Industry Life Cycles in General Equilibrium

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Motivation

- General purpose technologies (ICT, AI, ...) often trigger predictable industry life cycles (Klepper, 1996).
  1. **Initial phase:** Burst of entry, firms offering many different versions of the industry's product.
  2. **Middle phase:** Continued market growth but entry slows down, some firms exit.
  3. **Late phase:** Shakeout and consolidation, fewer product innovations, more process innovations.

- What are the macroeconomic implications?
  - Need a model where life-cycles emerge endogenously via product/process innovations.
  - Should industries be regulated after a technological revolution? If so, how?

What we do:

Provide a **quantitative macro model of industry life cycles**, which we use to...

(i) ... understand welfare implications of "GPT shocks"

(ii) ... study industrial policy in the aftermath of technological disruptions (e.g. ICTs, AI, ...)

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Roadmap

1. Theory: Overlapping Technologies + Oligopolistic Industries + Semi-Endogenous Growth

- Ingredients:
  1. Technological revolutions open up new innovation possibilities within an affected industry.
     - After the shock, a new generation of firms have to find their product through product innovation.
     - Once they have a product, they improve its efficiency via process innovation.
     - Fixed operating cost $\rightarrow$ Number of producing firms is endogenous and time-varying.
  2. Innovation decisions feature strategic interactions.

- Equilibrium: Shakeouts emerge when product innovation ("entry") costs $\ll$ process innovation costs.
  - Low entry costs imply burst of entry after the technological revolution.
  - Once some entrants are successful at process innovation, they push others out of the industry.
  - If process innovation costs are high enough, there's time for firm numbers to grow before the shakeout.

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2. Empirics: Aim to provide systematic evidence (complementing existing case studies)

- Interpret the ICT Revolution (mid-1980s) as a shock that generated new technological opportunities.
  - Compare dynamics (entry, exit, patenting) across industries with different exposure to the shock.
- Results: More exposed industries experience burst of entry, subsequent shakeout, process innovation $\uparrow$

3. Quantitative Analysis: Study optimal policy following a technological revolution

- Calibration:
  - Hit economy w/ shock that simultaneously disrupts multiple industries ("technological revolution").
  - Calibrate parameters so that transitional dynamics after the shock match those of the ICT shock.
- Policy analysis:
  - In calibrated BGP $\rightarrow$ Business-stealing externalities $\gg$ limited appropriability externalities.
  - In response to revolution $\rightarrow$ Optimal policy reaction is to subsidize entry and tax process innovation.

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Theory

A General-Equilibrium Model of Industry Life-Cycles

Environment

- **Preferences:** Over consumption of a single final good:

$$\max \int_0^{+\infty} e^{-\rho t} \ln(C_t)\, dt \qquad \text{s.t. } \dot{A}_t \leq r_t A_t + w_t L - C_t$$

- **Final good:** Continuum of industries, and each industry is populated by $\overline{N} < +\infty$ firms:

$$Y_t = \exp\left(\int_0^1 \ln\left(Y_{i,t}\right) di\right), \qquad \text{where } Y_{i,t} = \left(\sum_{n=1}^{\overline{N}} \left(y_{in,t}\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}, \ \varepsilon > 1$$

- **Technology:**
  - A firm $n = 1, \ldots, \overline{N}$ may not own a product (aka "potential entrant"), in which case $y_{in,t} = 0$.
  - If a firm owns a product (aka "incumbent"), it may produce it with productivity $q_{in,t}$ using labor $l_{in,t}$:

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  - To produce, an incumbent must pay a flow operating cost ϕY_t $\rightarrow$ Endogenous number of incumbents.

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Overlapping Technologies and Breakthroughs

- Two available technologies $\rightarrow$ **Traditional** (T) and **Modern** (M).
  - $\overline{N}/2$ firms are "traditional" (i.e. use only the T technology), and $\overline{N}/2$ firms are "modern".
- An industry experiences a **"technological breakthrough"** at an (exogenous) Poisson rate $a > 0$.
  1. ... the T technology becomes obsolete, and all of its users disappear with it.
  2. ... the old M technology becomes the new T technology.
  3. ... $\overline{N}/2$ firms are born who can use a new M technology $\rightarrow$ 1 as incumbent, $\overline{N}/2 - 1$ as pot. entrants.
- **Finite productivity ladders**:
  - Cohort of firms born with k-th revolution:

$$\mathcal{Q}^k \equiv \{0, q_1^k, q_2^k, \ldots, q_{j_{max}}^k\}, \quad \text{with } q_j^k = \lambda^{j-1} q_1^k,\ \lambda > 1$$

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- **Product innovation** (by potential entrants):
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The Life-Cycle of an Industry: Illustration

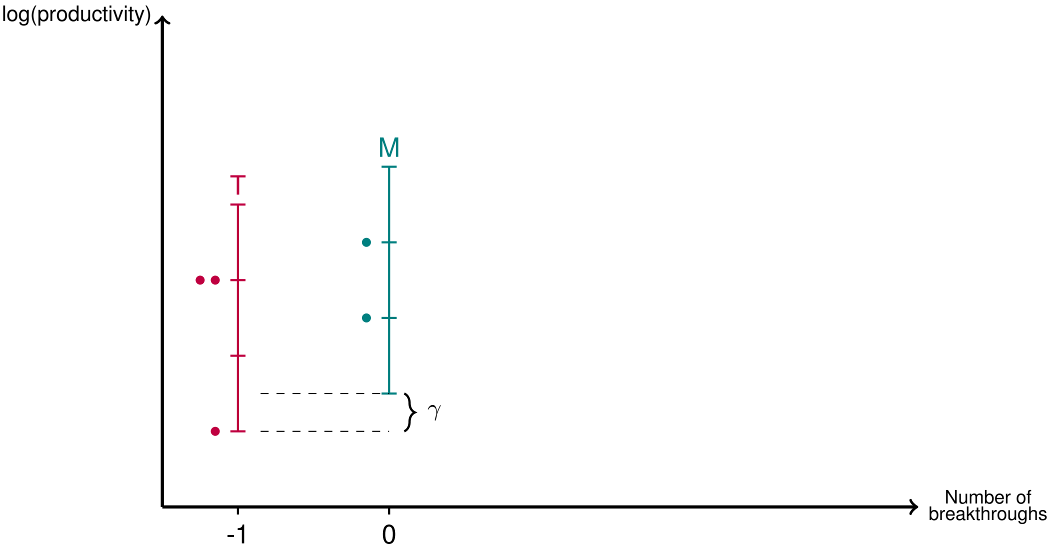

Figure: Overlapping generations of technologies within an industry as a function of the number of industry-level technological breakthroughs. *Note:* the ladders themselves need not overlap in the productivity space.

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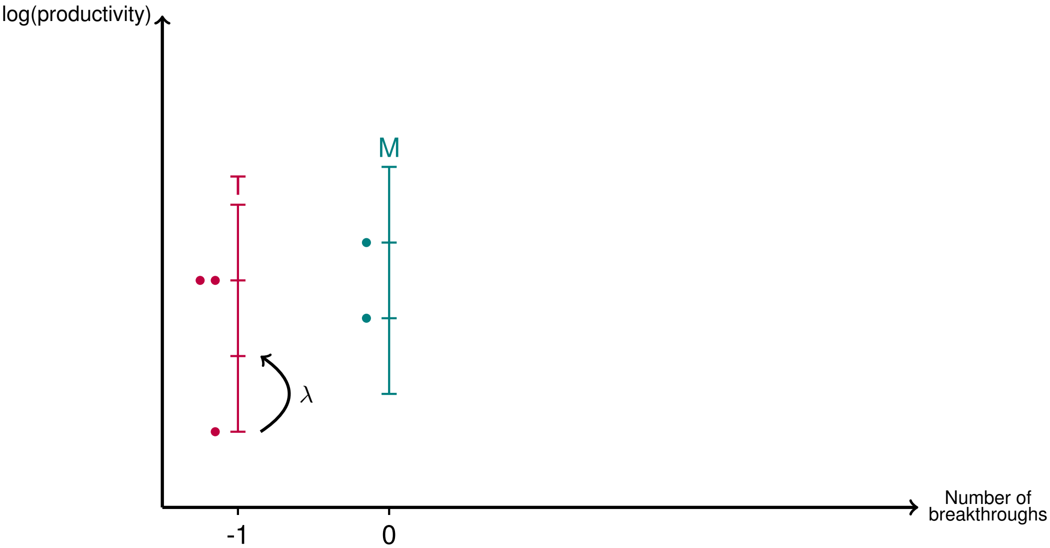

