ad-a or aa-a have $u^{(2,1)}\leq 0$ (cross-prudence in y), DMs with apportionment preferences dd-a, da-a, ad-d or aa-d have $u^{(2,1)}\geq 0$ (cross-imprudence in y).

(iii) DMs with apportionment preferences dd-d, da-a, ad-d or aa-a have $u^{(1,2)}\geq 0$ (cross-prudence in x), DMs with apportionment preferences dd-a, da-d, ad-a or aa-d have $u^{(1,2)}\leq 0$ (cross-imprudence in x).

(iv) DMs with apportionment preferences dd-d, da-a, ad-a or aa-d have $u^{(2,2)}\leq 0$ (cross-temperance), DMs with apportionment preferences dd-a, da-d, ad-d or aa-a have $u^{(2,2)}\geq 0$ (cross-intemperance).

Table 2 in the appendix organizes these signs by the DM's apportionment preferences. In Proposition 2(i), the signs for correlation aversion and correlation loving are flipped compared to Proposition 1(i). A lottery preference of $[(x-k,y);(x,y+\ell)]$ over $[(x-k,y+\ell);(x,y)]$ for all $(x,y)\in\mathcal{D}$ such that $x-k\geq 0$ and $y+\ell\leq\overline{y}$ can be equivalently stated as a lottery preference of $[(x-k,y'-\ell);(x,y')]$ over $[(x-k,y');(x,y'-\ell)]$ for all $(x,y')\in\mathcal{D}$ such that $x-k\geq 0$ and $y'-\ell\geq 0$. This is a simple change of variables by setting $y'=y+\ell$. But then we know from Eeckhoudt et al. (2007) that this lottery preference is equivalent to $u^{(1,1)}\geq 0$. Likewise, the signs for cross-prudence in y and cross-imprudence in y are flipped in Proposition 2(ii) compared to Proposition 1(ii). The same change of variables shows that a lottery preference of $[(x+\widetilde{\varepsilon},y);(x,y+\ell)]$ over $[(x+\widetilde{\varepsilon},y+\ell);(x,y)]$ for all $(x,y)\in\mathcal{D}$ such that $\text{Supp}[x+\widetilde{\varepsilon}]\subseteq[0,\overline{x}]$ and $y+\ell\leq\overline{y}$, is equivalent to a lottery preference of $[(x+\widetilde{\varepsilon},y'-\ell);(x,y')]$ over $[(x+\widetilde{\varepsilon},y');(x,y'-\ell)]$ for all $(x,y')\in\mathcal{D}$ such that $\text{Supp}[x+\widetilde{\varepsilon}]\subseteq[0,\overline{x}]$ and $y-\ell\geq 0$. Per Eeckhoudt et al. (2007), this is equivalent to $u^{(2,1)}\leq 0$. The signs for cross-prudence in x and cross-temperance are the same in Proposition 2 as in Proposition 1.

Our discussion leading up to Proposition 2 shows that, conceptually, nothing has changed when we adjust the characterizing lottery preferences accordingly. When we move to the utility representation, the signs of some cross-derivatives are flipped whereas others remain unchanged. Take correlation aversion as an example. In the **DD** case, both attributes are desirable. It is "riskier" to face a situation in which either both attributes are high or both are low at the same time instead of a situation in which low values of one attribute are compensated by high values of the other one. Intuitively, correlation averters should avoid positive correlation and seek negative correlation to hedge their bets. Now consider the **DU** case with one attribute being desirable and the other one undesirable. It is now better to face a situation in which either both attributes are high or both are low than a situation with one low and the other one high. When both are high, high values of the undesirable attribute are compensated by high values of the desirable attribute. When both are low, low values of the desirable attribute are compensated by low values of the undesirable attribute. Correlation averters now achieve hedging by avoiding negative correlation and seeking positive correlation.

3.3 Two undesirable attributes (case UU)

In a next step, we look at the **UU** case in which both x and y are undesirable. Then, a sure reduction of either attribute is a favorable change for the DM whereas a sure increase in either attribute is an unfavorable change. As shown in Section 2.3, a preference to combine good with bad on x implies that the introduction of a zero-mean risk on x is an unfavorable change compared to the status-quo while it is a favorable change if the DM prefers to combine good with good and bad with bad on x. The same holds for attribute y.

When both attributes are undesirable, we define a DM to be *correlation averse* if she prefers the lottery $[(x+k,y);(x,y+\ell)]$ over the lottery $[(x+k,y+\ell),(x,y)]$ for all $(x,y)\in\mathcal{D}$ such that $x+k\leq\overline{x}$ and $y+\ell\leq\overline{y}$, and *correlation loving* if she always has the reverse preference. Yet again correlation aversion represents a preference to disaggregate harms across attributes and correlation loving is consistent with aggregating harms across attributes. A harm is now a sure increase for either attribute. Only the apportionment preference across attributes matters so that *dd-d*, *da-d*, *ad-d* and *aa-d* DMs are correlation averters whereas *dd-a*, *da-a*, *ad-a* and *aa-a* DMs are correlation lovers. The apportionment preference on the individual attributes is irrelevant at this stage.

Let $\widetilde{\varepsilon}$ be a zero-mean risk on the first attribute and let ℓ be a sure increase of the second attribute. We call a DM *cross-prudent in y* if she prefers the lottery $[(x+\widetilde{\varepsilon}, y); (x, y+\ell)]$ over the lottery $[(x+\widetilde{\varepsilon}, y+\ell); (x, y)]$ for all $(x, y) \in \mathcal{D}$ such that $\mathrm{Supp}[x+\widetilde{\varepsilon}] \subseteq [0, \overline{x}]$ and $y + \ell \leq \overline{y}$, and *cross-imprudent in y* if she always has the reverse lottery preference. This lottery preference has two interpretations. If the DM prefers to disaggregate harms on x, the zero-mean risk is a harm relative to zero and cross-prudence in y is consistent with harms disaggregation across attributes. If the DM prefers to aggregate harms on x instead, the zero-mean risk on x is preferred over zero and cross-prudence in y represents a desire to aggregate harms across attributes. So *dd-d*, *da-d*, *ad-a* and *aa-a* DMs are cross-prudent in y whereas *dd-a*, *da-a*, *ad-d* and *aa-d* DMs are cross-imprudent in y. As before, the alignment between the apportionment preference on x and the apportionment preference across attributes matters while the apportionment preference on y plays no role.

Let $\widetilde{\delta}$ be a zero-mean risk on the second attribute and k be a sure increase of the first attribute. We call a DM *cross-prudent in x* if she prefers the lottery $[(x+k, y); (x, y+\widetilde{\delta})]$ over the lottery $[(x+k, y+\widetilde{\delta}); (x, y)]$ for all $(x, y) \in \mathcal{D}$ such that $x + k \leq \overline{x}$ and $\mathrm{Supp}[y+\widetilde{\delta}] \subseteq [0, \overline{y}]$, and *cross-imprudent in x* if she always has the reverse lottery preference. What matters is the alignment between the apportionment preference on y and the apportionment preference across attributes. We have cross-prudence in x when both are aligned and cross-imprudence in x when they are not. As a result, *dd-d*, *da-a*, *ad-d* and *aa-a* DMs are cross-prudent in x whereas *dd-a*, *da-d*, *ad-a* and *aa-d* DMs are cross-imprudent in x. The apportionment preference on x does not matter.

Consider two independent zero-mean risks, $\widetilde{\varepsilon}$ and $\widetilde{\delta}$, one on the first attribute x and the other one on the second attribute y. We call a DM *cross-temperate* if she prefers the lottery $[(x+\widetilde{\varepsilon}, y); (x, y+\widetilde{\delta})]$ over the lottery $[(x+\widetilde{\varepsilon}, y+\widetilde{\delta}); (x, y)]$ for all $(x, y) \in \mathcal{D}$ such that $\mathrm{Supp}[x+\widetilde{\varepsilon}] \subseteq [0, \overline{x}]$ and $\mathrm{Supp}[y+\widetilde{\delta}] \subseteq [0, \overline{y}]$, and *cross-intemperate* if she always has the reverse lottery preference. If the DM prefers to disaggregate harms on x and y individually, or if she prefers to aggregate harms on x and y individually, the lottery preference represents combining good with bad across attributes. If the DM's apportionment preferences on the individual attributes are not aligned, the lottery preference for cross-temperance is consistent with combining good with good and bad with bad across attributes. So *dd-d*, *da-a*, *ad-a* and *aa-d* DMs are cross-temperate whereas *dd-a*, *da-d*, *ad-d* and *aa-a* DMs are cross-intemperate.

The classification in the **UU** case is identical to the classification in the other two cases. We achieve this by adjusting the defining lottery preferences, specifically in those cases where a harm is now a sure increase of the attribute, not a sure reduction. In terms of the utility representation, the signs of some cross-derivatives are affected by these adjustments as follows.

Proposition 3. *Consider the case of two undesirable attributes (case **UU**).*

(i) DMs with apportionment preferences dd-d, da-d, ad-d or aa-d have $u^{(1,1)} \leq 0$ (correlation aversion), DMs with apportionment preferences dd-a, da-a, ad-a or aa-a have $u^{(1,1)} \geq 0$ (correlation loving).

(ii) DMs with apportionment preferences dd-d, da-d, ad-a or aa-a have $u^{(2,1)} \leq 0$ (cross-prudence in y), DMs with apportionment preferences dd-a, da-a, ad-d or aa-d have $u^{(2,1)} \geq 0$ (cross-imprudence in y).

(iii) DMs with apportionment preferences dd-d, da-a, ad-d or aa-a have $u^{(1,2)} \leq 0$ (cross-prudence in x), DMs with apportionment preferences dd-a, da-d, ad-a or aa-d have $u^{(1,2)} \geq 0$ (cross-imprudence in x).

(iv) DMs with apportionment preferences dd-d, da-a, ad-a or aa-d have $u^{(2,2)} \leq 0$ (cross-temperance), DMs with apportionment preferences dd-a, da-d, ad-d or aa-a have $u^{(2,2)} \geq 0$ (cross-intemperance).

Table 3 in the appendix organizes these signs according to the DM's apportionment preference. In Proposition 3(i), the signs for correlation aversion and correlation loving are flipped compared to Proposition 2(i) and are thus identical to Proposition 1(i). A preference of $[(x+k,y);(x,y+\ell)]$ over $[(x+k,y+\ell),(x,y)]$ for all $(x,y) \in \mathcal{D}$ such that $x+k \leq \overline{x}$ and $y+\ell \leq \overline{y}$ is equivalent to a preference of $[(x',y'-\ell);(x'-k,y')]$ over $[(x',y'),(x'-k,y'-\ell)]$ for all $(x',y') \in \mathcal{D}$ such that $x'-k \geq 0$ and $y'-\ell \geq 0$. Mathematically, this is a simple change of variables by letting $x' = x+k$ and $y' = y+\ell$. It renders the exact same characterizing lottery preference as in the **DD** case, which is why we find $u^{(1,1)} \leq 0$ for correlation aversion. The lotteries for cross-prudence in y are identical in the **DU** and **UU** cases, the lotteries for cross-prudence in x are different because the first attribute is desirable in the **DU** case but undesirable in the **UU** case. The cross-temperance lotteries are identical in all three cases.

Conceptually, we obtain the same classification in terms of the DM's underlying apportionment preference but only because we adjusted some of the characterizing lotteries. Consider correlation aversion again. In the **DD** and the **UU** case, correlation aversion obtains for $u^{(1,1)} \leq 0$. The underlying economic intuition is different. In both cases correlation averters hedge their bets by avoiding positive correlation and seeking negative correlation. In the **DD** case, positive correlation is unappealing because low values of one attribute tend to occur with low values of the other attribute. Negative correlation insulates the DM against this because low values of one attribute tend to be compensated by high values of the other one. For **UU**, positive correlation is unappealing because high values of one attribute tend to occur with high values of the other attribute. Negative correlation now helps because high values of one attribute tend to be compensated by low values of the other one. Even though a negative sign on $u^{(1,1)}$ characterizes correlation aversion in both cases, the roles of "high" and "'low" for the attribute values are reversed due to their different effects on the DM's welfare. While this is obvious for correlation attitude, the extension to higher orders is not immediate. The simple example of correlation attitude also shows that signing cross-derivatives and deriving economic intuition are two separate steps.

