Mortality Regularities in a Dependent Competing-Risk Setting

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CPop Competing-Risk Models as Mixtures

r.v.	T_1	T_2	$\dots$	T_n	– not necessarily independent
hazard	$h_1(x)$	$h_2(x)$	$\dots$	$h_n(x)$	– in the absence of other
p.d.f.	$f_1(x)$	$f_2(x)$	$\dots$	$f_n(x)$	competing risks

$\Updownarrow$ ▸ References

r.v.	$T = \min\{T_1, \dots, T_n\}$	– actual observed lifetime
hazard	$h(x) = h_1(x) + \dots + h_n(x)$	– additive risks
p.d.f.	$\boxed{f(x) = \sum_{i=1}^{n} \pi_i \, g_i(x)}$	$g_i(x) \neq f_i(x)$

$g_i(x)$ and π_i are, repspectively, the p.d.f. and prevalence of deaths from i-th risk, $i = 1, \dots, n$, in the presence of all others.

Additive-Risk Models as Mixtures: Benefits

- All additive-risk models are competing-risk models, whereby the competing risks are not necessarily independent
- Estimating the model, one can compute $g_i(x)$ and π_i to calculate the share of deaths from i-th risk (in the presence of all others) at every age x
- For specific functional forms of $h_1(x), \ldots, h_n(x)$, calculate different mortality indicators such as (remaining) life expectancy, the modal age of death, etc.

Patricio, S.C. and Missov, T.I. (2024). Makeham Mortality Models as Mixtures. *Demographic Research* (forthcoming). Preprint: https://arxiv.org/abs/2304.08920

Parametric Additive-Risk Mortality Models

All Makeham parametric mortality models are additive:

$$h(x) = \mu(x) + c$$

$$\begin{aligned}
\mu(x) &= ae^{bx} && \text{Gompertz} \\
\mu(x) &= \frac{ae^{bx}}{1 + \frac{a\gamma}{b}(e^{bx} - 1)} && \text{gamma-Gompertz} \\
\mu(x) &= \frac{ae^{bx}}{1 + kae^{bx}} && \text{Beard} \\
\mu(x) &= \frac{ae^{bx}}{1 + ae^{bx}} && \text{Kannisto} \\
\mu(x) &= a_1 e^{-b_1 x} + ae^{bx} && \text{Siler}
\end{aligned}$$

Death occurs either as a result of biological processes at early or late ages, or due to extrinsic risk c, whatever strikes first

Extending the Makeham Model

Predecessor: κ-Gompertz model (Vaupel and Wisser 2015):

$$\mu(x) = ae^{bx} + ce^{(b-\kappa)x}$$

Our extension:

$$h(x) = h_1(x) + h_2(x) + h_3(x)$$

- senescent: $h_1(x) = \frac{ae^{bx}}{1+\frac{a\gamma}{b}(e^{bx}-1)}$
  aging-related hazard
- behavior-related: $h_2(x) = \eta\, h_1(x) S(x)$
  age-decreasing inclination to act risky, captured by a survival function $S(x)$, interacting with age-increasing incurring damage (η is a scaling factor)
- external: $h_3(x) = c$
  non-aging-related hazard

Estimation Procedure

- Input: death counts $D(x)$ and exposures $E(x)$ for $x \geq 10$
- Assumption: $D(x) \sim \text{Poisson}(h(x)E(x))$
- Use a Bayesian procedure with inverse-gamma priors (and gamma hyper-priors) for the parameters (Patricio and Missov 2023)
  - Rationale: model is over-parameterized to apply standard ML + ML-estimators of parameters are highly correlated
- First step: choose appropriate $S(x)$
  - Assume gamma, Rayleigh, log-normal, and **skew normal** $S(x)$
  - Compare the goodness of fit by MSE, RMSE, RMSLE, RAE, MAE, MAPE, MedianAPE
- Estimate the three parts of mortality