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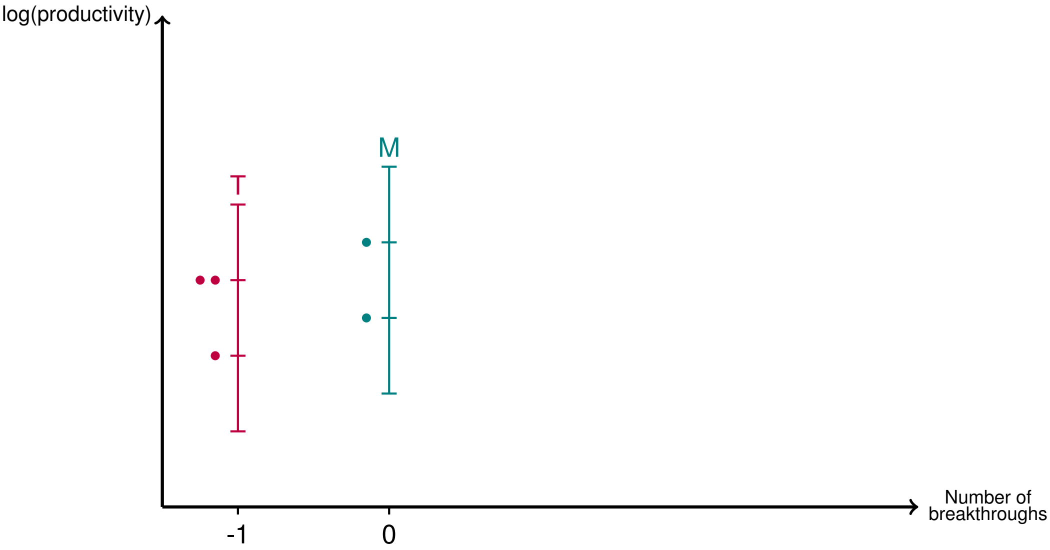

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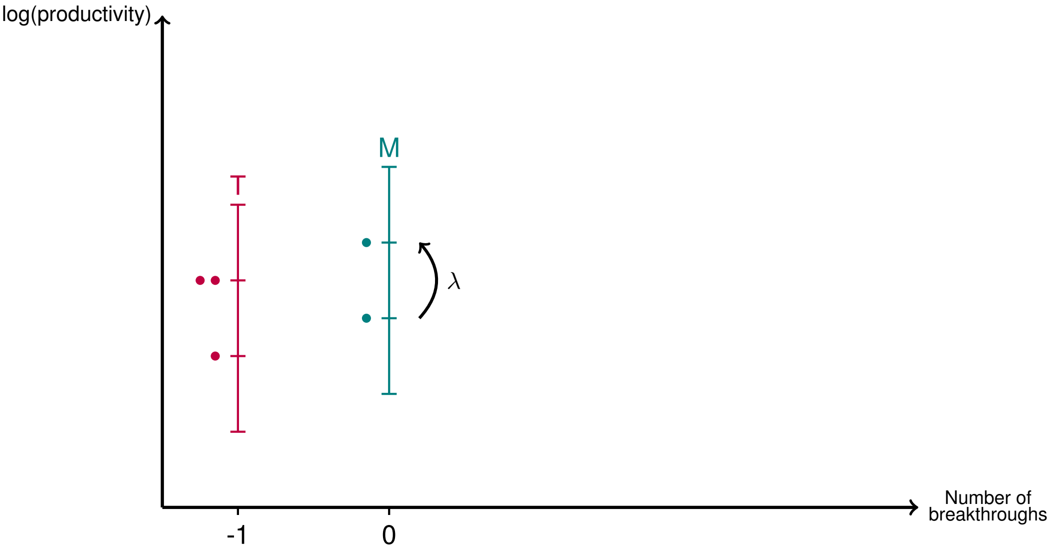

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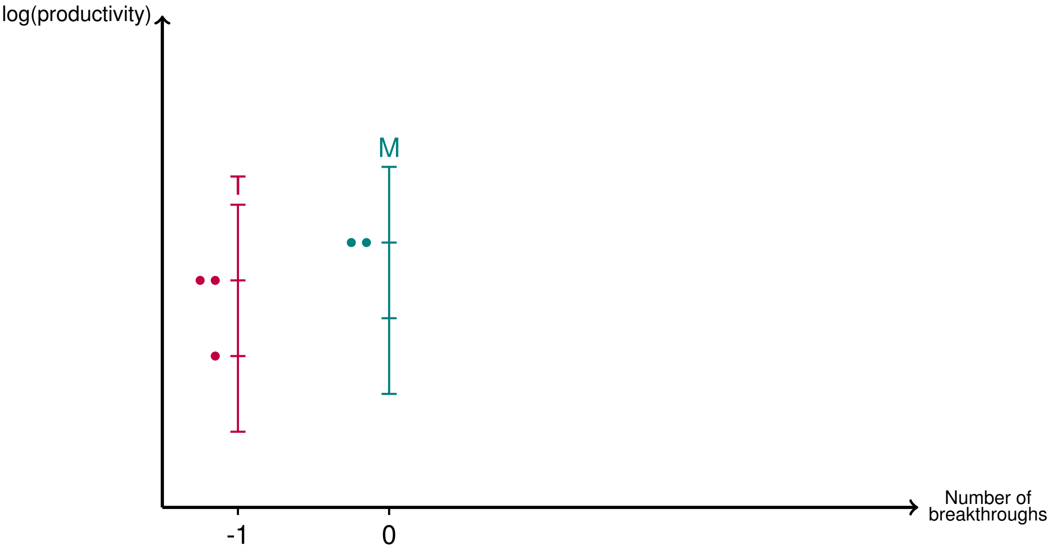

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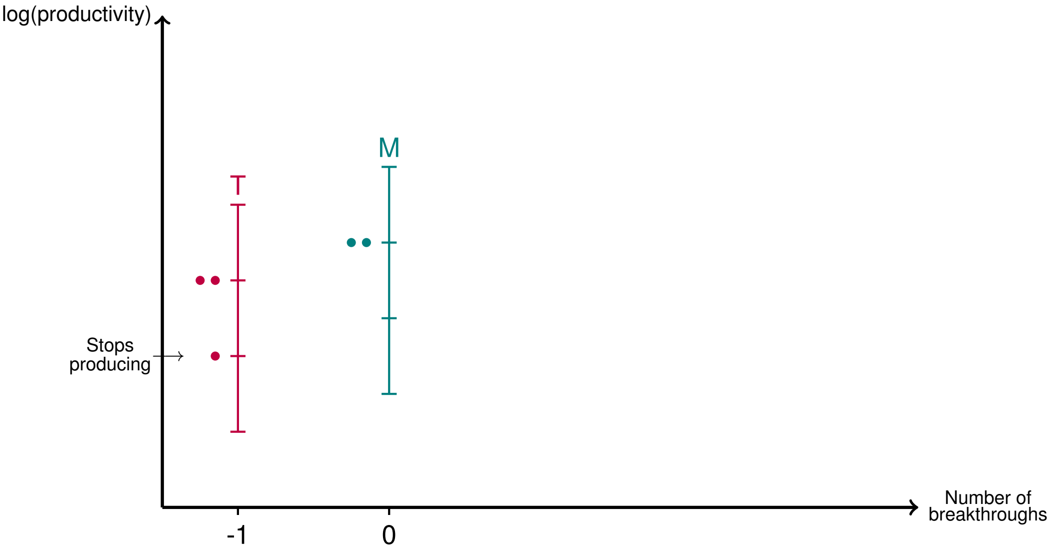