4 The general theory

4.1 Univariate risk apportionment

Eeckhoudt and Schlesinger (2006) define risk apportionment of any order via a specific lottery preference. Take the first attribute x and assume it is desirable. Let $\{\widetilde{\varepsilon}_i\}$ be an indexed set of zero-mean nondegenerate random variables, $i = 1, 2, 3, \ldots$, that are all mutually independent, and let k be a positive constant. Define $A_1 = [-k]$, $A_2 = [\widetilde{\varepsilon}_1]$, and $B_1 = B_2 = [0]$. Let $\text{Int}(z)$ denote the greatest-integer function. For $M \geq 3$, define the univariate lotteries

$$
\begin{aligned}
A_M &= [B_{M-2} + 0;\ A_{M-2} + \widetilde{\varepsilon}_{\text{Int}(M/2)}], \\
B_M &= [A_{M-2} + 0;\ B_{M-2} + \widetilde{\varepsilon}_{\text{Int}(M/2)}].
\end{aligned}
$$

A DM then prefers to combine good with bad on the first attribute if she prefers the lottery $[(x+B_M, y)]$ over the lottery $[(x+A_M, y)]$ for all $(x, y) \in \mathcal{D}$ and such that $\text{Supp}[x+A_M] \subseteq [0, \overline{x}]$ and $\text{Supp}[x+B_M] \subseteq [0, \overline{x}]$. She prefers combining good with good and bad with bad on the first attribute if she always has the reverse lottery preference. This is Eeckhoudt and Schlesinger's (2006) Definition 5 of risk apportionment of order M applied to the first attribute.[8]

Now assume that the first attribute is undesirable. When comparing A_1 and B_1, we now see that A_1 is preferred over B_1 because the DM appreciates a sure reduction of an undesirable attribute. To rectify this and maintain the iterative definition of higher-order risk preferences, all we need to do is to replace $A_1 = [-k]$ with $A_1 = [+k]$. For an undesirable attribute, a sure increase is now a harm relative to $B_1 = [0]$.

We can then proceed in a similar way regarding the second attribute y. Assume first that y is desirable. Let $\{\widetilde{\delta}_j\}$ be an indexed set of zero-mean nondegenerate random variables, $j = 1, 2, 3, \ldots$, that are all mutually independent and also mutually independent of the $\{\widetilde{\varepsilon}_i\}$. Let ℓ be a positive constant. Define $C_1 = [-\ell]$, $C_2 = [\widetilde{\delta}_1]$, and $D_1 = D_2 = [0]$. For $N \geq 3$, define the univariate lotteries

$$
\begin{aligned}
C_N &= [D_{N-2} + 0;\ C_{N-2} + \widetilde{\delta}_{\text{Int}(N/2)}], \\
D_M &= [C_{N-2} + 0;\ D_{N-2} + \widetilde{\delta}_{\text{Int}(N/2)}].
\end{aligned}
$$

[8] For ease of exposition, we take some liberty with the notation. The distribution of the lottery $[(x + B_M, y)]$ is the one that is induced by the distribution of the lottery B_M. In other words, the lottery $[(x + B_M, y)]$ has the outcome $(x + b, y)$ with probability $\mathbb{P}(B_M = b)$ for all $b \in \text{Supp}[B_M]$.

A DM prefers to combine good with bad on the second attribute if she prefers the lottery $[(x, y+D_N)]$ over the lottery $[(x, y+C_N)]$ for all $(x, y) \in \mathcal{D}$ such that $\mathrm{Supp}[y+C_N] \subseteq [0, \overline{y}]$ and $\mathrm{Supp}[y+D_N] \subseteq [0, \overline{y}]$. She prefers combining good with good and bad with bad on the second attribute if she always has the reverse lottery preference. This is Eeckhoudt and Schlesinger's (2006) Definition 5 of risk apportionment of order N applied to the second attribute. If y is undesirable instead, we need to replace $C_1 = [-\ell]$ with $C_1 = [+\ell]$ to keep the iterative definition of higher-order risk preferences intact.

The main advantage of Eeckhoudt and Schlesinger's (2006) risk apportionment approach is its simplicity and elegance. Specifically, higher-order risk preferences can be defined purely based on a simple lottery preference and no particular representation of preferences is used. This is what some refer to as "model-free" even though, of course, preferences themselves are an economic model of choice under risk and reduction of compound lotteries is implicit in the risk apportionment literature.

4.2 Risk apportionment across attributes

Building on these univariate risk apportionment lotteries, we can now define risk apportionment across attributes. For $M, N \geq 1$, we say that preferences satisfy risk apportionment of order (M, N) if the DM prefers the lottery $[(x + B_M, y + C_N); (x + A_M, y + D_N)]$ over the lottery $[(x+B_M, y+D_N); (x+A_M, y+C_N)]$ for all $(x, y) \in \mathcal{D}$ such that $\mathrm{Supp}[x+A_M] \subseteq [0, \overline{x}]$, $\mathrm{Supp}[x + B_M] \subseteq [0, \overline{x}]$, $\mathrm{Supp}[y + C_N] \subseteq [0, \overline{y}]$ and $\mathrm{Supp}[y + D_N] \subseteq [0, \overline{y}]$. If the DM always has the reverse lottery preference, we say that preferences exhibit anti-risk apportionment of order (M, N). Eeckhoudt and Schlesinger (2006) introduce a terminology for preferences consistent with risk apportionment and Deck and Schlesinger (2014) use the qualifier "anti" for the reverse preference.[9]

We can easily connect this to the analysis of correlation aversion, cross-prudence in x and y, and cross-temperance in Section 2. Take $M = N = 1$ and consider A_1, B_1, C_1 and D_1. We have $B_1 = [0]$ and $D_1 = [0]$; furthermore, we have $A_1 = [-k]$ and $C_1 = [-\ell]$ in the **DD** case, $A_1 = [-k]$ and $C_1 = [+\ell]$ in the **DU** case, and $A_1 = [+k]$ and $C_1 = [+\ell]$ in the **UU** case. Therefore, risk apportionment of order $(1, 1)$ is characterized as follows:

$$\begin{cases} [(x, y-\ell); (x-k, y)] \succsim [(x, y); (x-k, y-\ell)], & \text{in case of } \mathbf{DD}, \\ [(x, y+\ell); (x-k, y)] \succsim [(x, y); (x-k, y+\ell)], & \text{in case of } \mathbf{DU}, \\ [(x, y+\ell); (x+k, y)] \succsim [(x, y); (x+k, y+\ell)], & \text{in case of } \mathbf{UU}. \end{cases}$$

These are the lottery preferences we used in Section 2 to characterize correlation aversion in each case, with the reverse preference characterizing correlation loving. Now take $M = 2$ and

[9] The distribution of the lottery $[(x + B_M, y + C_N)]$ is the one that is induced by the joint distribution of (B_M, C_M). Due to independence, the lottery has outcome $(x+b, y+c)$ with probability $\mathbb{P}(B_M = b)\mathbb{P}(C_N = c)$ for all $b \in \mathrm{Supp}[B_M]$ and all $c \in \mathrm{Supp}[C_N]$.

$N = 1$ and consider A_2, B_2, C_1 and D_1. We have $A_2 = [\widetilde{\varepsilon}_1]$, $B_2 = [0]$ and $D_1 = [0]$; when y is desirable, we have $C_1 = [-\ell]$, when y is undesirable, we have $C_1 = [+\ell]$. Risk apportionment of order $(2, 1)$ is then characterized via the following lotteries:

$$\begin{cases} [(x, y-\ell); (x+\widetilde{\varepsilon}_1, y)] \succsim [(x, y); (x+\widetilde{\varepsilon}_1, y-\ell)], & \text{in case of } \mathbf{DD}, \\ [(x, y+\ell); (x+\widetilde{\varepsilon}_1, y)] \succsim [(x, y); (x+\widetilde{\varepsilon}_1, y+\ell)], & \text{in case of } \mathbf{DU} \text{ or } \mathbf{UU}. \end{cases}$$

These are the lottery preferences we used in Section 2 to characterize cross-prudence in y, with the reverse preference characterizing cross-imprudence in y. For $M = 1$ and $N = 2$, consider A_1, B_1, C_2 and D_2. We have $B_1 = [0]$, $C_2 = [\widetilde{\delta}_1]$ and $D_2 = [0]$; when x is desirable, we have $A_1 = [-k]$, when x is undesirable, we have $A_1 = [+k]$. Risk apportionment of order $(1, 2)$ is then characterized via the following lotteries:

$$\begin{cases} [(x, y+\widetilde{\delta}_1); (x-k, y)] \succsim [(x, y); (x-k, y+\widetilde{\delta}_1)], & \text{in case of } \mathbf{DD} \text{ or } \mathbf{DU}, \\ [(x, y+\widetilde{\delta}_1); (x-k, y)] \succsim [(x, y); (x-k, y+\widetilde{\delta}_1)], & \text{in case of } \mathbf{DU}. \end{cases}$$

These are the lottery preferences we used in Section 2 to characterize cross-prudence in x, with the reverse preference characterizing cross-imprudence in x. Finally, for $M = 2$ and $N = 2$, consider A_2, B_2, C_2 and D_2, that is, $A_2 = [\widetilde{\varepsilon}_1]$, $B_2 = [0]$, $C_2 = [\widetilde{\delta}_1]$ and $D_2 = [0]$. The distinction between **DD**, **DU** and **UU** is now irrelevant. We can then always characterize risk apportionment of order $(2, 2)$ via the following lottery preference:

$$\begin{cases} [(x, y+\widetilde{\delta}_1); (x+\widetilde{\varepsilon}_1, y)] \succsim [(x, y); (x+\widetilde{\varepsilon}_1, y+\widetilde{\varepsilon}_1)], & \text{in case of } \mathbf{DD}, \mathbf{DU} \text{ or } \mathbf{UU}. \end{cases}$$

This is the lottery preference we used in Section 2 for cross-temperance, with the reverse preference characterizing cross-intemperance.

Correlation aversion corresponds to risk apportionment of order $(1, 1)$, correlation loving to anti-risk apportionment of order $(1, 1)$, cross-prudence in y to risk apportionment of order $(2, 1)$, cross-imprudence in y to anti-risk apportionment of order $(2, 1)$, cross-prudence in x to risk apportionment of order $(1, 2)$, cross-imprudence in x to anti-risk apportionment of order $(1, 2)$, cross-temperance to risk apportionment of order $(2, 2)$, and cross-intemperance to anti-risk apportionment of order $(2, 2)$. When an attribute flips from desirable to undesirable, all we need to do is swap out the seed lottery and the iterative process and associated taxonomy stays fully intact. Specifically, if attribute x is undesirable, we need to replace $A_1 = [-k]$ with $A_1 = [+k]$. If attribute y is undesirable, we need to replace $C_1 = [-\ell]$ with $C_1 = [+\ell]$.

The lottery preference of $[(x+B_M, y+C_N); (x+A_M, y+D_N)]$ over $[(x+B_M, y+D_N); (x+A_M, y+C_N)]$ extends the notions of correlation aversion, cross-prudence in x and y, and cross-temperance to higher orders. Eeckhoudt et al. (2007) mention in their Footnote 12 that such an extension is possible but do not carry it out. What's more, their analysis focuses exclusively on the **DD** case. We show that, by defining the seed lotteries A_1 and C_1 accordingly, the entire risk apportionment machinery can also be applied to the **DU** and the **UU** case. As in the univariate analysis, no particular representation of preferences is necessary to define risk apportionment and anti-risk apportionment of order (M, N). Characterizations of these lottery preferences can be explored outside the confines of the expected-utility model. If the expected utility theorem holds, the stated lottery preference can be characterized by signing the corresponding cross-derivative of the utility function. We will provide this characterization in the next section based on risk apportionment via stochastic dominance.

5 Relation to utility - The general case

5.1 Two desirable attributes (case DD)

An alternative to the risk apportionment lotteries in Eeckhoudt and Schlesinger (2006) is the apportionment of risks via stochastic dominance in Eeckhoudt et al. (2009). Consider the four mutually independent random variables X_1, X_2, Y_1 and Y_2. Let X_2 have more Mth-degree risk than X_1, and Y_2 have more Nth-degree risk than Y_1. In the spirit of Eeckhoudt et al. (2009), we can then assess the DM's preference over the lotteries $[(X_1, Y_2); (X_2, Y_1)]$ and $[(X_1, Y_1); (X_2, Y_2)]$. The first lottery combines low Mth-degree risk on x with high Nth-degree risk on y, and high Mth-degree risk on x with low Nth-degree risk on y. The second lottery combines low Mth-degree risk on x with low Nth-degree risk on y, and high Mth-degree risk on x with high Nth-degree risk on y. When the DM always prefers the first lottery over the second one, we obtain $(-1)^{M+N+1}u^{(M,N)} \geq 0$ from Lemma 1. If she always has the reverse lottery preference instead, we obtain $(-1)^{M+N+1}u^{(M,N)} \leq 0$ from Lemma 1. We show this formally in Appendix A.2.