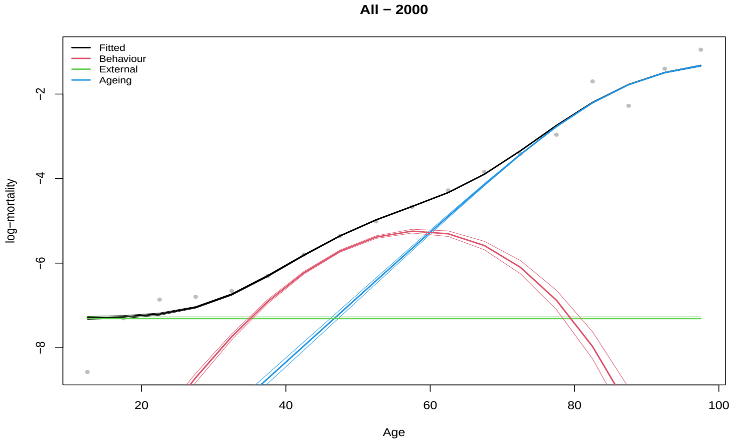
Fitting the Three-Component Model to COD Data

- France, females, years 2000–2015
- Human Cause-of-Death Database

All COD

Overall Mortality: Composition by Age

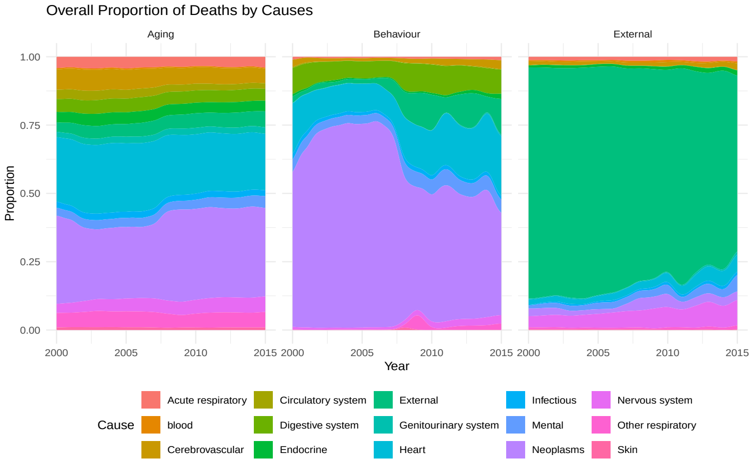
All COD



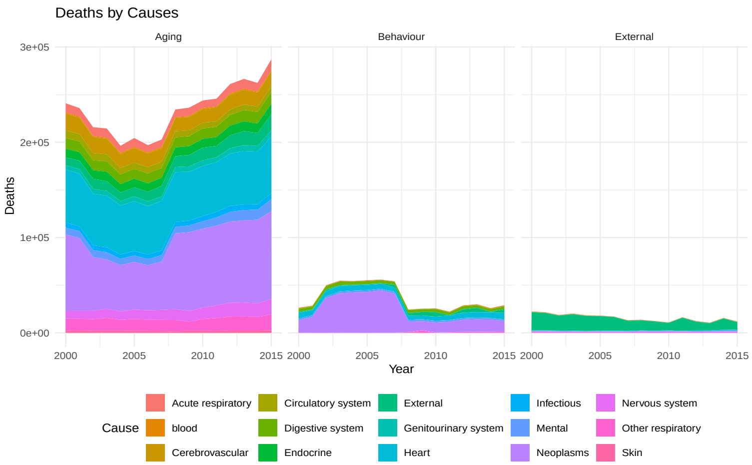
COD-specific Mortality: Convergence of MCMC

All	1.0000	Cerebrovascular	1.0000
Infectious	1.0000	Circulatory system	1.0000
Neoplasms	1.0000	Acute respiratory	1.0000
Blood	1.0000	Other respiratory	0.9375
Endocrine	0.9375	Digestive system	1.0000
Mental	1.0000	Skin	0.8750
Nervous system	1.0000	Genitourinary system	0.8750
Heart	1.0000	External	0.8125