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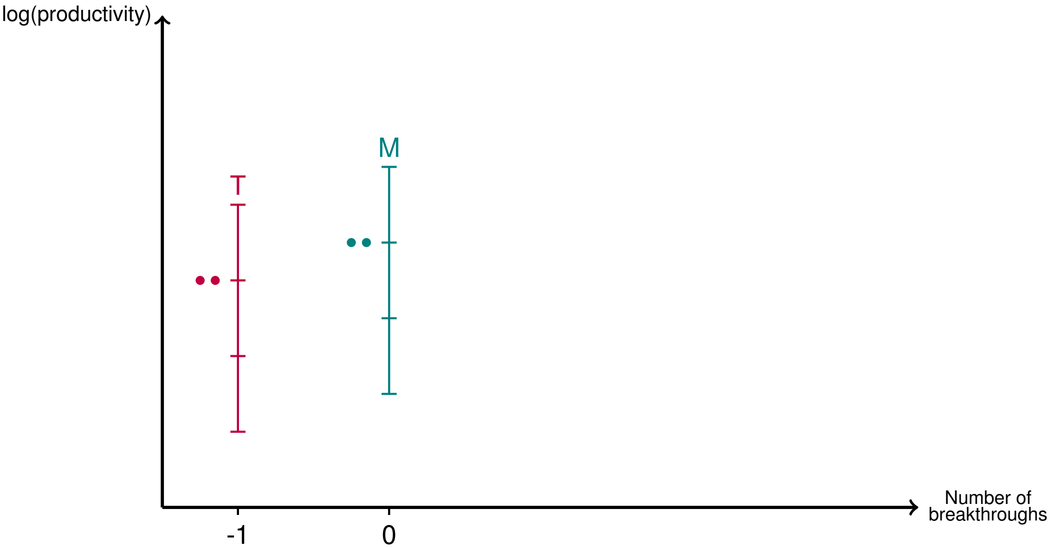

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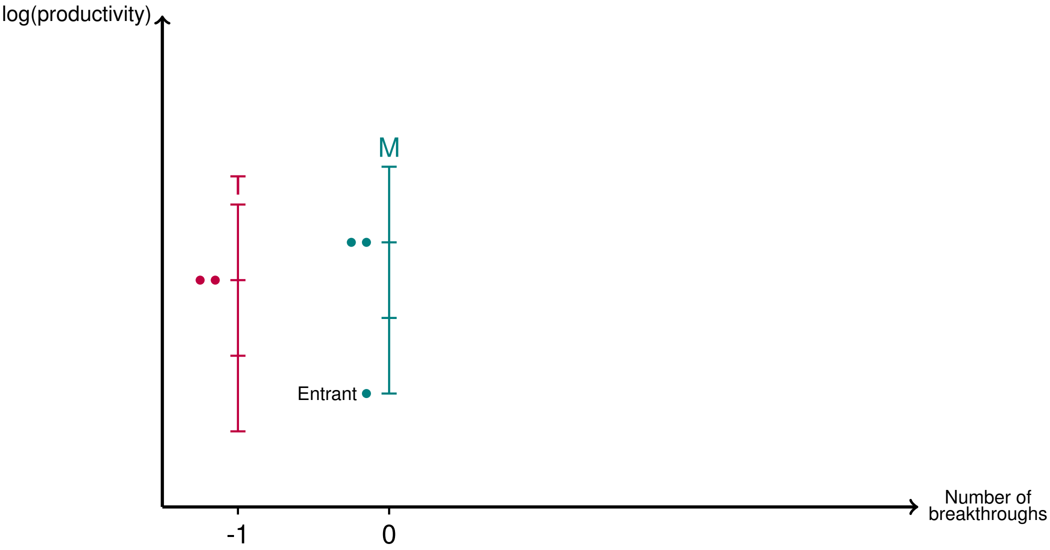

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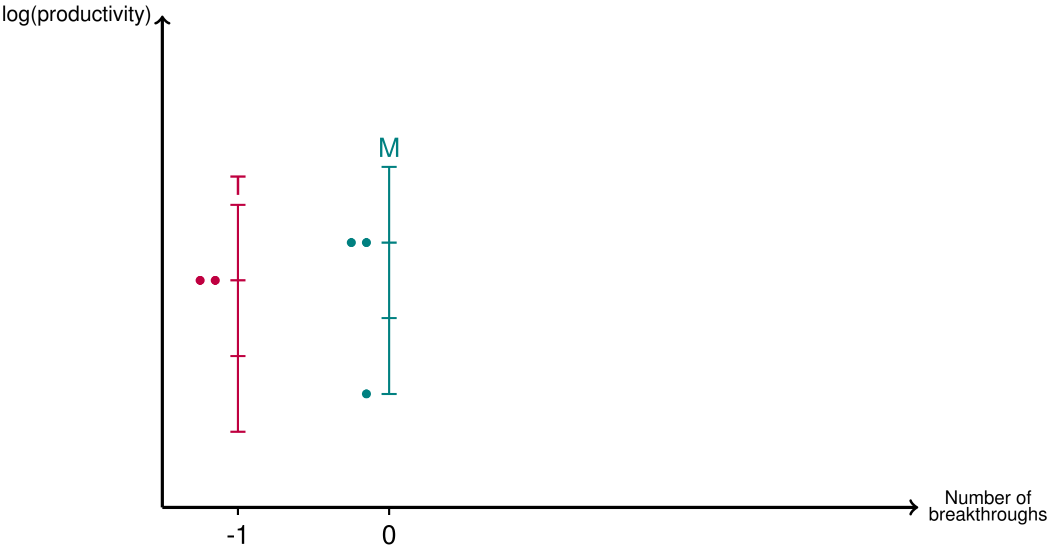

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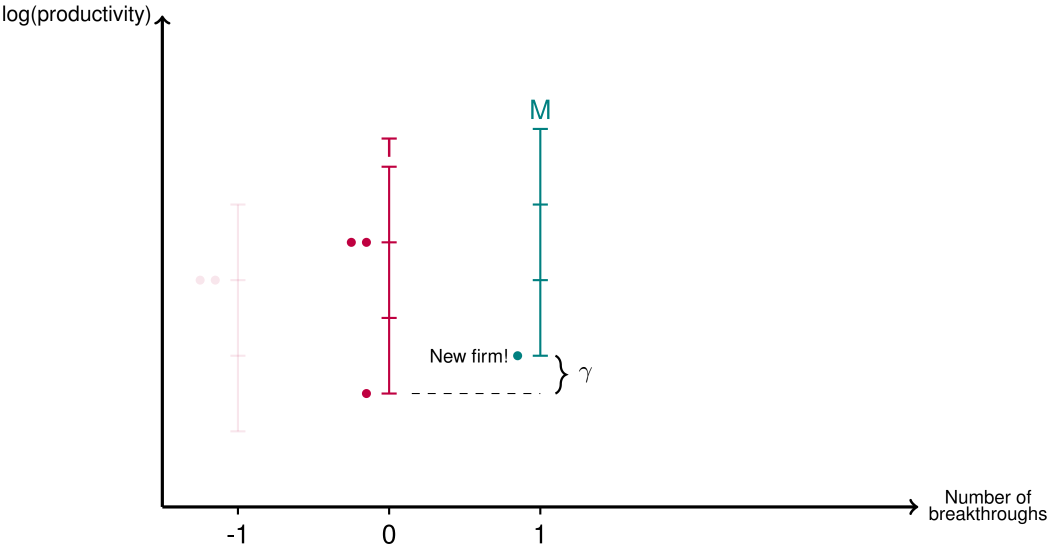

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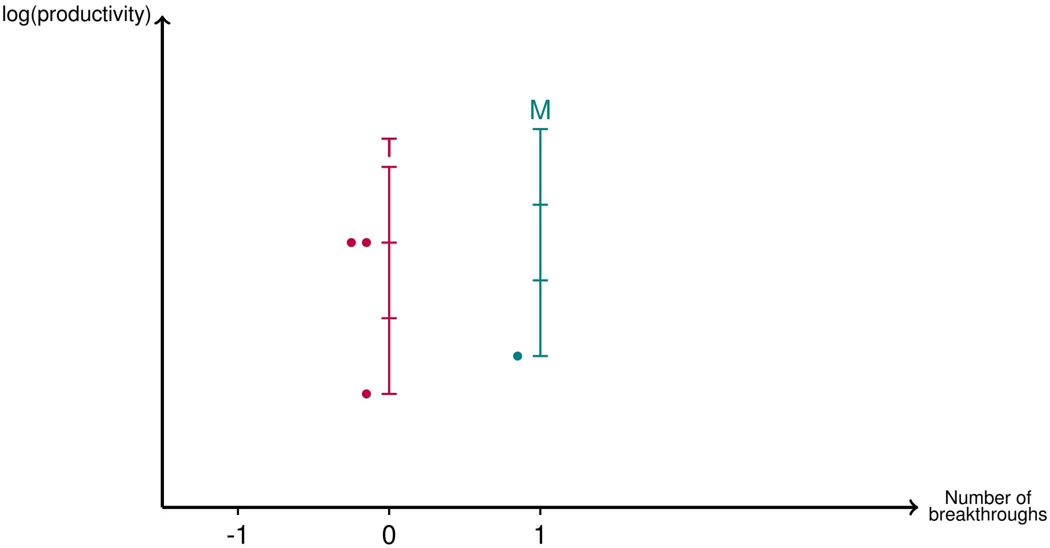