While the lottery preference of $[(X_1, Y_2); (X_2, Y_1)]$ over $[(X_1, Y_1); (X_2, Y_2)]$ pins down the sign of $(-1)^{M+N+1}u^{(M,N)}$ unambiguously, the interpretation of this sign depends on the DM's apportionment preference on the individual attributes, her apportionment preference across attributes, as well as on the parity of the orders (i.e., whether M and N are odd or even). The following result organizes the signs by the DM's risk apportionment preference.

Theorem 1 (Case **DD**). *Consider the case of two desirable attributes and let $M, N \geq 1$.*

(i) DMs with apportionment preference dd-d have $(-1)^{M+N+1}u^{(M,N)} \geq 0$, DMs with apportionment preference dd-a have $(-1)^{M+N+1}u^{(M,N)} \leq 0$.

(ii) DMs with apportionment preference da-d have $(-1)^{M}u^{(M,N)} \geq 0$, DMs with apportionment preference da-a have $(-1)^{M}u^{(M,N)} \leq 0$.

(iii) DMs with apportionment preference ad-d have $(-1)^{N}u^{(M,N)} \geq 0$, DMs with apportionment preference ad-a have $(-1)^{N}u^{(M,N)} \leq 0$.

(iv) DMs with apportionment preference aa-d have $u^{(M,N)} \leq 0$, DMs with apportionment preference aa-a have $u^{(M,N)} \geq 0$.

Appendix A.3 provides the proof. Proposition 1 is a special case of Theorem 1. When the DM prefers to disaggregate harms on x and y (*dd-a* and *dd-a*), her apportionment preference across attributes depends on whether the total order $M+N$ is odd or even. For $M+N$ odd, a positive sign on $u^{(M,N)}$ indicates a preference to disaggregate harms across attributes and a negative sign a preference to aggregate harms across attributes. When $M+N$ is even, the interpretation of the signs flips. The total order $M+N$ is decisive because DMs who prefer to combine good with bad on x and y individually will always view $[(X_1, Y_2); (X_2, Y_1)]$ as the lottery that combines good with bad across attributes and $[(X_1, Y_2); (X_2, Y_1)]$ as the lottery that combines good with good and bad with bad across attributes.

When the DM prefers to disaggregate harms on x but aggregate harms on y (*da-d* and *da-a*), her apportionment preference across attributes depends on the parity of M, the order of the risk change on the first attribute. Similarly, when she prefers to aggregate harms on x but disaggregate harms on y (*ad-d* and *ad-a*), the parity of N is decisive, the order of the risk change on the second attribute. For these DMs, lottery $[(X_1, Y_2); (X_2, Y_1)]$ is not always the one that combines good with bad across attributes relative to lottery $[(X_1, Y_2); (X_2, Y_1)]$. As we move up the orders, high Nth-degree risk on y is a good thing for *da-d* and *da-a* DMs when N is even, and high Mth-degree risk on x is a good thing for *ad-d* and *ad-a* DMs when M is even. This alternating pattern causes $(-1)^{N+1}$ to cancel from the condition for *da-d* and *da-a* DMs, and $(-1)^{M+1}$ to cancel from the condition for *ad-d* and *ad-a* DMs.

Finally, when the DM prefers to aggregate harms on x and on y (*aa-d* and *aa-a*), the parity of M, N and $M+N$ are all irrelevant because a negative sign on $u^{(M,N)}$ always indicates a preference to disaggregate harms across attributes and a positive sign on $u^{(M,N)}$ always represents a preference to aggregate them. The alternating pattern for each attribute implies that now both orders vanish and the sign of $u^{(M,N)}$ alone determines the DM's apportionment preference across attributes.

5.2 One desirable and one undesirable attribute (case DU)

In a next step, we provide the signs of the cross-derivatives of the utility function when one attribute is desirable and the other one is undesirable. Section 2.3 provides the link between risk apportionment preferences on individual attributes and the signs of the unidirectional derivatives of the utility function. When looking at cross-derivatives, the **DU** case shows a different pattern than the **DD** case. Here is our result.

Theorem 2 (Case **DU**)**.** *Consider the case in which the first attribute is desirable and the second attribute is undesirable, and let $M, N \geq 1$.*

(i) DMs with apportionment preference dd-d have $(-1)^M u^{(M,N)} \leq 0$, DMs with apportionment preference dd-a have $(-1)^M u^{(M,N)} \geq 0$.

(ii) DMs with apportionment preference da-d have $(-1)^{M+N+1} u^{(M,N)} \leq 0$, DMs with apportionment preference da-a have $(-1)^{M+N+1} u^{(M,N)} \geq 0$.

(iii) *DMs with apportionment preference ad-d have* $u^{(M,N)} \geq 0$*, DMs with apportionment preference ad-a have* $u^{(M,N)} \leq 0$*.*

(iv) *DMs with apportionment preference aa-d have* $(-1)^N u^{(M,N)} \leq 0$*, DMs with apportionment preference aa-a have* $(-1)^N u^{(M,N)} \geq 0$*.*

Appendix A.4 gives the proof. As in the **DD** case, we obtain different criteria on the utility function depending on the DM's apportionment preference on the individual attributes, her apportionment preference across attributes, and the parity of the risk changes. How these criteria are assigned to the DM's apportionment preference has changed. When the DM prefers to disaggregate harms on x and on y (*dd-d* and *dd-a*), her apportionment preference across attributes now depends on the parity of M and not the parity of $M+N$ as was the case under **DD**. When M is odd, a positive sign on $u^{(M,N)}$ now indicates a preference to disaggregate harms across attributes and a negative sign a preference to aggregate harms across attributes. When M is even, the interpretation of the signs flips. Comparing Theorems 1(i) and 2(i), the two criteria are different when N is odd and identical when N is even.

When the DM prefers to disaggregate harms on x and aggregate harms on y (*da-d* and *da-a*), her apportionment preference across attributes depends on the parity of the total order $M+N$ in the **DU** case. For **DD**, the parity of M was decisive. Yet again, the two criteria differ for N odd and coincide for N even. When the DM prefers to aggregate harms on x and disaggregate harms on y (*ad-d* and *ad-a*), a positive sign on $u^{(M,N)}$ indicates a preference to disaggregate harms across attributes in the **DU** case. For **DD**, the parity of N was critical. The two criteria differ for N odd and coincide for N even. Finally, when the DM prefers to aggregate harms on x and on y (*aa-d* and *aa-a*), her apportionment preference across attributes depends on the parity of N in the **DU** case. For **DD**, a negative sign on $u^{(M,N)}$ always indicated a preference to disaggregate harms across attributes. As before the two criteria differ for N odd and coincide for N even.

This observation extends the comparison of Propositions 1 and 2. For correlation aversion and cross-prudence in y, the signs flip when going from **DD** to **DU** because we have $N=1$, an odd number. For cross-prudence in x and cross-temperance, the signs stay the same because we have $N=2$, an even number. To understand why the parity of N determines whether the sign on the cross-derivative needs to be flipped or not, we examine Nth-degree risk attitudes in the **DD** and **DU** case. Consider DMs who prefer to disaggregate harms on y. For **DD**, they are Nth-degree risk-averse for all $N \geq 1$. For **DU**, they are Nth-degree risk-loving for N odd and Nth-degree risk-averse for N even. They agree on the fact that even-order risk increases on y are unfavorable but disagree on odd-order risk increases. DMs who prefer to aggregate harms on y are Nth-degree risk-averse for N odd and Nth-degree risk-loving for N even in the **DD** case. For **DU**, they are always Nth-degree risk-loving. Yet again, they agree that even-order risk increases on y are favorable but disagree on odd-order risk increases. Regardless of whether the apportionment preference on y is combining good with bad or combining good with good and bad with bad, the signs of the cross-derivative remain unchanged when N is even but need to be flipped when N is odd.

5.3 Two undesirable attributes (case UU)

Finally, we consider the **UU** case with two undesirable attributes, $u^{(1,0)} \leq 0$ and $u^{(0,1)} \leq 0$. Section 2.3 provides the signs of the unidirectional derivatives of the utility function depending on the DM's risk apportionment preferences regarding the individual attributes x and y. We will now look at the signs of the cross-derivative of the utility function. The next result summarizes how the DM's risk apportionment preference determines these signs.

Theorem 3 (Case **UU**). *Consider the case of two undesirable attributes and let $M, N \geq 1$.*

(i) DMs with apportionment preference dd-d have $u^{(M,N)} \leq 0$, DMs with apportionment preference dd-a have $u^{(M,N)} \geq 0$.

(ii) DMs with apportionment preference da-d have $(-1)^N u^{(M,N)} \geq 0$, DMs with apportionment preference da-a have $(-1)^N u^{(M,N)} \leq 0$.

(iii) DMs with apportionment preference ad-d have $(-1)^M u^{(M,N)} \geq 0$, DMs with apportionment preference ad-a have $(-1)^M u^{(M,N)} \leq 0$.

(iv) DMs with apportionment preference aa-d have $(-1)^{M+N+1} u^{(M,N)} \geq 0$, DMs with apportionment preference aa-a have $(-1)^{M+N+1} u^{(M,N)} \leq 0$.

Appendix A.5 provides the proof. The comparison between Theorems 2 and 3 follows along the same lines as the comparison between Theorems 1 and 2. When going from **DU** to **UU**, the two criteria are different for M odd and identical for M even. The reason is that DMs agree on whether an Mth-degree risk increase on x is favorable or unfavorable for M even but disagree when M is odd.

To compare Theorems 1 and 3, we start with a DM who prefers to disaggregate harms on x and y (*dd-d* and *dd-a*). For **DD**, her apportionment preference across attributes depends on the parity of $M + N$ while for **UU**, it is determined by the sign of $u^{(M,N)}$ directly and the parity of neither M, N nor $M + N$ matter. The two criteria differ for $M + N$ odd and coincide for $M + N$ even. When the DM prefers to disaggregate harms on x and aggregate harms on y (*da-d* and *da-a*), her apportionment preference across attributes depends on the parity of N in the **UU** case and on the parity of M in the **DD** case. When the DM prefers to aggregate harms on x and disaggregate harms on y (*ad-d* and *ad-a*), her apportionment preference across attributes depends on the parity of M in the **UU** case and on the parity of N in the **DD** case. In both cases the two criteria differ for $M + N$ odd and coincide for $M + N$ even. Finally, when the DM prefers to aggregate harms on x and y (*aa-d* and *aa-a*), her apportionment preference across attributes depends on the parity of $M + N$ in the **UU** case and is determined by the sign of $u^{(M,N)}$ in the **DD** case. Yet again, the two criteria differ for $M + N$ odd and coincide for $M + N$ even.

This observation extends the comparison of Propositions 1 and 3. For cross-prudence in x and cross-prudence in y, the signs flip when going from **DD** to **UU** because we have $M + N = 1 + 2 = 2 + 1 = 3$, an odd number. For correlation aversion and cross-temperance, the signs stay the same because we have $M + N = 1 + 1 = 2$ and $M + N = 2 + 2 = 4$, two even numbers. To explain why it is now the parity of the total order that determines whether the criterion needs to be adjusted, we examine the DM's Mth- and Nth-degree risk attitudes in the **DD** and **UU** cases. Consider a DM who prefer to disaggregate harms on x and y (*dd-d* and *dd-a*). For **DD**, she is Mth-degree risk-averse for all $M \geq 1$ and Nth-degree risk-averse for all $N \geq 1$. For **UU**, she is Mth-degree risk-loving for M odd, Mth-degree risk-averse for M even, Nth-degree risk-loving for N odd, and Nth-degree risk-averse for N even. When both M and N are even, the two DMs agree that an Mth-degree risk increase on x and an Nth-degree risk increase on y are both unfavorable. When both M and N are odd, the two risk increases are unfavorable in the **DD** case but favorable in the **UU** case. Given that two reversals occur when going from **DD** to **UU**, they cancel each other out and no adjustment to the sign of the cross-derivative is necessary.[10] When M is even and N odd or when M is odd and N even, only one of the risk increases becomes favorable when moving from **DD** to **UU**, and the sign of the cross-derivative flips. The reasoning is analogous for the other apportionment preferences on individual attributes.