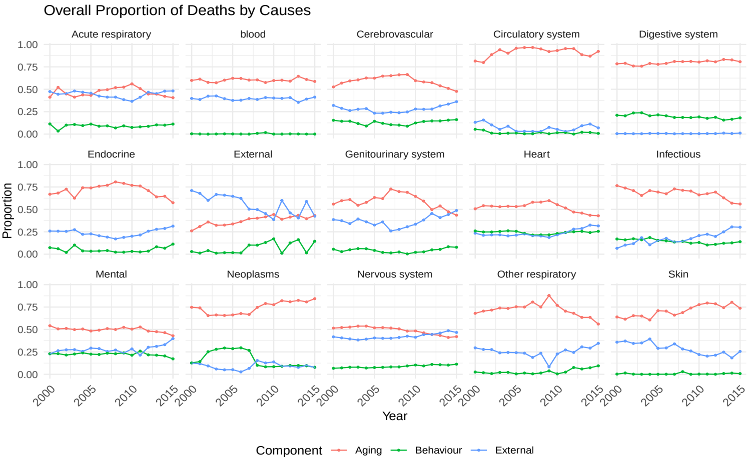
COD-share by Components at Different Ages



COD-deaths by Components at Different Ages



Component-share by COD at Different Ages



Further Steps and Concluding Remarks

- Competing-risk models are additive-risk models that can be represented as mixtures
- One can characterize the distribution of deaths for all subpopulations stratified by competing risks
- Estmating a three-component additive-risk model aids identifying age-specific regularities for COD
- Forecasting COD may be carried out component-wise in a CoDA setting.

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References: Competing Dependent Risks

Gail, M. (1975). A review and critique of some models used in competing risk analysis. *Biometrics* 31(1): 209–222.

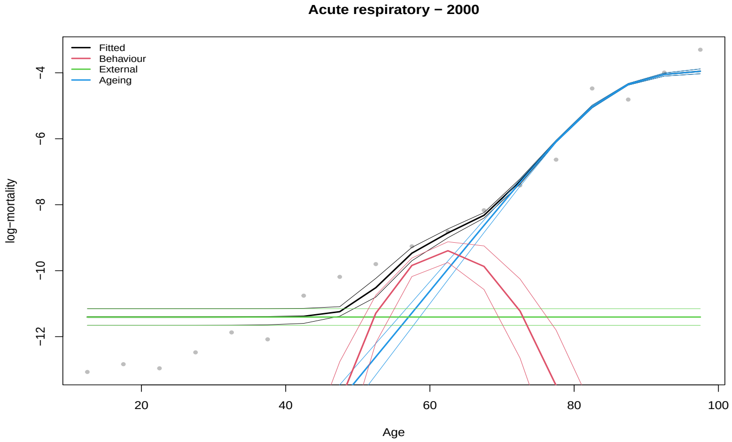
Elandt-Johnson, R.C. (1976). Conditional failure time distributions under competing risk theory with dependent failure times and proportional hazard rates. *Scandinavian Actuarial Journal* 1976(1): 37–51.

Hakulinen, T. and Rahiala, M. (1977). An example on the risk dependence and additivity of intensities in the theory of competing risks. *Biometrics* 33(3): 557–559.

▸ Competing Risks as Mixtures

Acute Respiratory: Composition by Age

back



Blood: Composition by Age

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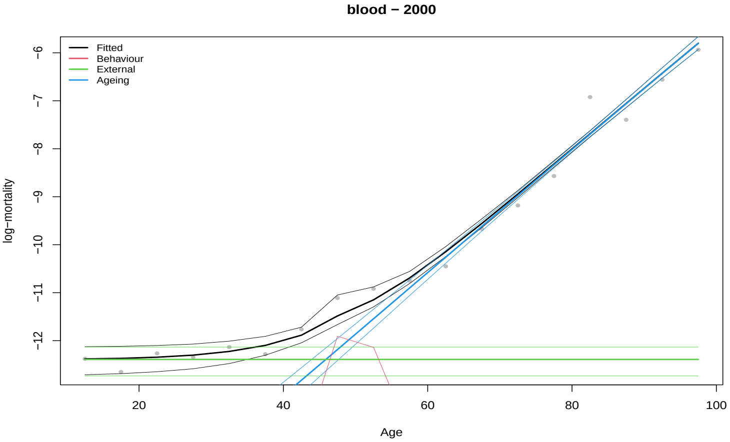