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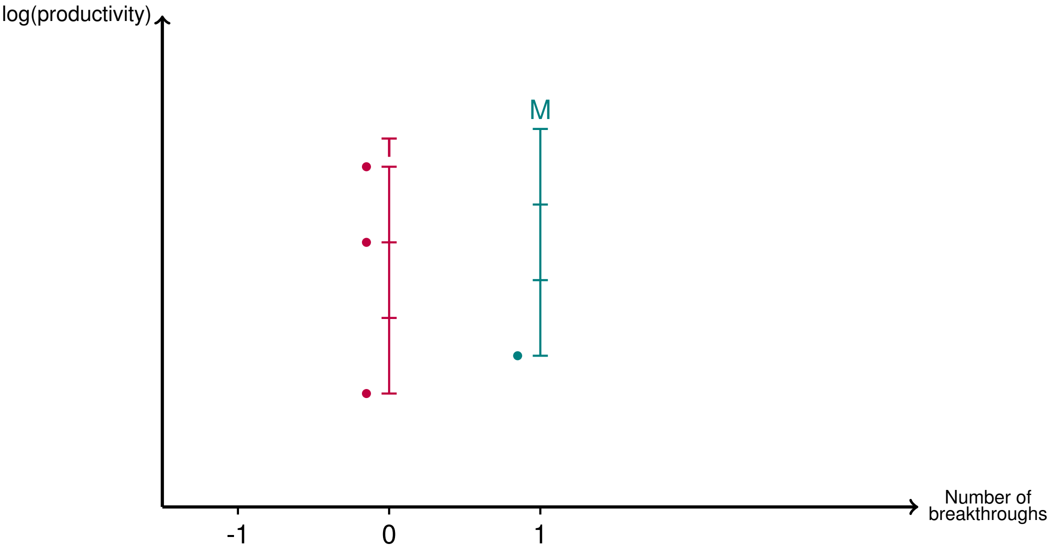

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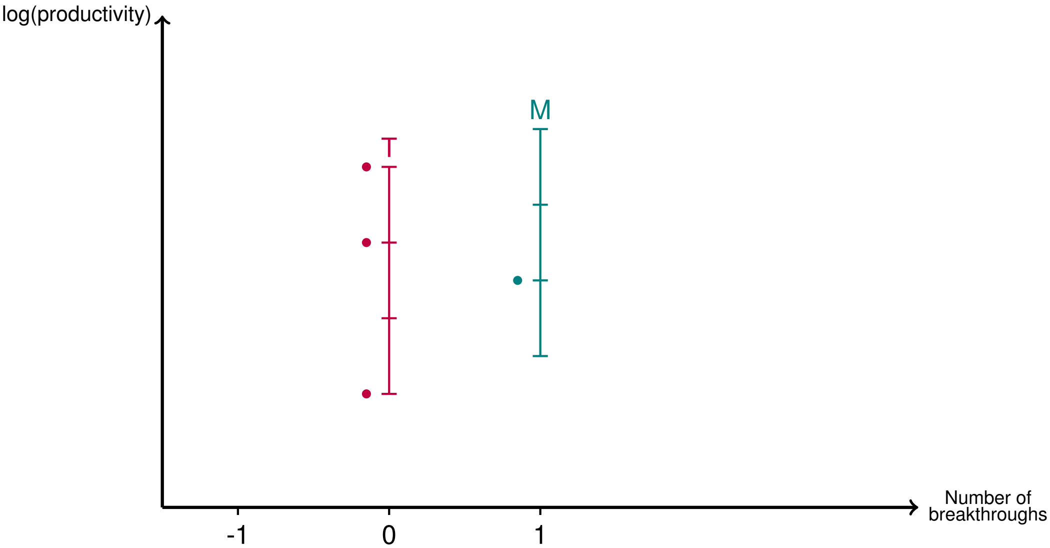

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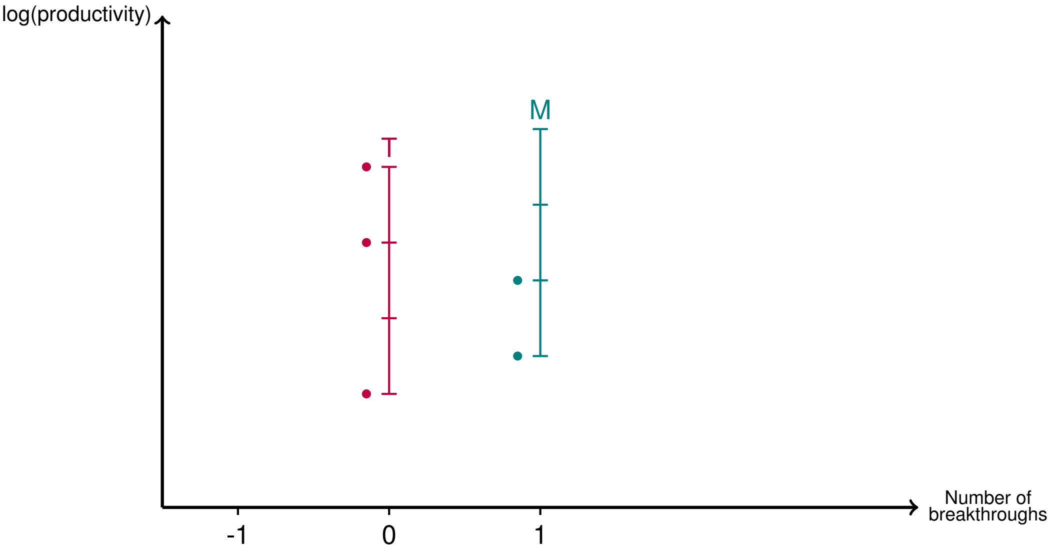

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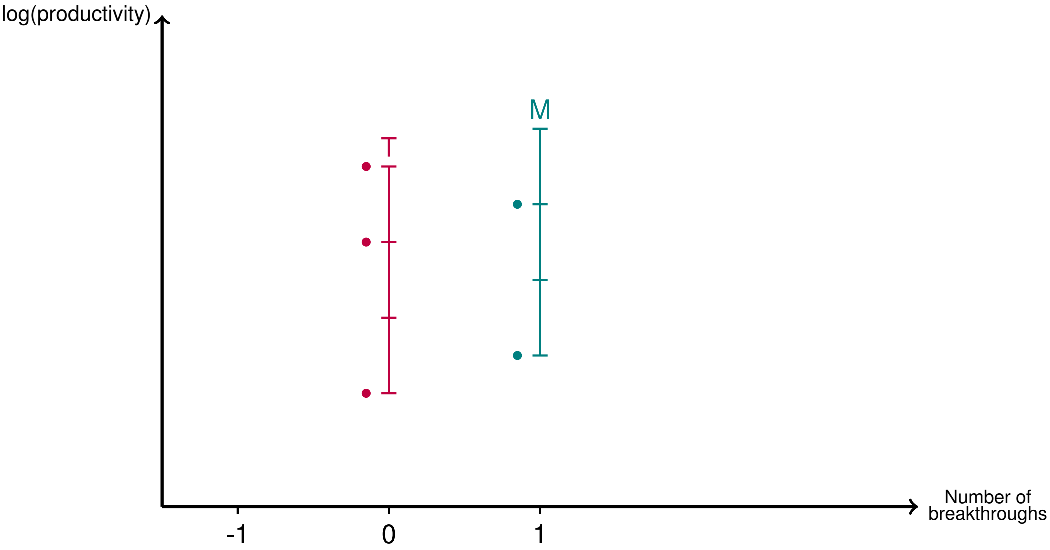