5.4 A simple mathematical reconciliation

We will now show the consistency between the criteria stated in Theorems 1, 2 and 3 directly. Take the case of **DU** and let preferences be represented by utility function $u(x, y)$ for $(x, y) \in \mathcal{D} = [0, \overline{x}] \times [0, \overline{y}]$. We have $u^{(1,0)} \geq 0$ and $u^{(0,1)} \leq 0$. Define utility function $v(x, y) = u(x, \overline{y} - y)$ for $(x, y) \in \mathcal{D}$. Obviously, we have $v^{(1,0)} = u^{(1,0)} \geq 0$ and $v^{(0,1)} = -u^{(0,1)} \geq 0$ so that utility function v represents the **DD** case. More generally, we find that $v^{(M,N)} = (-1)^N u^{(M,N)}$.

As a consequence, when going from Theorem 1 to Theorem 2, we need to multiply each of the criteria by $(-1)^N$. For example, *dd-d* is characterized by $(-1)^{M+N+1} u^{(M,N)} \geq 0$ in the **DD** case. Multiplying by $(-1)^N$ yields $(-1)^{M+1} u^{(M,N)} \geq 0$ or, equivalently, $(-1)^M u^{(M,N)} \leq 0$, the criterion for *dd-d* in the **DU** case. Similarly, *da-d* is characterized by $(-1)^M u^{(M,N)} \geq 0$ in the **DD** case. Multiplying by $(-1)^N$ yields $(-1)^{M+N} u^{(M,N)} \geq 0$ or, equivalently, $(-1)^{M+N+1} u^{(M,N)} \leq 0$, the criterion for *da-d* in the **DU** case.

We also show the consistency between Theorems 1 and 3. When utility function u represents preferences in case of **UU**, we define $v(x, y) = u(\overline{x} - x, \overline{y} - y)$ and obtain a utility function for the **DD** case. We have $v^{(M,N)} = (-1)^{M+N} u^{(M,N)}$. Therefore, when going from Theorem 1 to Theorem 3, each of the criteria needs to be multiplied by $(-1)^{M+N}$. Finally, if u represents preferences in case of **UU**, then $v(x, y) = u(\overline{x} - x, y)$ is a utility function for the

[10] More specifically, a lottery that allocates the risk increases to different states combines good with bad for either DM, the only difference being that the labels "good" and "bad" need to be switched for both risk increases when going from **DD** to **UU**.

DU case. We have $v^{(M,N)} = (-1)^M u^{(M,N)}$ so that each of the criteria needs to be multiplied by $(-1)^M$ when going from Theorem 2 to Theorem 3.

Mathematically, it is easy to see the equivalence between the criteria in Theorems 1 to 3 even though nothing is learned about the underlying economic intuition. Some problems are more naturally formulated in terms of undesirable attributes, and our results show how to make the entire arsenal of the risk apportionment literature available in those situations. While the mathematical equivalence of Theorems 1 to 3 is easy to see if a utility representation exists, we emphasize that the concepts of correlation aversion, cross-prudence, cross-temperance and their higher-order extensions can be defined with the help of simple lotteries, and thus do not require the existence of a utility representation. As we showed in Section 4, an appropriate adjustment to the seed lotteries ensures that the definitions stay intact when going from the **DD** case to the **DU** and **UU** cases. The definition based on simple lotteries allows researchers to utilize multivariate risk preferences outside the narrow confines of the expected-utility model and regardless of whether desirable or undesirable attributes are studied. This flexibility broadens the scope of our results significantly.

6 Gollier's (2021) generalized risk apportionment theory

Recently, Gollier (2021) provides a generalization of Eeckhoudt et al.'s (2009) risk apportionment approach. He assumes that the Mth-degree riskiness of X is uncertain and that the Nth-degree riskiness of Y is uncertain. In his model, X is parameterized by random variable Θ, and Y is parameterized by random variable Ψ. Then, for realizations $\theta_2 > \theta_1$ of Θ, $X(\theta_2)$ has more Mth-degree risk than $X(\theta_1)$, and for realizations $\psi_2 > \psi_1$ of Ψ, $Y(\psi_2)$ has more Nth-degree risk than $Y(\psi_1)$. The uncertainty over the riskiness of X and Y is represented by a joint distribution function for (Θ, Ψ).

In the original approach by Eeckhoudt and Schlesinger (2006) and Eeckhoudt et al. (2009), Θ and Ψ are both limited to a support of $\{1, 2\}$ so that the level of riskiness can either be low or high. Furthermore, state probabilities are equal because only 50-50 lotteries are considered. Thirdly, only perfect negative or perfect positive correlation between Θ and Ψ are considered. Gollier's (2021) contribution is to show that all these appendages can be removed. He accomplishes this by utilizing the following notion of dependence.

Definition 2 (Tchen et al.,1980; Epstein and Tanny,1980)**.** For two pairs of random variables (Θ_1, Ψ_1) and (Θ_2, Ψ_2) with joint cumulative distribution functions H_1 and H_2, we say that (Θ_2, Ψ_2) is more concordant than (Θ_1, Ψ_1) if H_1 and H_2 have the same marginal distributions and $H_2(\theta, \psi) \geq H_1(\theta, \psi)$ for all (θ, ψ) in the relevant domain.

Tchen et al. (1980) use the term concordance for this change in the joint distribution. Epstein and Tanny (1980) show in their Theorem 1 that, for discrete random variables, an increase in concordance is obtained as a sequence of correlation-increasing transformations. An increase in concordance implies higher correlation, a higher Kendall's τ, and a higher

Spearman's ρ, see Tchen et al. (1980). We use the notion of concordance and apply it to the uncertainty over the riskiness of X and Y.

Definition 3 (Gollier,2021)**.** Let Θ be an index of the Mth-degree riskiness of X and Ψ be an index of the Nth-degree riskiness of Y. Then, $(X(\Theta_2), Y(\Psi_2))$ is an (M, N)-degree risk increase over $(X(\Theta_1), Y(\Psi_1))$ if (Θ_2, Ψ_2) is more concordant than (Θ_1, Ψ_1).

Gollier (2021) goes on to show that a change in the joint distribution of (X, Y) is an (M, N)-degree risk increase if and only if it reduces the expectation of $u(X, Y)$ for any utility function u whose (M, N) cross-derivative has the same sign as $(-1)^{M+N+1}$.[11] We can use this result and our Theorems 1 to 3 to assess a DM's attitude towards increases in (M, N)-degree risk. We call a DM (M, N)-degree risk-averse if she dislikes any increase in (M, N)-degree risk and (M, N)-degree risk-loving if she appreciates any increase in (M, N)-degree risk. We formulate our results as corollaries and dissociate the three cases for readability.

Corollary 1 (Case **DD**)**.** *Consider the case of two desirable attributes and let $M, N \geq 1$.*

(i) DMs with dd-d (dd-a) are (M, N)-degree risk-averse (risk-loving).

(ii) DMs with da-d (da-a) are (M, N)-degree risk-averse (risk-loving) for N odd and (M, N)-degree risk-loving (risk-averse) for N even.

(iii) DMs with ad-d (ad-a) are (M, N)-degree risk-averse (risk-loving) for M odd and (M, N)-degree risk-loving (risk-averse) for M even.

(iv) DMs with aa-d (aa-a) are (M, N)-degree risk-loving (risk-averse) for $M + N$ odd and (M, N)-degree risk-averse (risk-loving) for $M + N$ even.

The only risk apportionment preference that implies (M, N)-degree risk aversion throughout is *dd-d* with *dd-a* yielding universal (M, N)-degree risk loving. In all other cases, the DM's (M, N)-degree risk attitude flips as we look at different orders, and either the parity of N, M or $M + N$ is decisive. Let us look at the **DU** case next.

Corollary 2 (Case **DU**)**.** *Consider the case in which the first attribute is desirable and the second attribute is undesirable, and let $M, N \geq 1$.*

(i) DMs with dd-d (dd-a) are (M, N)-degree risk-loving (risk-averse) for N odd and (M, N)-degree risk-averse (risk-loving) for N even.

(ii) DMs with da-d (da-a) are (M, N)-degree risk-loving (risk-averse).

[11] In his Theorem 1, Gollier (2021) uses the sign criterion on the cross-derivative of the utility function to define an (M, N)-degree risk increase and then shows the equivalence to an increase in the concordance between the index of the Mth-degree riskiness of X and the index of the Nth-degree riskiness of Y. Given the equivalence, we go the reverse route, which makes it easier to connect our results to his.

(iii) *DMs with ad-d (ad-a) are (M, N)-degree risk-averse (risk-loving) for $M + N$ odd and (M, N)-degree risk-loving (risk-averse) for $M + N$ even.*

(iv) *DMs with aa-d (aa-a) are (M, N)-degree risk-loving (risk-averse) for M odd and (M, N)-degree risk-averse (risk-loving) for M even.*

Now the only risk apportionment preferences that implies (M, N)-degree risk aversion throughout is *da-a*. When the DM prefers to combine good with good and bad with bad on y, she is Nth-degree risk-loving on the second attribute for all N. If she prefers to aggregate harms across attributes, higher concordance between the Mth-degree riskiness of x and the Nth-degree riskiness of y makes her worse off. She would rather face low Mth-degree risk on x together with high Nth-degree risk on y (two good things) or high Mth-degree risk on x together with low Nth-degree risk on y (two bad things) instead of low Mth-degree risk on x (a good thing) together with low Nth-degree risk on y (a bad thing) or high Mth-degree risk on x (a bad thing) toghether with high Nth-degree risk on y (a good thing). While both *dd-d* DMs in the **DD** case and *da-a* DMs in the **DU** case are consistently (M, N)-degree risk-averse, the reasons for their preference are quite different.

Corollary 3 (Case **UU**)**.** *Consider the case of two undesirable attributes and let $M, N \geq 1$.*

(i) *DMs with dd-d (dd-a) are (M, N)-degree risk-loving (risk-averse) for $M + N$ odd and (M, N)-degree risk-averse (risk-loving) for $M + N$ even.*

(ii) *DMs with da-d (da-a) are (M, N)-degree risk-averse (risk-loving) for M odd and (M, N)-degree risk-loving (risk-averse) for M even.*

(iii) *DMs with ad-d (ad-a) are (M, N)-degree risk-averse (risk-loving) for N odd and (M, N)-degree risk-loving (risk-averse) for N even.*

(iv) *DMs with aa-d (aa-a) are (M, N)-degree risk-averse (risk-loving).*

It is now DMs with *aa-d* who are always (M, N)-degree risk-averse. Yet again the intuition for this preference differs from the previous discussion. When the DM prefers to combine good with good and bad wit bad on x and y individually, she is Mth-degree risk-loving on x and Nth-degree risk-loving on y for all M and N. If she prefers to disaggregate harms across attributes, higher concordance between the Mth-degree riskiness of x and the Nth-degree riskiness of y makes her worse off. The DM would rather face high Mth-degree risk on x (a good thing) together with low Nth-degree risk on y (a bad thing) or low Mth-degree risk on x (a bad thing) together with high Nth-degree risk on y (a good thing) instead of high Mth-degree risk on x and high Nth-degree risk on y (two good things) or low Mth-degree risk on x and low Nth-degree risk on y (two bad things). At the surface, the resulting lottery preference is the same as for *dd-d* DMs in the **DD** case and *da-a* DMs in the **DU** case. How we obtain this lottery preference is entirely different.

Table 4 in the appendix provides a compact overview of the (M, N)-degree risk attitudes implied by different apportionment preferences in the three cases. We fully agree with Gollier's (2021) conclusion that a DM's (M, N)-degree risk attitude can be characterized without knowledge of any of her lower-degree risk attitudes by signing $u^{(M,N)}$. Our point here is that, if one imposes a consistent apportionment preference on the individual attributes and across attributes, this implies specific (M, N)-degree risk attitudes. Corollaries 1 to 3 detail what these (M, N)-degree risk attitudes are. Our results also highlight that the underlying reasons for a particular (M, N)-degree risk attitude can vary considerably within each case and across cases. By making the apportionment preference explicit, we can uncover these reasons and provide economic intuition.[12]

7 Some special multivariate models

7.1 Multiplicative separability

The utility function is multiplicatively separable if we can write it as $u(x, y) = v(x)z(y)$ for univariate utility functions v and z. Bleichrodt and Quiggin (1999) use this utility function to assess the consistency of quality-adjusted life years with life-cycle preferences when both consumption and health are arguments of the utility function. One might think that the separability assumption is constraining and restricts the types of risk apportionment preferences one can model. Our next result shows that this is not the case. To the contrary, multiplicatively separable utility is quite flexible and can be used to model any of the eight combinations of apportionment preferences discussed in this paper.