Cerebrovascular: Composition by Age

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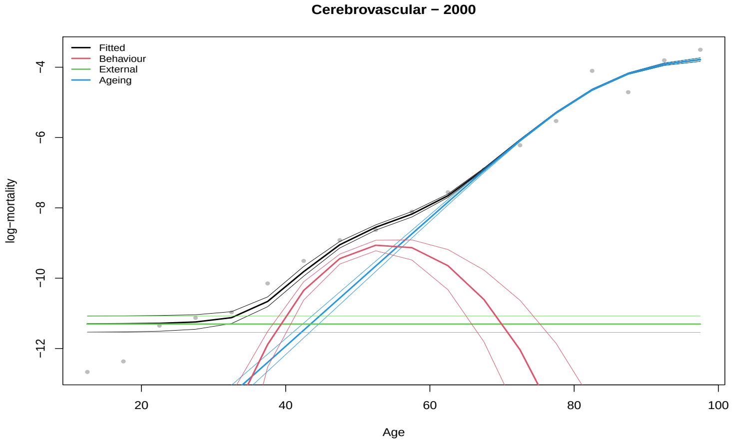
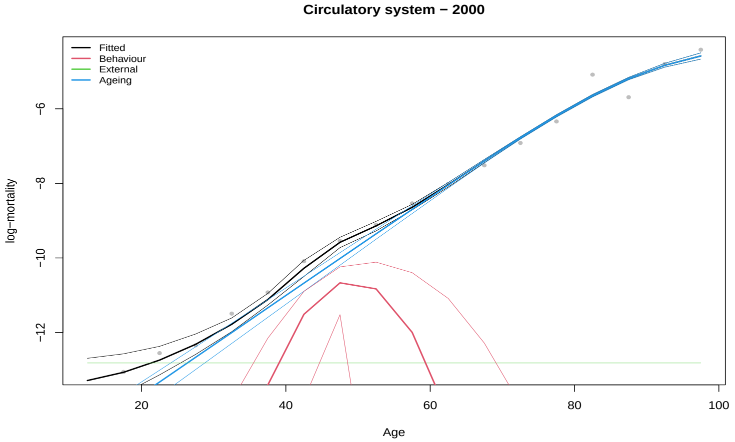


Circulatory system: Composition by Age ▸ back



Digestive system: Composition by Age

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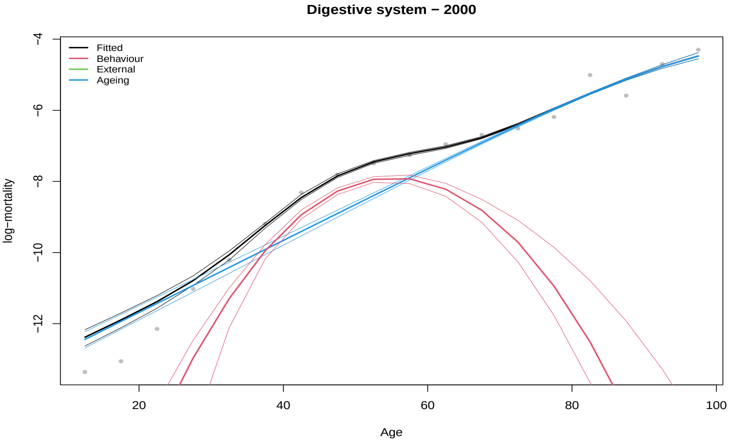