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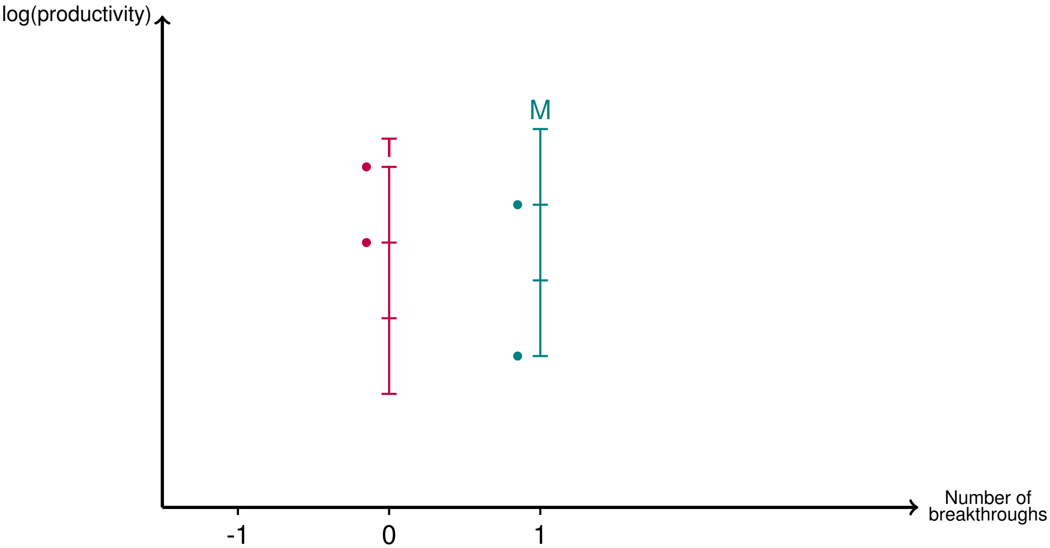

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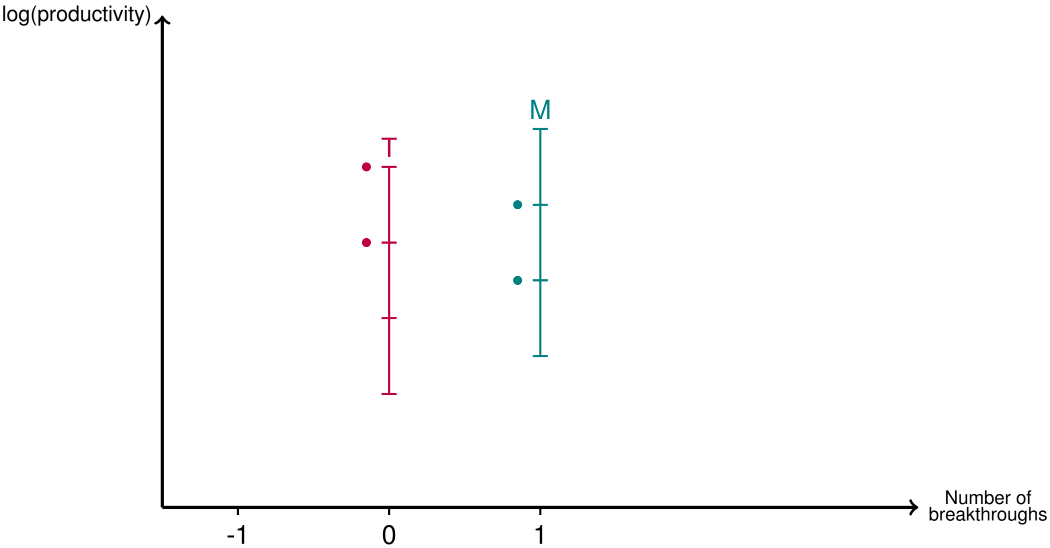

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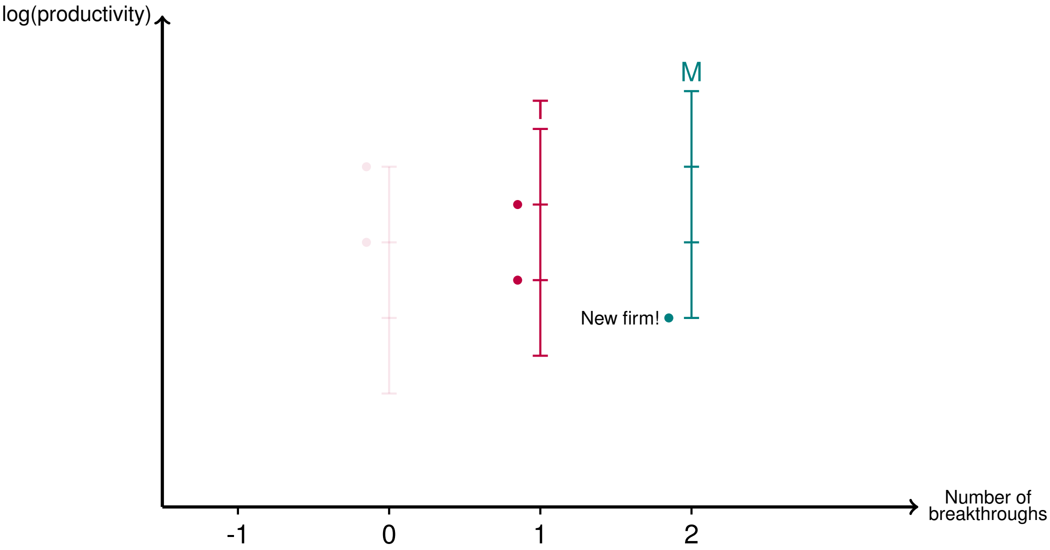

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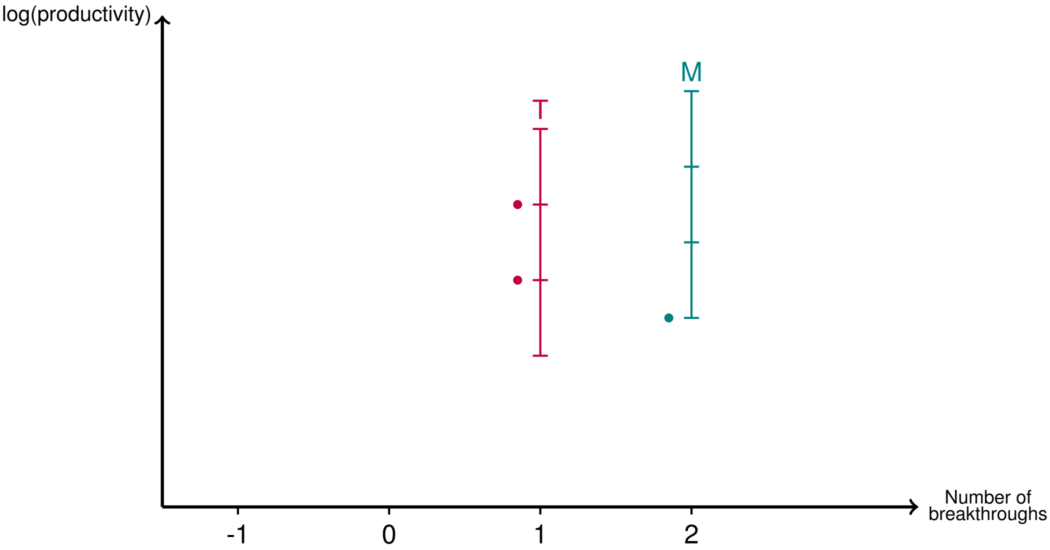

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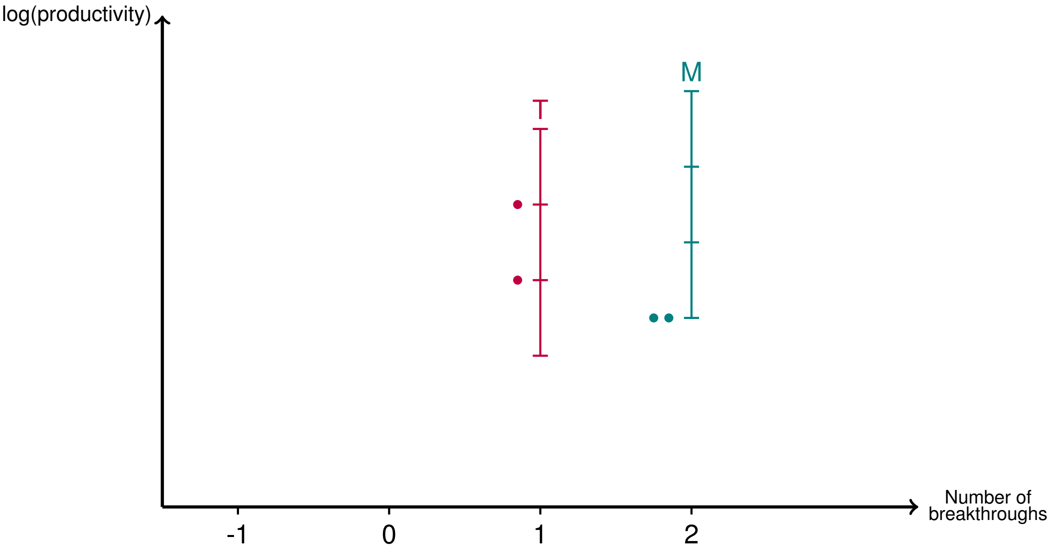