Proposition 4. *Let the utility function be multiplicatively separable, $u(x, y) = v(x)z(y)$, for univariate utility functions v of the first attribute and z of the second attribute. In each of the three cases,* ***DD****,* ***DU*** *or* ***UU****, we find the following:*

- *If $sgn(v) = sgn(z)$, the utility function can accommodate dd-a, da-a, ad-a and aa-a. The DM always prefers to aggregate harms across attributes.*
- *If $sgn(v) \neq sgn(z)$, the utility function can accommodate dd-d, da-d, ad-d and aa-d. The DM always prefers to disaggregate harms across attributes.*

Appendix A.6 provides the proof. For multiplicatively separable utility, the DM's apportionment preference across attributes is simply determined by the signs of the univariate utility functions v and z. If the two signs are the same, either both positive or both negative, the DM necessarily prefers to combine good with good and bad with bad across attributes. If

[12] Of course, DMs with a given (M, N)-degree risk attitude may not belong to any of the eight groups of apportionment preferences considered in this paper. In the univariate context, some prudent DMs are risk-averse, some prudent DMs are risk-loving, and some prudent DMs may be neither risk-averse nor risk-loving.

the two signs are different with one being positive and the other one negative, the DM prefers to combine good with bad across attributes.

We can use Proposition 4 to construct any of the eight combinations of apportionment preferences studied in this paper. Consider the univariate utility function $v(x)$ for $x \in [0, \overline{x}]$. If attribute x is desirable and the DM prefers to combine good with bad on x, then v is mixed risk-averse, $(-1)^{M+1}v^{(M)} \geq 0$ for all $M \geq 1$. The class of utility functions with harmonic absolute risk aversion (HARA) provides specific examples. Let

$$v(x) = \begin{cases} \zeta \cdot \left(\eta + \frac{x}{\gamma}\right)^{1-\gamma}, & \text{for } \gamma \neq 1, \\ \zeta \cdot \log(\eta + x), & \text{for } \gamma = 1, \end{cases}$$

with $\eta > 0$, $\gamma > 0$, and $\zeta > 0$ for $\gamma \leq 1$ and $\zeta < 0$ for $\gamma > 1$. Then, v is mixed risk-averse and increases from $\underline{v} = v(0)$ to $\overline{v} = v(\overline{x})$. If v is negative, then $\hat{v}(x) = v(x) - \underline{v} + 1$ is mixed risk-averse and positive. If v is positive, then $\hat{v}(x) = v(x) - \overline{v} - 1$ is mixed risk-averse and negative. In general, a utility function displays mixed risk aversion if and only if it is the mixture of negative exponential functions (see Caballé and Pomansky, 1996).

Let $v(x)$ be a mixed risk-averse utility function for a desirable attribute $x \in [0, \overline{x}]$. Then, $\check{v}(x) = -v(\overline{x} - x)$ is a mixed risk-loving utility function for the desirable attribute x. Indeed, $\check{v}^{(M)}(x) = (-1)^{M+1}v^M(\overline{x} - x) \geq 0$ for all $M \geq 1$. Similarly, define $\breve{v}(x) = v(\overline{x} - x)$. This utility function satisfies $\breve{v}^{(M)}(x) = (-1)^M v^{(M)}(\overline{x} - x) \leq 0$ for all $M \geq 1$, and thus represents a preference for combining good with bad for an undesirable attribute x. Along the lines of Ebert (2020), we label this as anti-mixed risk-loving. If we set $\dot{v}(x) = -v(x)$, utility function $\dot{v}(x)$ satisfies $(-1)^{M+1}\dot{v}^{(M)}(x) = (-1)^M v^{(M)}(x) \leq 0$ for all $M \geq 1$, and thus represents a preference for combining good with good and bad with bad for an undesirable attribute x. Following Ebert (2020), we call this anti-mixed risk-averse.[13]

We can thus use the large class of mixed risk-averse utility functions to construct mixed risk-loving, anti-mixed risk-averse, and anti-mixed risk-loving utility functions. Shifting a utility function up or down ensures the desired sign. This puts us in a position to construct any of the eight combinations of apportionment preferences with the help of Proposition 4. To economize on space, we carry this out in Appendix B. Applied decision theorists can use this as a toolbox to construct utility functions with desired properties.

7.2 Equivalent monetary utility

We only consider a simple case of equivalent monetary utility, which is $u(x, y) = v(x + Ay)$ for $A \neq 0$. In this case, parameter A measures the marginal rate of substitution of attribute

[13] For an undesirable attribute, combining good with bad is characterized by a consistent negative sign while combining good with good and bad with bad is characterized by alternating signs, starting with a negative one. The pattern is thus reversed compared to the case of a desirable attribute, for which good with bad has the alternating sign pattern, and good with good and bad with bad a consistent positive sign.

y for attribute x. We assume for simplicity that A is constant and does not depend on the levels of x and y. Our next result shows that this restricts the DM's apportionment preference considerably.

Proposition 5. *Consider an equivalent monetary utility function with a constant marginal rate of substitution between attributes. In each of the three cases,* ***DD****,* ***DU*** *or* ***UU****, the utility function can accommodate either dd-d or aa-a.*

Appendix A.7 states the proof. In other words, equivalent monetary utility with a constant marginal rate of substitution imposes a strong consistency assumption on the DM's risk apportionment preference. She either prefers to combine good with bad on the individual attributes as well as across attributes, or she prefers to combine good with good and bad with bad on the individual attributes as well as across attributes. The other six combinations of apportionment preferences are excluded per assumption with this specification. This illustrates clearly that some simplifying assumptions that are sometimes made for convenience or tractability, can have far-reaching economic implications.[14]

8 Conclusion

Risk apportionment has revolutionized our understanding of higher-order risk preferences and accelerated their use in economics and finance. In this paper, we advanced the theory of risk apportionment for multivariate risks along several dimensions. We defined the concepts of correlation aversion, cross-prudence and cross-temperance in terms of simple lotteries when one or both attributes are undesirable. We characterized risk apportionment preferences across attributes by signing cross-derivatives of the utility function. We related our results to popular multivariate models and explained how to construct any of the eight combinations of apportionment preferences studied in this paper. It is our hope that these tools will help improve our understanding of risk-taking behavior in the many situations in which people face several attributes, some of which may be undesirable.

[14] Equivalent monetary utility is more flexible if we allow the marginal rate of substitution to depend on the levels of the attributes. We leave it for future research to determine how flexible this specification can be.

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A Proofs

A.1 Signs of $u^{(0,N)}$ for all $N \geq 1$ in the DU case

Lemma 2. *Consider the **DU** case so that attribute y is undesirable and let preferences be represented by a smooth utility function $u(x, y)$.*

(i) If the DM prefers to combine good with bad on y, then $u^{(0,N)} \leq 0$ for all $N \geq 1$.

(ii) If the DM prefers to combine good with good and bad with bad on y, then $(-1)^{N+1}u^{(0,N)} \leq 0$ for all $N \geq 1$.

Proof. We show the two statements by mathematical induction. For $N = 1$, $u^{(0,1)} \leq 0$ holds by assumption because y is undesirable.

Now assume the statements are true for a given $N \geq 1$. Let Y_1, Y_2, Y_1' and Y_2' be four mutually independent random variables with Y_2 having more first-degree risk than Y_1, and Y_2' having more Nth-degree risk than Y_1'. The DM prefers Y_2 over Y_1 because of $u^{(0,1)} \leq 0$. If N is odd and she prefers to combine good with bad, she is Nth-degree risk-loving and thus prefers Y_2' over Y_1'. Combining good with bad implies that she also prefers the 50-50 lottery $[(x, Y_2 + Y_1'); (x, Y_2' + Y_1)]$ over the 50-50 lottery $[(x, Y_2 + Y_2'); (x, Y_1 + Y_1')]$ because the first lottery combines high first-degree risk (a good thing) with low Nth-degree risk (a bad thing) and high Nth-degree risk (a good thing) with low first-degree risk (a bad thing) whereas the second lottery combines high first-degree risk with high Nth-degree risk (two good things) and low first-degree risk with low Nth-degree risk (two bad things). If the DM always has said lottery preference, we know from Eeckhoudt et al. (2009) that $(-1)^{N+2}u^{(0,N+1)} \geq 0$. For N odd, this simplifies to $u^{(0,N+1)} \leq 0$ as claimed in statement (i).

If N is odd and the DM prefers to combine good with good and bad with bad, she is also Nth-degree risk-loving but now prefers the 50-50 lottery $[(x, Y_2 + Y_2'); (x, Y_1 + Y_1')]$ over the 50-50 lottery $[(x, Y_2 + Y_1'); (x, Y_2' + Y_1)]$ because of combining good with good and bad with bad. We know from Eeckhoudt et al. (2009) that this lottery preference is characterized by $(-1)^{N+2}u^{(0,N+1)} \leq 0$, as claimed in statement (ii).

If N is even and the DM prefers to combine good with bad, she is Nth-degree risk-averse and thus prefers Y_1' over Y_2'. Combining good with bad now implies that she always prefers the 50-50 lottery $[(x, Y_2+Y_2'); (x, Y_1+Y_1')]$ over the 50-50 lottery $[(x, Y_2+Y_1'); (x, Y_2'+Y_1)]$ because the first lottery combines high first-degree risk (a good thing) with high Nth-degree risk (a bad thing) and low first-degree risk (a bad thing) with low Nth-degree risk (a good thing) whereas the second lottery combines high first-degree risk with low Nth-degree risk (two good things) and high Nth-degree risk with low first-degree risk (two bad things). It follows from Eeckhoudt et al. (2009) that $(-1)^{N+2}u^{(0,N+1)} \leq 0$, which simplifies to $u^{(0,N+1)} \leq 0$ because N is even. This verifies statement (i) .

If N is even and the DM prefers to combine good with good and bad with bad, she is Nth-degree risk-loving and thus prefers Y_2' over Y_1'. Combining good with good and bad with bad leads to a lottery preference of $[(x, Y_2+Y_2'); (x, Y_1+Y_1')]$ over $[(x, Y_2+Y_1'); (x, Y_2'+Y_1)]$, which is characterized by $(-1)^{N+2}u^{(0,N+1)} \leq 0$, as claimed in statement (ii). So if statements (i) and (ii) are true for a given $N \geq 1$, they also hold for $N+1$, which completes the proof. $\square$

A.2 Sign of $u^{(M,N)}$ based on Eeckhoudt et al.'s (2009) approach

Assume the DM prefers lottery $[(X_1, Y_2); (X_2, Y_1)]$ over lottery $[(X_1, Y_1); (X_2, Y_2)]$ for all sets of four mutually independent random variables X_1, X_2, Y_1 and Y_2 such that X_2 has more Mth-degree risk than X_1, and Y_2 has more Nth-degree risk than Y_1. In terms of expected utility, the DM's lottery preference reads

$$\tfrac{1}{2}\mathbb{E}u(X_1, Y_2) + \tfrac{1}{2}\mathbb{E}u(X_2, Y_1) \geq \tfrac{1}{2}\mathbb{E}u(X_1, Y_1) + \tfrac{1}{2}\mathbb{E}u(X_2, Y_2),$$

which is equivalent to

$$\mathbb{E}u(X_2, Y_1) - \mathbb{E}u(X_2, Y_2) \geq \mathbb{E}u(X_1, Y_1) - \mathbb{E}u(X_1, Y_2).$$

Define auxiliary function $v(x) = \mathbb{E}u(x, Y_1) - \mathbb{E}u(x, Y_2)$; the last inequality can then be rewritten as $\mathbb{E}v(X_2) \geq \mathbb{E}v(X_1)$. If this inequality holds for every Mth-degree risk increase from X_1 to X_2, it follows from Lemma 1 that $-v$ must be Mth-degree risk-averse, that is, $(-1)^M v^{(M)}(x) \geq 0$. Using the definition of v, this is equivalent to

$$(-1)^M \mathbb{E}u^{(M,0)}(x, Y_1) \geq (-1)^M \mathbb{E}u^{(M,0)}(x, Y_2).$$

Per Lemma 1, this inequality holds for every Nth-degree risk increase from Y_1 to Y_2 if and only if $(-1)^M u^{(M,0)}$ is Nth-degree risk-averse in y, that is, if and only if

$$(-1)^{M+N+1}u^{(M,N)} \geq 0.$$

A.3 Proof of Theorem 1

Let $\mathcal{L}_1 = [(X_1, Y_2); (X_2, Y_1)]$ be the lottery where the Mth-degree risk increase on x and the Nth-degree risk increase on y occur in different states, and $\mathcal{L}_2 = [(X_1, Y_1); (X_2, Y_2)]$ be the lottery where they occur in the same state. For *dd-d* and *dd-a* DMs, both risk increases are unfavorable changes so that lottery $\mathcal{L}_1$ represents combining good with bad across attributes whereas lottery $\mathcal{L}_2$ represents combining good with good and bad with bad across attributes. A universal preference of $\mathcal{L}_1 \succsim \mathcal{L}_2$ is equivalent to $(-1)^{M+N+1}u^{(M,N)} \geq 0$ whereas a universal preference of $\mathcal{L}_2 \succsim \mathcal{L}_1$ is equivalent to $(-1)^{M+N+1}u^{(M,N)} \leq 0$. This shows (i).