Endocrine: Composition by Age

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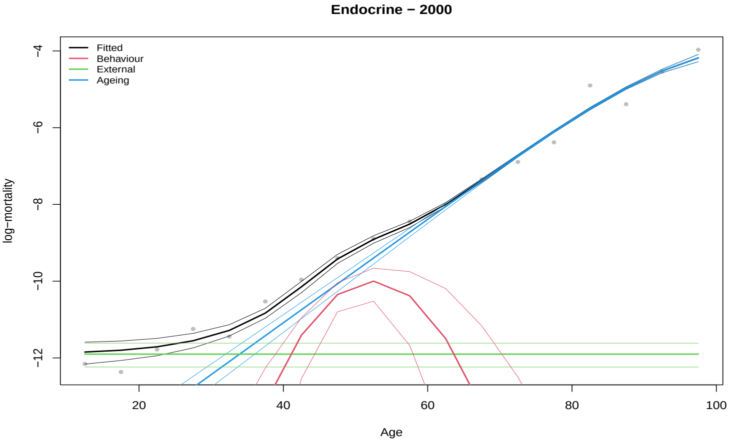

External: Composition by Age

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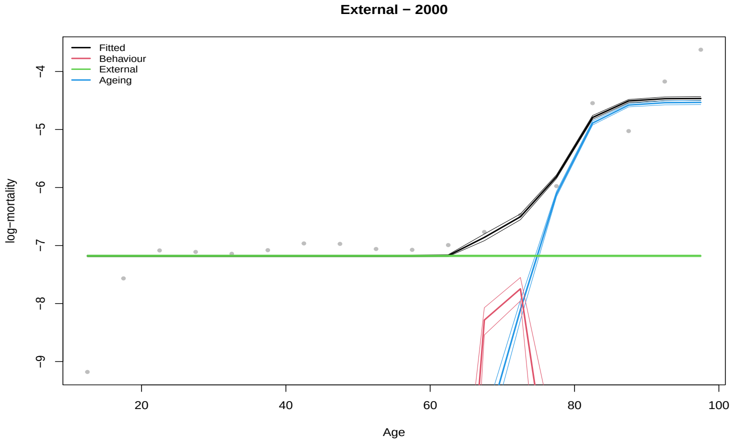

Genitourinary system: Composition by Age

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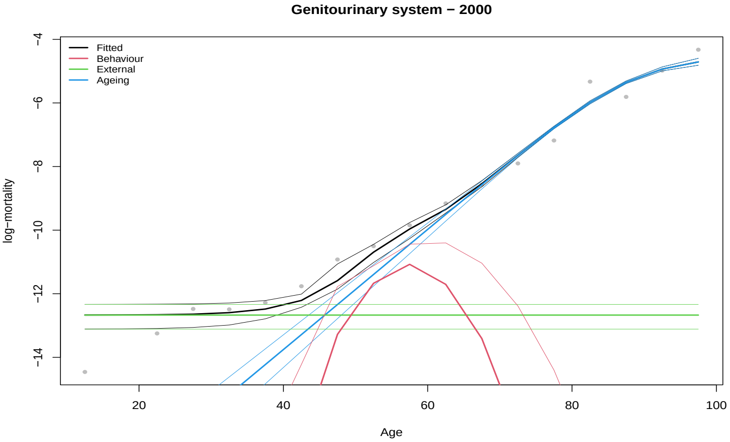

Heart: Composition by Age

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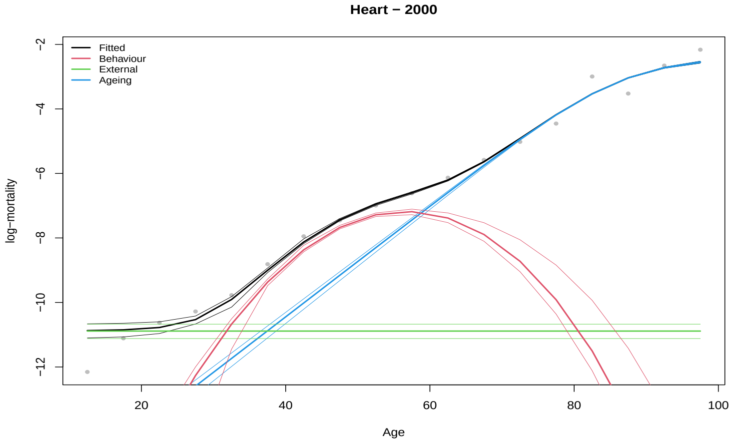