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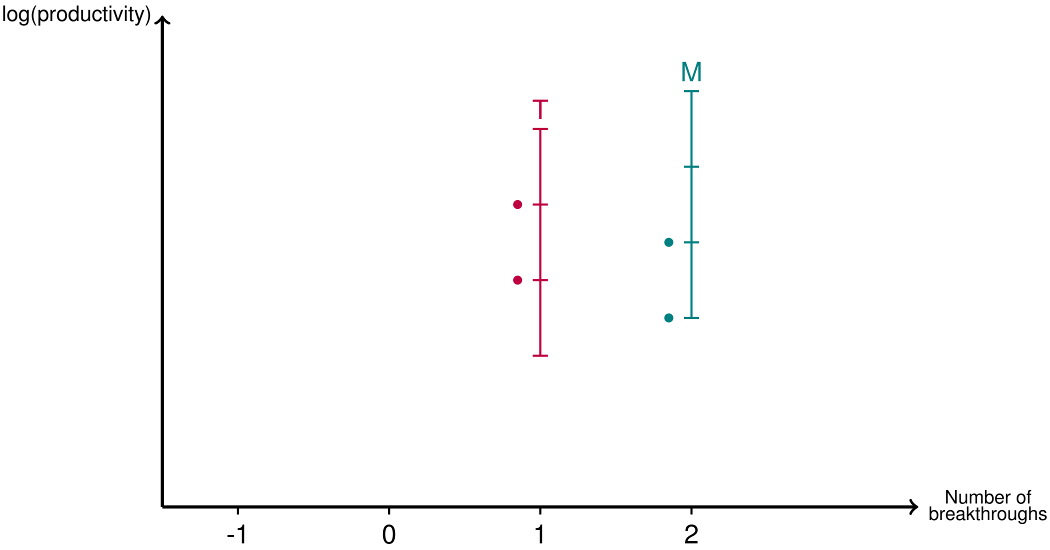

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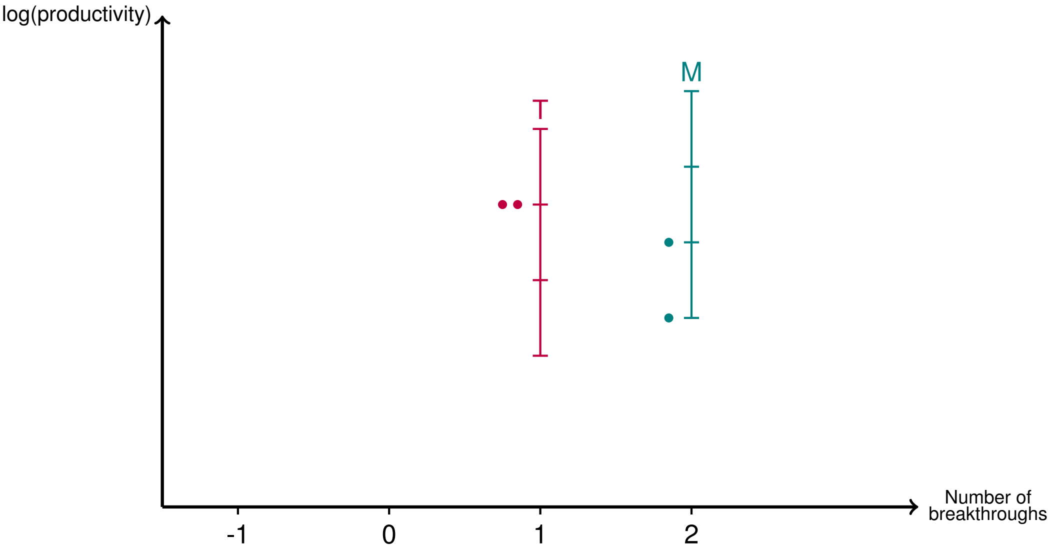

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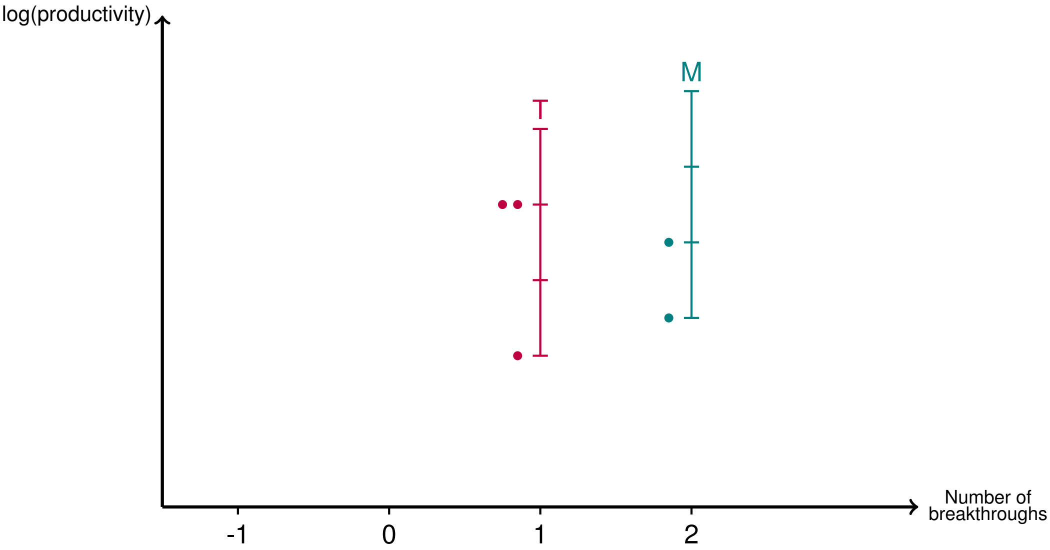

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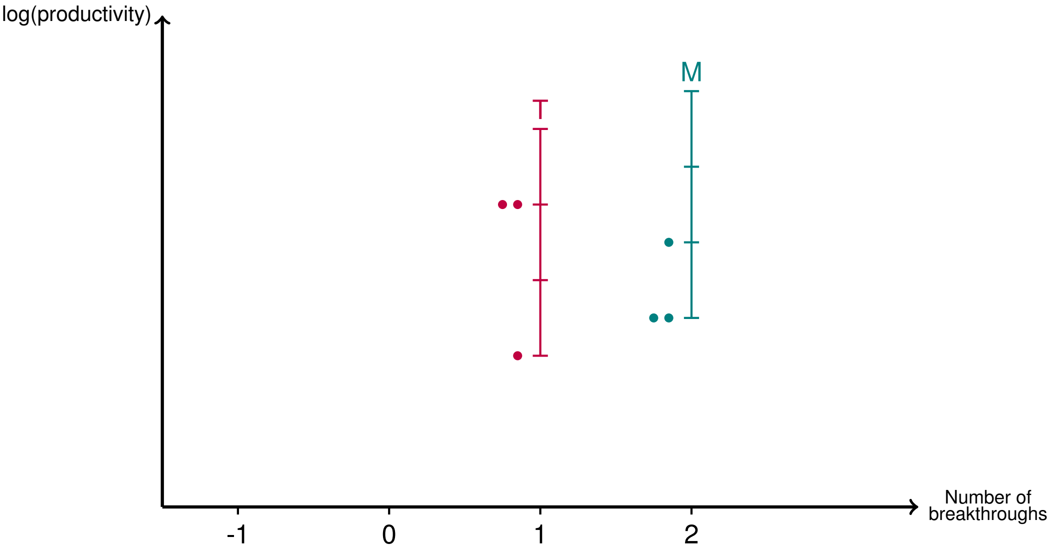