For *da-d* and *da-a* DMs, the Mth-degree risk increase on x is always an unfavorable change. However, the Nth-degree risk increase on y is an unfavorable change when N is odd and a favorable change when N is even. The lottery $\mathcal{L}_1$ then represents combining good with bad when N is odd. It represents combining good with good and bad with bad when N is even. So a preference to disaggregate harms across attributes leads to $\mathcal{L}_1 \succsim \mathcal{L}_2$ for N odd and to $\mathcal{L}_2 \succsim \mathcal{L}_1$ for N even. In terms of the utility function, this means $(-1)^{M+N+1}u^{(M,N)} \geq 0$ for N odd and $(-1)^{M+N+1}u^{(M,N)} \leq 0$ for N even. When N is odd, $(-1)^{M+N+1} = (-1)^M$, and when N is even, $(-1)^{M+N+1} = (-1)^{M+1}$. So the criterion on the utility function can be consolidated to $(-1)^M u^{(M,N)} \geq 0$ for *da-d* DMs, and to $(-1)^M u^{(M,N)} \leq 0$ for *da-a* DMs. This proves (*ii*). Result (*iii*) follows with the same argument replacing M by N.

For *aa-d* and *aa-a* DMs, the Mth-degree risk increase on x is an unfavorable change when M is odd and a favorable change when M is even. Likewise, the Nth-degree risk increase on y is an unfavorable change when N is odd and a favorable change when N is even. Lottery $\mathcal{L}_1$ thus represents combining good with bad when both M and N are odd or when both M and N are even. When M is odd and N is even or M is even and N is odd, lottery $\mathcal{L}_1$ corresponds to combining good with good and bad with bad. This implies $(-1)^{M+N+1}u^{(M,N)} \geq 0$ when both M and N are odd or both are even, which can be simplified to $u^{(M,N)} \leq 0$. For M odd and N even or M even and N odd, we obtain $(-1)^{M+N+1}u^{(M,N)} \leq 0$, which can also be simplified to $u^{(M,N)} \leq 0$. So regardless of the parity of M and N, *aa-d* DMs have $u^{(M,N)} \leq 0$ whereas *aa-a* DMs have $u^{(M,N)} \geq 0$

A.4 Proof of Theorem 2

Let X_1, X_2, Y_1 and Y_2 be four mutually independent random variables with X_2 having more Mth-degree risk than X_1 and Y_2 having more Nth-degree risk than Y_1. Let $\mathcal{L}_1 = [(X_1, Y_2); (X_2, Y_1)]$ be the lottery where the Mth-degree risk increase on x and the Nth-degree risk increase on y occur in different states, and $\mathcal{L}_2 = [(X_1, Y_1); (X_2, Y_2)]$ be the lottery where they occur in the same state. For *dd-d* and *dd-a* DMs, the Mth-degree risk increase on x is always an unfavorable change whereas the Nth-degree risk increase on y is a favorable change when N is odd and an unfavorable change when N is even. So *dd-d* DMs prefer $\mathcal{L}_2$ over $\mathcal{L}_1$ when N is odd, leading to $(-1)^{M+N+1}u^{(M,N)} \leq 0$, while they prefer $\mathcal{L}_1$ over $\mathcal{L}_2$ when N is even, leading to $(-1)^{M+N+1}u^{(M,N)} \geq 0$. Taking the parity of N into account, the condition on the utility function can be condensed to $(-1)^M u^{(M,N)} \leq 0$ for *dd-d* DMs, and to $(-1)^M u^{(M,N)} \geq 0$ for *dd-a* DMs.

For *da-d* and *da-a* DMs, the Mth-degree risk increase on x is always an unfavorable change and the Nth-degree risk increase on y is always a favorable change. Therefore, *da-d* DMs have a preference of $\mathcal{L}_2$ over $\mathcal{L}_1$ because $\mathcal{L}_2$ combines low Mth-degree risk on x (a good thing) with low Nth-degree risk on y (a bad thing) and high Mth-degree risk on x (a bad thing) with high Nth-degree risk on y (a good thing) whereas $\mathcal{L}_1$ combines low Mth-degree risk on x with high Nth-degree risk on y (two good things) and high Mth-degree risk on x with low Nth-degree risk on y (two bad things). This leads to $(-1)^{M+N+1}u^{(M,N)} \leq 0$ for *da-d* DMs and to $(-1)^{M+N+1}u^{(M,N)} \geq 0$ for *da-a* DMs.

For *ad-d* and *ad-a* DMs, the Mth-degree risk increase on x is an unfavorable change when M is odd and a favorable change when M is even whereas the Nth-degree risk increase on

y is a favorable change when N is odd and an unfavorable change when N is even. For M odd, *ad-d* DMs then prefer $\mathcal{L}_2$ over $\mathcal{L}_1$ when N is odd, and $\mathcal{L}_1$ over $\mathcal{L}_2$ when N is even. This leads to $(-1)^M u^{(M,N)} \leq 0$, which can be further simplified to $u^{(M,N)} \geq 0$ because M is odd. When M is even instead, *ad-d* DMs prefers $\mathcal{L}_1$ over $\mathcal{L}_2$ when N is odd, and $\mathcal{L}_2$ over $\mathcal{L}_1$ when N is even. This yields $(-1)^M u^{(M,N)} \geq 0$, which is also equivalent to $u^{(M,N)} \geq 0$ because M is even. Therefore, *ad-d* DMs have $u^{(M,N)} \geq 0$ and *ad-a* DMs have $u^{(M,N)} \leq 0$.

For *aa-d* and *aa-a* DMs, the Mth-degree risk increase on x is an unfavorable change when M is odd and a favorable change when M is even whereas the Nth-degree risk increase on y is always a favorable change. So for M odd, *aa-d* DMs prefers $\mathcal{L}_2$ over $\mathcal{L}_1$, leading to $(-1)^{M+N+1} u^{(M,N)} \leq 0$. This can be simplified to $(-1)^N u^{(M,N)} \leq 0$. When M is even, *aa-d* DMs prefer $\mathcal{L}_1$ over $\mathcal{L}_2$, leading to $(-1)^{M+N+1} u^{(M,N)} \geq 0$. This can also be simplified to $(-1)^N u^{(M,N)} \leq 0$. For *aa-a* DMs, matters are reversed so that $(-1)^N u^{(M,N)} \geq 0$.

A.5 Proof of Theorem 3

Let X_1, X_2, Y_1 and Y_2 be four mutually independent random variables with X_2 having more Mth-degree risk than X_1 and Y_2 having more Nth-degree risk than Y_1. Let $\mathcal{L}_1 = [(X_1, Y_2); (X_2, Y_1)]$ be the lottery where the Mth-degree risk increase on x and the Nth-degree risk increase on y occur in different states, and $\mathcal{L}_2 = [(X_1, Y_1); (X_2, Y_2)]$ be the lottery where they occur in the same state. For *dd-d* and *dd-a* DMs, the Mth-degree risk increase on x is a favorable change when M is odd and an unfavorable change when M is even, and the same is the case for the Nth-degree risk increase on y. In this case, a *dd-d* DM prefers lottery $\mathcal{L}_1$ over lottery $\mathcal{L}_2$ when M and N are both odd or both even, and has the reverse lottery preference otherwise. This implies $(-1)^{M+N+1} u^{(M,N)} \geq 0$ when M and N are both odd or both even, which simplifies the condition to $u^{(M,N)} \leq 0$, and $(-1)^{M+N+1} u^{(M,N)} \leq 0$ when M is odd and N even or M is even and N odd, which again simplifies to $u^{(M,N)} \leq 0$. The lottery preference is reversed for *dd-a* DMs, which leads to $u^{(M,N)} \geq 0$.

For *da-d* and *da-a* DMs, the Mth-degree risk increase on x is a favorable change when M is odd and an unfavorable change when M is even but the Nth-degree risk increase on y is always a favorable change. Consequently, *da-d* DMs prefer $\mathcal{L}_1$ over $\mathcal{L}_2$ when M is odd and $\mathcal{L}_2$ over $\mathcal{L}_1$ when M is even. This leads to $(-1)^{M+N+1} u^{(M,N)} \geq 0$ for M odd and to $(-1)^{M+N+1} u^{(M,N)} \leq 0$ for M even. The condition then simplifies to $(-1)^N u^{(M,N)} \geq 0$ for *da-d* DMs. Reversing the lottery preferences shows $(-1)^N u^{(M,N)} \leq 0$ for *da-a* DMs. Result (*iii*) follows with the same argument replacing M by N.

For *aa-d* and *aa-a* DMs, the Mth-degree risk increase on x and the Nth-degree risk increase on y are both always favorable changes. Hence, a *aa-d* DM always prefers $\mathcal{L}_1$ over $\mathcal{L}_2$, implying $(-1)^{M+N+1} u^{(M,N)} \geq 0$, whereas a *aa-a* DM always prefers $\mathcal{L}_2$ over $\mathcal{L}_1$, implying $(-1)^{M+N+1} u^{(M,N)} \leq 0$

A.6 Proof of Proposition 4

For multiplicatively separable utility, we have $u^{(M,N)} = v^{(M)} \cdot z^{(N)}$ for $M, N \geq 1$, and in particular $u^{(M,0)} = v^{(M)} \cdot z$ and $u^{(0,N)} = v \cdot z^{(N)}$ for the unidirectional derivatives. Let us start with the **DD** case and assume $v(x) > 0$ for $x \in [0, \overline{x}]$ and $z(y) > 0$ for $y \in [0, \overline{y}]$ so that $\text{sgn}(v) = \text{sgn}(z)$. Combining good with bad on x is equivalent to $(-1)^{M+1}u^{(M,0)} \geq 0$ for all $M \geq 1$ or $(-1)^{M+1}v^{(M)} \geq 0$ for all $M \geq 1$. Combining good with good and bad with bad on x is equivalent to $u^{(M,0)} \geq 0$ for all $M \geq 1$ or $v^{(M)} \geq 0$ for all $M \geq 1$. Combining good with bad on y is equivalent to $(-1)^{N+1}u^{(0,N)} \geq 0$ for all $N \geq 1$ or $(-1)^{N+1}z^{(N)} \geq 0$ for all $N \geq 1$. Combining good with good and bad with bad on y is equivalent to $u^{(0,N)} \geq 0$ for all $N \geq 1$ or $z^{(N)} \geq 0$ for all $N \geq 1$.

When the DM prefers to combine good with bad on both attributes individually, we obtain

$$(-1)^{M+N+1}u^{(M,N)} = (-1) \cdot \underbrace{(-1)^{M+1}v^{(M)}}_{\geq 0} \cdot \underbrace{(-1)^{N+1}z^{(N)}}_{\geq 0} \leq 0,$$

which characterizes *dd-a* according to Theorem 1(i). When she prefers to combine good with bad on x but good with good and bad with bad on y, we obtain

$$(-1)^{M}u^{(M,N)} = (-1) \cdot \underbrace{(-1)^{M+1}v^{M}}_{\geq 0} \cdot \underbrace{z^{N}}_{\geq 0} \leq 0,$$

which characterizes *da-a* according to Theorem 1(ii). When she prefers to combine good with good and bad with bad on x but good with bad on y, we obtain

$$(-1)^{N}u^{(M,N)} = (-1) \cdot \underbrace{v^{(M)}}_{\geq 0} \cdot \underbrace{(-1)^{N+1}z^{N}}_{\geq 0} \leq 0,$$

which characterize *ad-a* according to Theorem 1(iii). When she prefers to combine good with good and bad with bad on both attributes individually, we obtain

$$u^{(M,N)} = \underbrace{v^{(M)}}_{\geq 0} \cdot \underbrace{z^{(N)}}_{\geq 0} \geq 0,$$

which characterize *aa-a* according to Theorem 1(iv). Regardless of her apportionment preference on the individual attributes, she always prefers to aggregate harms across attributes.