Infectious: Composition by Age

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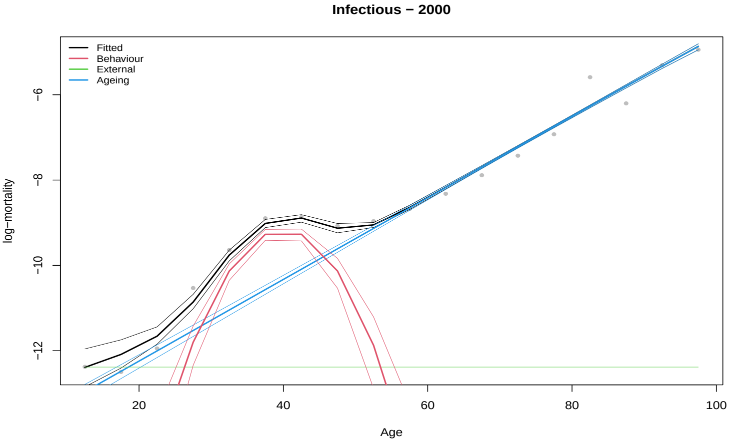

Mental: Composition by Age

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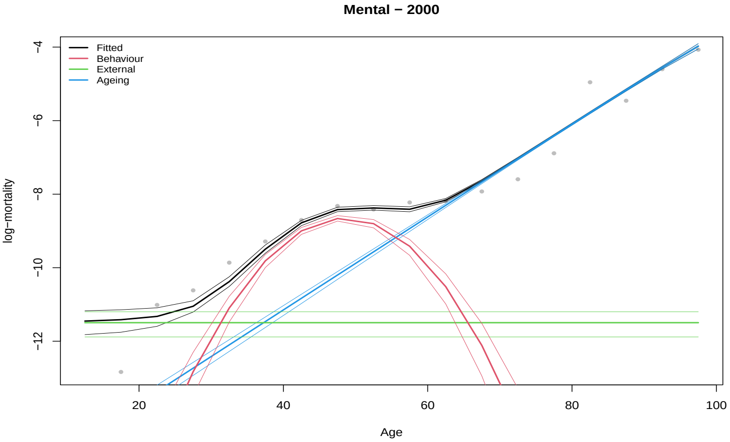

Neoplasms: Composition by Age

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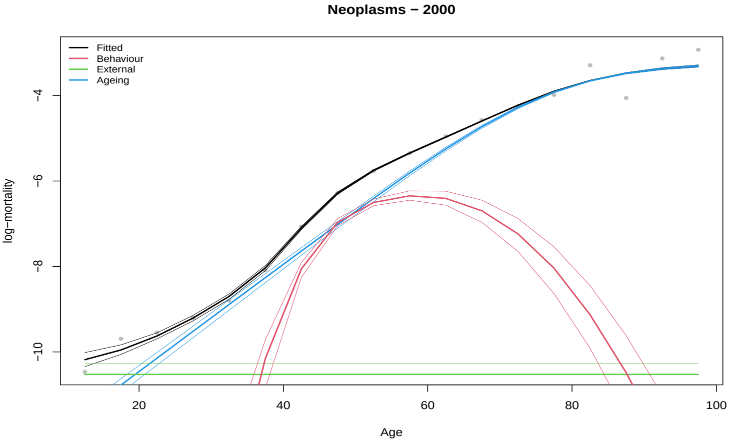