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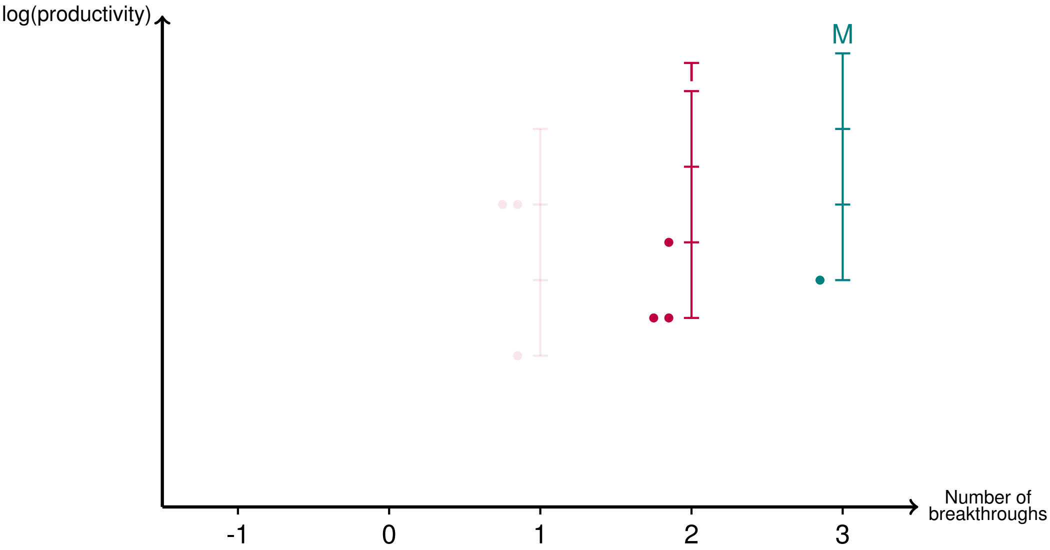

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Static Equilibrium Conditions

- Let $\mathcal{N}_{i,t} \subseteq \{1, \ldots, \overline{N}\}$ be the endogenous set of producing firms ("incumbents") in industry i at time t.
- **Markups:** Conditional on producing, firm $n \in \mathcal{N}_{i,t}$ sets a markup:

$$m_{in,t} = 1 + \frac{1}{(\varepsilon - 1)(1 - \sigma_{in,t})}, \quad \text{where} \quad \sigma_{in,t} \equiv \frac{p_{in,t} y_{in,t}}{P_{i,t} Y_{i,t}} = \left(\frac{q_{in,t}/Q_{i,t}}{m_{in,t}/M_{i,t}} \right)^{\varepsilon - 1}$$

where $M_{i,t} \equiv \left(\sum_{n \in \mathcal{N}_{i,t}} (m_{in,t})^{1-\varepsilon} \left(\frac{q_{in,t}}{Q_{i,t}} \right)^{\varepsilon - 1} \right)^{\frac{1}{1-\varepsilon}}$ and $Q_{i,t} \equiv \left(\sum_{n \in \mathcal{N}_{i,t}} (q_{in,t})^{\varepsilon - 1} \right)^{\frac{1}{\varepsilon - 1}}$.

- **Profits:** Increasing and concave in (i) firm's markup $m_{in,t}$, and (ii) relative productivity, $q_{in,t}/Q_{i,t}$. Example

$$\pi_{in,t} = \left[\left(1 - \frac{1}{m_{in,t}} \right) \sigma_{in,t} - \phi \right] Y_t$$

  - Firm needs high enough $\sigma_{in,t}$ to find it profitable to produce!
- **Dynamic part:** Nash equilibrium in product and process innovation policies (*next*). More: Macro Aggregates

Dynamic Equilibrium Conditions

- There are three state variables for the firm:
  1. Own technology vintage $\tau = 1, 2$ ($\tau = 1$ means “T”, $\tau = 2$ means “M”)
  2. Own productivity step on the τ-vintage ladder, $j \in \{0, 1, 2, \ldots, j_{\max}\}$.
  3. Industry state $\mathbf{N}$: matrix giving # firms w/ some technology vintage (*rows*) and productivity step (*cols*).

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- Firm's value: (where $\chi_0 \equiv \chi^E$, and $\chi_j \equiv \chi^I$ for $j \geq 1$)

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Transitional and Long-Run Growth

- **Aggregate productivity:** Can be decomposed into

$$Q_t = Q_t^{\text{Ladder}} \times \widehat{Q}_t$$

where

$$\underbrace{Q_t^{\text{Ladder}} \equiv \exp\left(\sum_{s=1}^{\bar{S}} h_t(s)\ln\left(q_t^{\max}(s)\right)\right)}_{\text{Highest attainable productivity, } q^{\max} \equiv \max_{k=T,M} \mathcal{Q}^k} \quad \text{and} \quad \underbrace{\widehat{Q}_t \equiv \exp\left(\sum_{s=1}^{\bar{S}} h_t(s)\ln\left(\sum_{n\in\mathcal{N}_t(s)}\left(\frac{q_{n,t}}{q_{n,t}^{\max}}(s)\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}\right)}_{\text{Firms climbing the ladder}}$$

- Sources of growth:
  - **Along the transition:** Both Q_t^{Ladder} (via breakthroughs) and $\widehat{Q}_t$ (via process innovation) grow.
  - **On the BGP:** $\dot{\widehat{Q}}_t/\widehat{Q}_t = 0$, so agg. growth is only due to frequency and magnitude of breakthroughs.

$$\frac{\dot{Y}_t}{Y_t} = \frac{\dot{Q}_t^{\text{Ladder}}}{Q_t^{\text{Ladder}}} = \boxed{a\ln(\gamma)}$$

Empirics

Life-Cycle Patterns After the ICT Revolution

Empirical Strategy

- **Empirical question:**
  - How do industry dynamics respond to changes in technological possibilities?
  - Endogeneity issues $\rightarrow$ Firm entry/exit/innovation are endogenous to other industry-level shifters.

- **Premises:**
  1. ICT Revolution generated heterogeneous technological opportunities across "downstream" industries.
  2. Extent of these effects can be predicted by ex-ante industry characteristics (ex-ante exposure).
  3. ICT shock was largely exogenous to other idiosyncratic shifters that differently affected industries.

- **Empirical strategy:** Data sources
  1. Restrict sample to non-ICT industries only (Goldschlag and Miranda (2020), 250 4-dig NAICS).
     List of ICT NAICS codes
  2. Build exposure index $\rightarrow$ Share of patents in non-ICT industries that are ICT-related in 1975-1979.
     - We identify CPC technology classes as ICT-related from list in Braguinsky et al. (2023)

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Empirical Specification

- How did the arrival of technological opportunities affect entry/exit/innovation patterns?
- Estimate a series of local projection models:

$$y_{i,t} = \alpha_{s(i),t} + \beta_t \, \text{High ICT-exposure}_{i,1975-79} + \gamma_t \boldsymbol{X}_{i,1975-79} + \epsilon_{i,t}$$

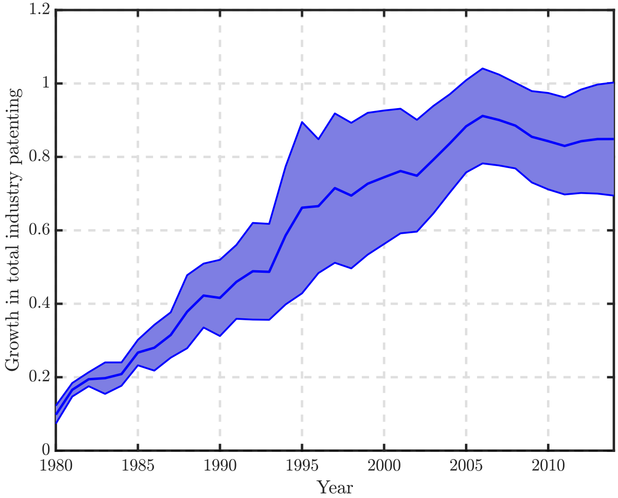
  where

  1. $y_{i,t}$ $\rightarrow$ Outcomes for industry i in year t.
  2. $\text{High ICT-exposure}_{i,1975-79}$ $\rightarrow$ Equals 1 if share of ICT patenting in initial period (1975-79) $\geq$ 10%.
  3. $\boldsymbol{X}_{i,1975-79}$ $\rightarrow$ Initial number of patents (logs).
  4. $\alpha_{s(i),t}$ $\rightarrow$ 2-digit sector-time fixed effect (captures sectoral trends).

- Coefficients $\{\beta_t\}_{t=1980}^{2015}$ capture effect of the ICT shock by industry's exposure to the shock.