Consider now that $v(x) > 0$ for $x \in [0, \overline{x}]$ but $z(y) < 0$ for $y \in [0, \overline{y}]$ so that $\text{sgn}(v) \neq \text{sgn}(z)$. Combining good with bad on x is now equivalent to $(-1)^{M+1}v^{(M)} \leq 0$ for all $M \geq 1$, and combining good with good and bad with bad on x is now equivalent to $v^{(M)} \leq 0$ for all $M \geq 1$. The signs of higher-order derivatives of utility function z are as in the case of both utility functions positive. As a result, all signs of the cross-derivatives flip and we now obtain *dd-d*, *da-d*, *ad-d* and *aa-d*. The DM now prefers to disaggregate harms across attributes. Similarly, if $v(x) < 0$ for $x \in [0, \overline{x}]$ and $z(y) > 0$ for $y \in [0, \overline{y}]$, combining good with bad on

y is equivalent to $(-1)^{N+1}z^{(N)} \leq 0$ for all $N \geq 1$, and combining good with good and bad with bad on y is equivalent to $z^{(N)} \leq 0$ for all $N \geq 1$. The signs of higher-order derivatives of utility function z are as in the case of both utility functions positive. Yet again, all signs of the cross-derivatives flip compared to the case with $\mathrm{sgn}(v) = \mathrm{sgn}(z)$, and we thus obtain *dd-d*, *da-d*, *ad-d* and *aa-d*. If both $v(x) < 0$ for $x \in [0, \overline{x}]$ and $z(y) < 0$ for $y \in [0, \overline{y}]$, the signs of higher-order derivatives of both utility functions flip and we obtain the same signs as in the case with $\mathrm{sgn}(v) = \mathrm{sgn}(z)$. In other words, we find *dd-a*, *da-a*, *ad-a* and *aa-a*.

The **DU** and **UU** cases follow a similar logic. We briefly look at the **DU** case. When $v(x) > 0$ for $x \in [0, \overline{x}]$ and $z(y) > 0$ for $y \in [0, \overline{y}]$, combining good with bad on x is equivalent to $(-1)^{M+1}v^{(M)} \geq 0$ for all $M \geq 1$, and combining good with good and bad with bad on x is equivalent to $v^{(M)} \geq 0$ for all $M \geq 1$. Combining good with bad on y is equivalent to $u^{(0,N)} \leq 0$ for all $N \geq 1$ or $z^{(N)} \leq 0$ for all $N \geq 1$. Combining good with good and bad with bad on y is equivalent to $(-1)^{N+1}u^{(0,N)} \leq 0$ for all $N \geq 1$ or $(-1)^{N+1}z^{(N)} \leq 0$ for all $N \geq 1$. Using Theorem 2, we find

$$(-1)^M u^{(M,N)} = (-1) \cdot \underbrace{(-1)^{M+1}v^{(M)}}_{\geq 0} \cdot \underbrace{z^{(N)}}_{\leq 0} \geq 0$$

for *dd-a*,

$$(-1)^{M+N+1} u^{(M,N)} = (-1) \cdot \underbrace{(-1)^{M+1}v^{(M)}}_{\geq 0} \cdot \underbrace{(-1)^{N+1}z^{(N)}}_{\leq 0} \geq 0$$

for *da-a*,

$$u^{(M,N)} = \underbrace{v^{(M)}}_{\geq 0} \cdot \underbrace{z^{(N)}}_{\leq 0} \leq 0$$

for *ad-a*, and

$$(-1)^N u^{(M,N)} = (-1) \cdot \underbrace{v^{(M)}}_{\geq 0} \cdot \underbrace{(-1)^{N+1}z^{(N)}}_{\leq 0} \geq 0$$

for *aa-a*. The DM always prefers to aggregate harms across attributes regardless of her apportionment preference on the individual attributes. When the sign of v switches from positive to negative, all signs of the higher-order derivatives of z flip and so do the signs of the cross-derivatives. If instead the sign of z switches from positive to negative and the sign of v is positive, all signs of the higher-order derivatives of v flip and so do the signs of the cross-derivatives. Regardless, as soon as $\mathrm{sgn}(v) \neq \mathrm{sgn}(z)$, we have *dd-d*, *da-d*, *ad-d* or *aa-d*, and the DM prefers to disaggregate harms across attributes. When both v and z are negative, the two sign reversals cancel each other out and we are back to *dd-a*, *da-a*, *ad-a* or *aa-a*, as in the case of both v and z positive.

A.7 Proof of Proposition 5

Let $u(x, y) = v(x + Ay)$ and consider the **DD** case first. The first attribute is desirable so that $v' \geq 0$. For the second attribute to be desirable, we then have $A \geq 0$ (except in the uninteresting case of $v' = 0$). If the DM prefers to combine good with bad on x, we obtain $(-1)^{M+1}v^{(M)} \geq 0$ for all $M \geq 1$. This implies $(-1)^{N+1}u^{(0,N)} = (-1)^{N+1}A^N v^{(N)} \geq 0$ for all $N \geq 1$ so that she prefers to combine good with bad on y. Furthermore, $(-1)^{M+N+1}u^{(M,N)} = (-1)^{M+N+1}A^N v^{(M+N)} \geq 0$ for all $M, N \geq 1$ so that she prefers to combine good with bad across attributes according to Theorem 1(i). The DM's preference is thus *dd-d*. If she prefers to combine good with good and bad with bad on x instead, we obtain $v^{(M)} \geq 0$ for all $M \geq 1$, which implies $u^{(0,N)} = A^N v^{(N)} \geq 0$ for all $N \geq 1$, and $u^{(M,N)} = A^N v^{M+N} \geq 0$ for all $M, N \geq 1$. From Theorem 1(iv), her preference is then *aa-a*.

In the **DU** case, we have $v' \geq 0$ and $A \leq 0$. If the DM prefers to combine good with bad on x, we obtain $(-1)^{M+1}v^{(M)} \geq 0$ for all $M \geq 1$. This implies $u^{(0,N)} = A^N v^{(N)} = (-1)(-A)^N(-1)^{N+1}v^{(N)} \geq 0$ for all $N \geq 1$ so that she prefers to combine good with bad on y. In addition we find $(-1)^M u^{(M,N)} = (-1)^M A^N v^{(M+N)} = (-1)(-A)^N(-1)^{M+N+1}v^{(M+N)} \leq 0$ so that she prefers to combine good with bad across attributes according to Theorem 2(i). The DM's preference is *dd-d*. If she prefers to combine good with good and bad with bad on x instead, we have $v^{(M)} \geq 0$ for all $M \geq 1$, which implies $(-1)^{N+1}u^{(0,N)} = (-1)(-A)^N v^{(N)} \leq 0$ for all $N \geq 1$, and $(-1)^N u^{(M,N)} = (-A)^N v^{(M+N)} \geq 0$ for all $M, N \geq 1$. Using Theorem 2(iv), her preference is then *aa-a*.

In the **UU** case, we have $v' \leq 0$ and $A \geq 0$. If the DM prefers to combine good with bad on x, we have $v^{(M)} \leq 0$ for all $M \geq 1$. This implies $u^{(0,N)} = A^N v^{(N)} \leq 0$ for all $N \geq 1$ so that she prefers to combine good with bad on y. We obtain $u^{(M,N)} = A^N v^{(M+N)} \leq 0$ for all $M, N \geq 1$ so that she prefers to combine good with bad across attributes, see Theorem 3(i). Her preference is *dd-d*. If she prefers to combine good with good and bad with bad on x instead, we have $(-1)^{M+1}v^{(M)} \leq 0$ for all $M \geq 1$. This implies $(-1)^{N+1}u^{(0,N)} = A^N(-1)^{N+1}v^{(N)} \leq 0$ for all $N \geq 1$ and $(-1)^{M+N+1}u^{(M,N)} = A^N(-1)^{M+N+1}v^{(M+N)} \leq 0$. According to Theorem 3(iv), the DM's preference is then *aa-a*.

In either one of the three cases, we either find *dd-d* or *aa-a* for monetary equivalent utility $u(x, y) = v(x + Ay)$ with a constant marginal rate of substitution between attributes.

order	***dd-d* and *dd-a***	***dd-d***	***dd-a***
$M+N=2$	$u^{(2,0)} \leq 0, u^{(0,2)} \leq 0$	$u^{(1,1)} \leq 0$	$u^{(1,1)} \geq 0$
$M+N=3$	$u^{(3,0)} \geq 0, u^{(0,3)} \geq 0$	$u^{(2,1)} \geq 0,\ u^{(1,2)} \geq 0$	$u^{(2,1)} \leq 0,\ u^{(1,2)} \leq 0$
$M+N=4$	$u^{(4,0)} \leq 0, u^{(0,4)} \leq 0$	$u^{(3,1)} \leq 0,\ u^{(2,2)} \leq 0,\ u^{(1,3)} \leq 0$	$u^{(3,1)} \geq 0,\ u^{(2,2)} \geq 0,\ u^{(1,3)} \geq 0$

order	***da-d* and *da-a***	***da-d***	***da-a***
$M+N=2$	$u^{(2,0)} \leq 0, u^{(0,2)} \geq 0$	$u^{(1,1)} \leq 0$	$u^{(1,1)} \geq 0$
$M+N=3$	$u^{(3,0)} \geq 0, u^{(0,3)} \geq 0$	$u^{(2,1)} \geq 0,\ u^{(1,2)} \leq 0$	$u^{(2,1)} \leq 0,\ u^{(1,2)} \geq 0$
$M+N=4$	$u^{(4,0)} \leq 0, u^{(0,4)} \geq 0$	$u^{(3,1)} \leq 0,\ u^{(2,2)} \geq 0,\ u^{(1,3)} \leq 0$	$u^{(3,1)} \geq 0,\ u^{(2,2)} \leq 0,\ u^{(1,3)} \geq 0$

order	***ad-d* and *ad-a***	***ad-d***	***ad-a***
$M+N=2$	$u^{(2,0)} \geq 0, u^{(0,2)} \leq 0$	$u^{(1,1)} \leq 0$	$u^{(1,1)} \geq 0$
$M+N=3$	$u^{(3,0)} \geq 0, u^{(0,3)} \geq 0$	$u^{(2,1)} \leq 0,\ u^{(1,2)} \geq 0$	$u^{(2,1)} \geq 0,\ u^{(1,2)} \leq 0$
$M+N=4$	$u^{(4,0)} \geq 0, u^{(0,4)} \leq 0$	$u^{(3,1)} \leq 0,\ u^{(2,2)} \geq 0,\ u^{(1,3)} \leq 0$	$u^{(3,1)} \geq 0,\ u^{(2,2)} \leq 0,\ u^{(1,3)} \geq 0$

order	***aa-d* and *aa-a***	***aa-d***	***aa-a***
$M+N=2$	$u^{(2,0)} \geq 0, u^{(0,2)} \geq 0$	$u^{(1,1)} \leq 0$	$u^{(1,1)} \geq 0$
$M+N=3$	$u^{(3,0)} \geq 0, u^{(0,3)} \geq 0$	$u^{(2,1)} \leq 0,\ u^{(1,2)} \leq 0$	$u^{(2,1)} \geq 0,\ u^{(1,2)} \geq 0$
$M+N=4$	$u^{(4,0)} \geq 0, u^{(0,4)} \geq 0$	$u^{(3,1)} \leq 0,\ u^{(2,2)} \leq 0,\ u^{(1,3)} \leq 0$	$u^{(3,1)} \geq 0,\ u^{(2,2)} \geq 0,\ u^{(1,3)} \geq 0$

Table 1: All signs up to order 4 for the **DD** case with two desirable attributes x and y, $u^{(1,0)} \geq 0$ and $u^{(0,1)} \geq 0$. Our classification distinguishes whether the DM prefers to disaggregate (in short: d) or aggregate (in short: a) harms on the first attribute (first letter), on the second attribute (second letter), and across attributes (third letter). Correlation aversion, cross-prudence in x and y, and cross-temperance are highlighted in blue, correlation loving, cross-imprudence in x and y, and cross-intemperance are highlighted in green. The signs are collected in Proposition 1.