Nervous system: Composition by Age

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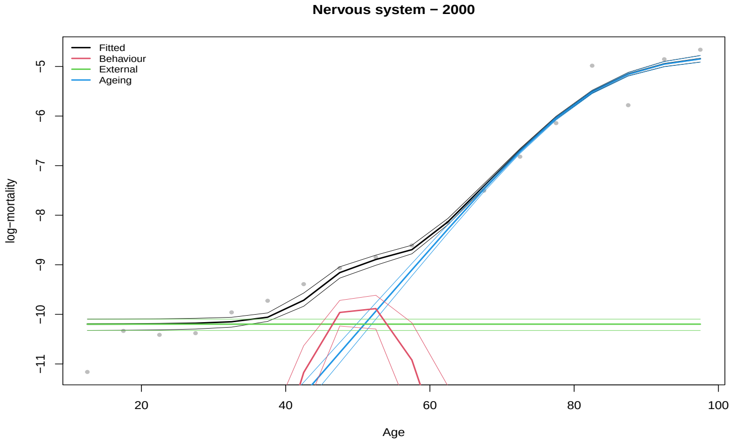

Other respiratory: Composition by Age

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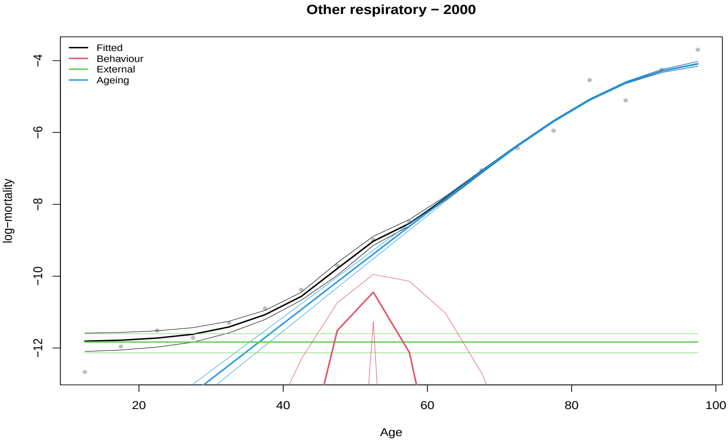

Skin: Composition by Age

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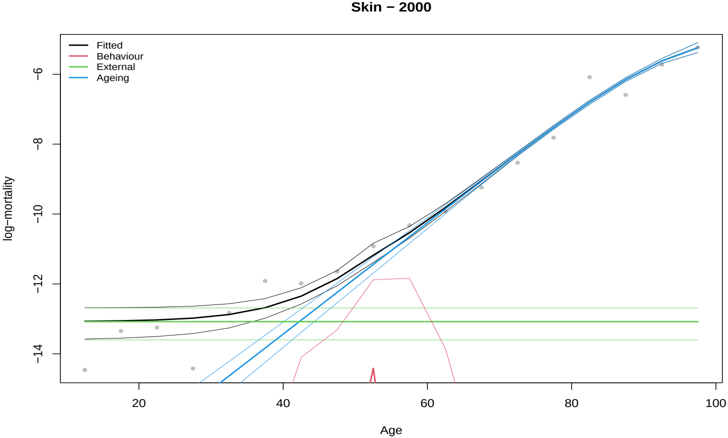
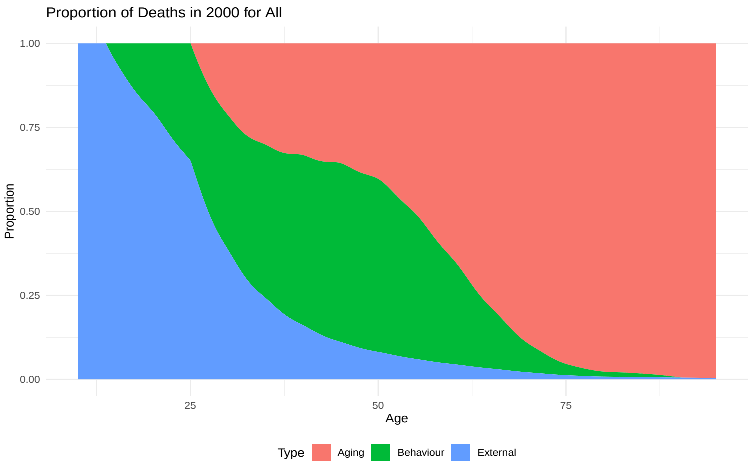


CPop COD-Overall Mortality: Composition by Age ▸ back



Overall Mortality: Composition by Age

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