order	***dd-d* and *dd-a***	***dd-d***	***dd-a***
$M+N=2$	$u^{(2,0)} \leq 0, u^{(0,2)} \leq 0$	$u^{(1,1)} \geq 0$	$u^{(1,1)} \leq 0$
$M+N=3$	$u^{(3,0)} \geq 0, u^{(0,3)} \leq 0$	$u^{(2,1)} \leq 0$, $u^{(1,2)} \geq 0$	$u^{(2,1)} \geq 0$, $u^{(1,2)} \leq 0$
$M+N=4$	$u^{(4,0)} \leq 0, u^{(0,4)} \leq 0$	$u^{(3,1)} \geq 0$, $u^{(2,2)} \leq 0$, $u^{(1,3)} \geq 0$	$u^{(3,1)} \leq 0$, $u^{(2,2)} \geq 0$, $u^{(1,3)} \leq 0$
order	***da-d* and *da-a***	***da-d***	***da-a***
$M+N=2$	$u^{(2,0)} \leq 0, u^{(0,2)} \geq 0$	$u^{(1,1)} \geq 0$	$u^{(1,1)} \leq 0$
$M+N=3$	$u^{(3,0)} \geq 0, u^{(0,3)} \leq 0$	$u^{(2,1)} \leq 0$, $u^{(1,2)} \leq 0$	$u^{(2,1)} \geq 0$, $u^{(1,2)} \geq 0$
$M+N=4$	$u^{(4,0)} \leq 0, u^{(0,4)} \geq 0$	$u^{(3,1)} \geq 0$, $u^{(2,2)} \geq 0$, $u^{(1,3)} \geq 0$	$u^{(3,1)} \leq 0$, $u^{(2,2)} \leq 0$, $u^{(1,3)} \leq 0$
order	***ad-d* and *ad-a***	***ad-d***	***ad-a***
$M+N=2$	$u^{(2,0)} \geq 0, u^{(0,2)} \leq 0$	$u^{(1,1)} \geq 0$	$u^{(1,1)} \leq 0$
$M+N=3$	$u^{(3,0)} \geq 0, u^{(0,3)} \leq 0$	$u^{(2,1)} \geq 0$, $u^{(1,2)} \geq 0$	$u^{(2,1)} \leq 0$, $u^{(1,2)} \leq 0$
$M+N=4$	$u^{(4,0)} \geq 0, u^{(0,4)} \leq 0$	$u^{(3,1)} \geq 0$, $u^{(2,2)} \geq 0$, $u^{(1,3)} \geq 0$	$u^{(3,1)} \leq 0$, $u^{(2,2)} \leq 0$, $u^{(1,3)} \leq 0$
order	***aa-d* and *aa-a***	***aa-d***	***aa-a***
$M+N=2$	$u^{(2,0)} \geq 0, u^{(0,2)} \geq 0$	$u^{(1,1)} \geq 0$	$u^{(1,1)} \leq 0$
$M+N=3$	$u^{(3,0)} \geq 0, u^{(0,3)} \leq 0$	$u^{(2,1)} \geq 0$, $u^{(1,2)} \leq 0$	$u^{(2,1)} \leq 0$, $u^{(1,2)} \geq 0$
$M+N=4$	$u^{(4,0)} \geq 0, u^{(0,4)} \geq 0$	$u^{(3,1)} \geq 0$, $u^{(2,2)} \leq 0$, $u^{(1,3)} \geq 0$	$u^{(3,1)} \leq 0$, $u^{(2,2)} \geq 0$, $u^{(1,3)} \leq 0$

Table 2: All signs up to order 4 for **DU** with a desirable attribute x, $u^{(1,0)} \geq 0$, and an undesirable attribute y, $u^{(0,1)} \leq 0$. Our classification distinguishes whether the DM prefers to disaggregate (in short: d) or aggregate (in short: a) harms on the first attribute (first letter), on the second attribute (second letter), and across attributes (third letter). Correlation aversion, cross-prudence in x and y, and cross-temperance are highlighted in blue, correlation loving, cross-imprudence in x and y, and cross-intemperance are highlighted in green. The signs are collected in Proposition 2.

order	*dd-d* and *dd-a*	*dd-d*	*dd-a*
$M+N=2$	$u^{(2,0)} \leq 0, u^{(0,2)} \leq 0$	$u^{(1,1)} \leq 0$	$u^{(1,1)} \geq 0$
$M+N=3$	$u^{(3,0)} \leq 0, u^{(0,3)} \leq 0$	$u^{(2,1)} \leq 0,\ u^{(1,2)} \leq 0$	$u^{(2,1)} \geq 0,\ u^{(1,2)} \geq 0$
$M+N=4$	$u^{(4,0)} \leq 0, u^{(0,4)} \leq 0$	$u^{(3,1)} \leq 0,\ u^{(2,2)} \leq 0,\ u^{(1,3)} \leq 0$	$u^{(3,1)} \geq 0,\ u^{(2,2)} \geq 0,\ u^{(1,3)} \geq 0$

order	*da-d* and *da-a*	*da-d*	*da-a*
$M+N=2$	$u^{(2,0)} \leq 0, u^{(0,2)} \geq 0$	$u^{(1,1)} \leq 0$	$u^{(1,1)} \geq 0$
$M+N=3$	$u^{(3,0)} \leq 0, u^{(0,3)} \leq 0$	$u^{(2,1)} \leq 0,\ u^{(1,2)} \geq 0$	$u^{(2,1)} \geq 0,\ u^{(1,2)} \leq 0$
$M+N=4$	$u^{(4,0)} \leq 0, u^{(0,4)} \geq 0$	$u^{(3,1)} \leq 0,\ u^{(2,2)} \geq 0,\ u^{(1,3)} \leq 0$	$u^{(3,1)} \geq 0,\ u^{(2,2)} \leq 0,\ u^{(1,3)} \geq 0$

order	*ad-d* and *ad-a*	*ad-d*	*ad-a*
$M+N=2$	$u^{(2,0)} \geq 0, u^{(0,2)} \leq 0$	$u^{(1,1)} \leq 0$	$u^{(1,1)} \geq 0$
$M+N=3$	$u^{(3,0)} \leq 0, u^{(0,3)} \leq 0$	$u^{(2,1)} \geq 0,\ u^{(1,2)} \leq 0$	$u^{(2,1)} \leq 0,\ u^{(1,2)} \geq 0$
$M+N=4$	$u^{(4,0)} \geq 0, u^{(0,4)} \leq 0$	$u^{(3,1)} \leq 0,\ u^{(2,2)} \geq 0,\ u^{(1,3)} \leq 0$	$u^{(3,1)} \geq 0,\ u^{(2,2)} \leq 0,\ u^{(1,3)} \geq 0$

order	*aa-d* and *aa-a*	*aa-d*	*aa-a*
$M+N=2$	$u^{(2,0)} \geq 0, u^{(0,2)} \geq 0$	$u^{(1,1)} \leq 0$	$u^{(1,1)} \geq 0$
$M+N=3$	$u^{(3,0)} \leq 0, u^{(0,3)} \leq 0$	$u^{(2,1)} \geq 0,\ u^{(1,2)} \geq 0$	$u^{(2,1)} \leq 0,\ u^{(1,2)} \leq 0$
$M+N=4$	$u^{(4,0)} \geq 0, u^{(0,4)} \geq 0$	$u^{(3,1)} \leq 0,\ u^{(2,2)} \leq 0,\ u^{(1,3)} \leq 0$	$u^{(3,1)} \geq 0,\ u^{(2,2)} \geq 0,\ u^{(1,3)} \geq 0$

Table 3: All signs up to order 4 for **UU** with two undesirable attributes x and y, $u^{(1,0)} \geq 0$ and $u^{(0,1)} \geq 0$. Our classification distinguishes whether the DM prefers to disaggregate (in short: d) or aggregate (in short: a) harms on the first attribute (first letter), on the second attribute (second letter), and across attributes (third letter). Correlation aversion, cross-prudence in x and y, and cross-temperance are highlighted in blue, correlation loving, cross-imprudence in x and y, and cross-intemperance are highlighted in green. The signs are collected in Proposition 3.

Case	app. pref.	(M,N)-deg. risk att.	app. pref.	(M,N)-deg. risk att.	condition
DD	*dd-d*	averse	*dd-a*	loving	
	da-d	averse loving	*da-a*	loving averse	if N odd if N even
	ad-d	averse loving	*ad-a*	loving averse	if M odd if M even
	aa-d	loving averse	*aa-a*	averse loving	if $M+N$ odd if $M+N$ even
DU	*dd-d*	loving averse	*dd-a*	averse loving	if N odd if N even
	da-d	loving	*da-a*	averse	
	ad-d	averse loving	*ad-a*	loving averse	if $M+N$ odd if $M+N$ even
	aa-d	loving averse	*aa-a*	averse loving	if M odd if M even
UU	*dd-d*	loving averse	*dd-a*	averse loving	if $M+N$ odd if $M+N$ even
	da-d	averse loving	*da-a*	loving averse	if M odd if M even
	ad-d	averse loving	*ad-a*	loving averse	if N odd if N even
	aa-d	averse	*aa-a*	loving	

Table 4: Attitudes towards an increase in the (M,N)-degree riskiness of (X,Y) organized by the DM's apportionment preference. The table distinguishes the different combinations of apportionment preferences, states the implied (M,N)-degree risk attitude, and provides a condition if required. The first, second and third panel summarizes the results of Corollary 1, 2 and 3 for the case **DD**, **DU** and **UU**, respectively.

B Construction of multiplicatively separable utility functions with desired risk apportionment preferences

Table 5 shows how to construct any of the eight combinations of apportionment preferences (rows) in any of the three cases (columns) when the utility function is multiplicatively separable, $u(x, y) = v(x)z(y)$. The acronym "mra" stands for mixed risk-averse, "mrl" for mixed risk-loving, "amra" for anti-mixed risk-averse, and "amrl" for an anti-mixed risk-loving. Each cell contains two possibilities depending on the signs of the factor utility functions v and z.

	DD	**DU**	**UU**
dd-d	$v > 0$ amra & $z < 0$ mra $v < 0$ mra & $z > 0$ amra	$v > 0$ amra & $z < 0$ amrl $v < 0$ mra & $z > 0$ mrl	$v > 0$ mrl & $z < 0$ amrl $v < 0$ amrl & $z > 0$ mrl
dd-a	$v > 0$ mra & $z > 0$ mra $v < 0$ amra & $z < 0$ amra	$v > 0$ mra & $z > 0$ amrl $v < 0$ amra & $z < 0$ mrl	$v > 0$ amrl & $z > 0$ amrl $v < 0$ mrl & $z < 0$ mrl
da-d	$v > 0$ amra & $z < 0$ mrl $v < 0$ mra & $z > 0$ amrl	$v > 0$ amra & $z < 0$ amra $v < 0$ mra & $z > 0$ mra	$v > 0$ mrl & $z < 0$ amra $v < 0$ amrl & $z > 0$ mra
da-a	$v > 0$ mra & $z > 0$ mrl $v < 0$ amra & $z < 0$ amrl	$v > 0$ mra & $z > 0$ amra $v < 0$ amra & $z < 0$ mra	$v > 0$ amrl & $z > 0$ amra $v < 0$ mrl & $z < 0$ mra
ad-d	$v > 0$ amrl & $z < 0$ mra $v < 0$ mrl & $z > 0$ amra	$v > 0$ amrl & $z < 0$ amrl $v < 0$ mrl & $z > 0$ mrl	$v > 0$ mra & $z < 0$ amrl $v < 0$ amra & $z > 0$ mrl
ad-a	$v > 0$ mrl & $z > 0$ mra $v < 0$ amrl & $z < 0$ amra	$v > 0$ mrl & $z > 0$ amrl $v < 0$ amrl & $z < 0$ mrl	$v > 0$ amra & $z > 0$ amrl $v < 0$ mra & $z < 0$ mrl
aa-d	$v > 0$ amrl & $z < 0$ mrl $v < 0$ mrl & $z > 0$ amrl	$v > 0$ amrl & $z < 0$ amra $v < 0$ mrl & $z > 0$ mra	$v > 0$ mra & $z < 0$ amra $v < 0$ amra & $z > 0$ mra
aa-a	$v > 0$ mrl & $z > 0$ mrl $v < 0$ amrl & $z < 0$ amrl	$v > 0$ mrl & $z > 0$ amra $v < 0$ amrl & $z < 0$ mra	$v > 0$ amra & $z > 0$ amra $v < 0$ mra & $z < 0$ mra

Table 5: Construction of multiplicatively separable utility functions with desired apportionment preferences in the three cases. The acronyms "mra", "mrl", "amra" and "amrl" abbreviate mixed risk-averse, mixed risk-loving, anti-mixed risk-averse and anti-mixed risk-loving utility functions, respectively. Together with the sign of v and z, this establishes the univariate apportionment preferences, see Section 2.3. The apportionment preference across attributes follows from the alignment or misalignment of the signs of v and z, see Proposition 5.