Results: Total Industry Patenting

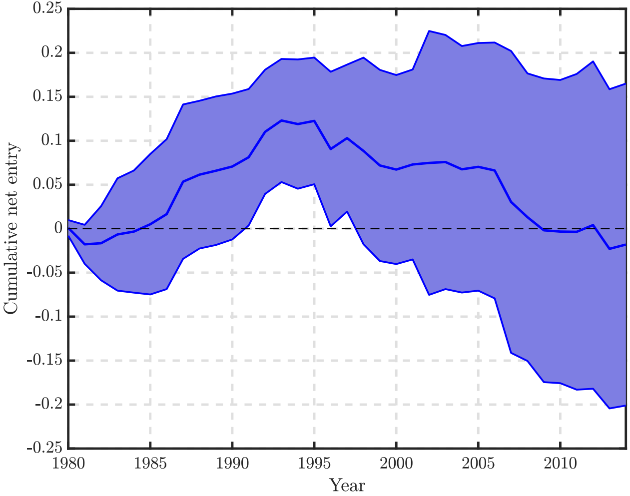
→ More exposed industries experience a prolonged increase in total industry patenting



Notes: Estimates of β_t when outcome variable is log-industry patenting in year t. The error bands denote 95% confidence intervals. All regressions weighted by the number of patents in 1975-79.

Results: Cumulative Net Entry

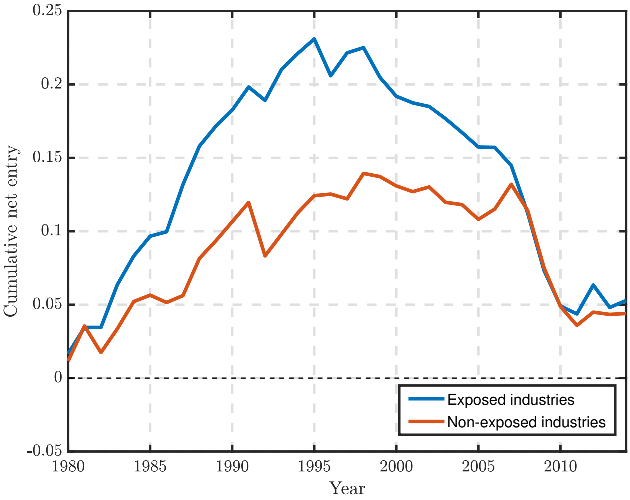
→ More exposed industries experience an increase in net entry, followed by a shakeout



Notes: Estimates of β_t when outcome variable is cumulative net-entry (i.e. the growth rate in the total number of firms) between 1979 and year t. The error bands denote 95% confidence intervals. All regressions weighted by the number of patents in 1975-79.

Results: Cumulative Net Entry (Exposed vs Non-Exposed)

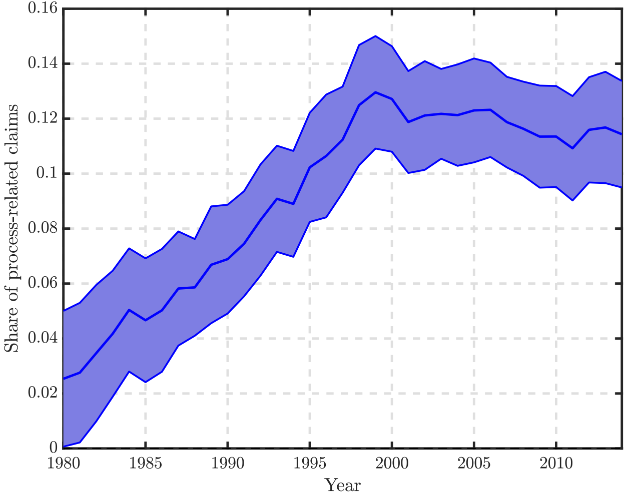
→ Profile is more pronounced for the exposed industries.



Notes: Observations are weighted by total industry patenting in 1975-79.

Results: Process Innovation

→ More exposed industries shift from product to process innovation over time.



Notes: Estimates of β_t when outcome variable is the share of process-related claims (Bena and Simintzi 2023) in patents in year t. The error bands denote 95% confidence intervals. All regressions weighted by the number of patents in 1975-79.

Calibration

Bringing the Model to the Data

Calibration Strategy

- Shock the model with a **"technological revolution"**:
  - An "MIT"-shock that simultaneously creates a technological breakthrough in 21.9% of industries.
- Solve for the full transition path, and calibrate parameters to match industry life-cycle seen in the data.
- **Result:** Low entry costs (initial burst of entry) and process inn. costs high (no rebounds after shakeout).

Param.	Value	Description	Source/Target	Model	Data
Externally calibrated					
ρ	0.03	Discount rate	4% annual interest rate		
$\overline{N}$	8	Max number of firms per industry			
$j_{\max}$	3	Max number of process innovations			
ψ	2	Innovation cost curvature	Akcigit and Kerr (2018)		
s_{GPT}	21.9%	% industries affected by GPT	Our data		
Internally calibrated					
ε	7.8	EoS within industries	Agg. markup (Edmond et al., 2023)	26.8%	25%
a	0.033	Frequency of breakthroughs	R&D share (NSF)	3.8%	2.70%
γ	1.318	Distance between ladders	BGP growth (BEA, 2005-2019)	0.92%	1.07%
χ^I	18.3	Process innovation cost	Estimated life cycle		
χ^E	0.962	Product innovation cost	Estimated life cycle		
ϕ	0.030	Fixed cost of production	Estimated life cycle		
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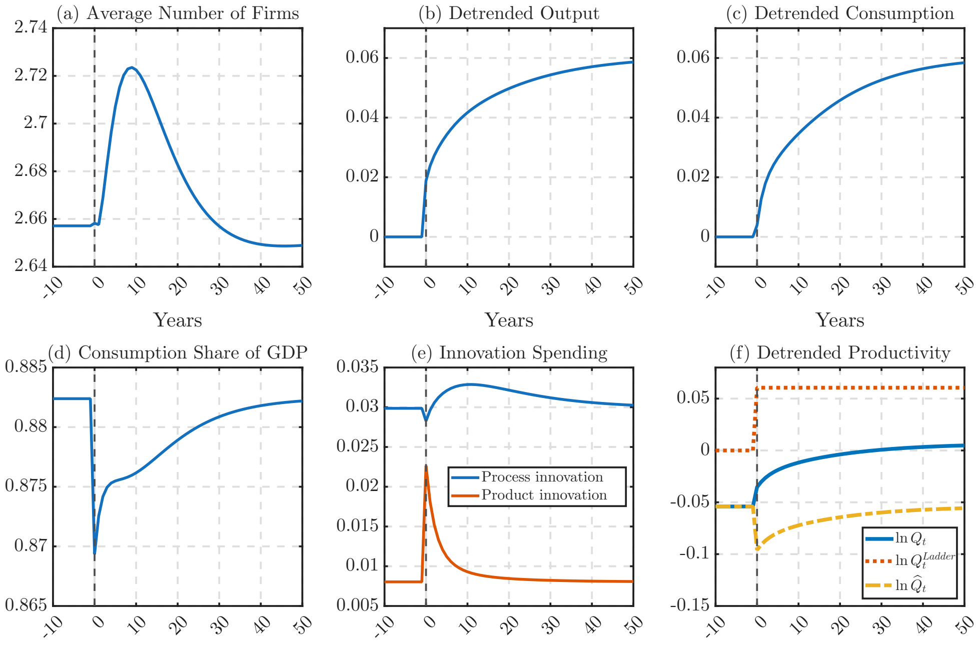
Transitional Dynamics after the Revolution

Figure: Cumulative net-entry (i.e. the growth rate in the total number of firms) in disrupted industries relative to non-disrupted industries, between year 0 (time of the shock) and year *t*. The red line is obtained from the model's transitional dynamics in response to an unforeseen technological breakthrough that simultaneously disrupts 21.9% of industries. The blue line reproduces the results from our empirical regressions.

Transitional Dynamics after the Revolution



Policy Analysis

Optimal Policy Reaction to a Revolution

Should We Tax or Subsidize Innovation?

- **Static inefficiencies:** misallocation of labor between producing firms.
  - Oligopolistic competition generates markup dispersion (productive firms are too small).
- **Dynamic inefficiencies:** misallocation of goods between consumption and R&D investment.
  - Limited appropriability: Firms do not capture the full social value of their innovations.
  - Business stealing: transferring rents from one firm to another has no social value.
- **Today:** A simple policy exercise:
  - At $t = 0$, the government starts to subsidize/tax product and process innovation.
  - We look for the two constant policy rates $(\xi_{\text{product}}, \xi_{\text{process}})$ that maximize CE welfare.
  - **Result:** A 12.5% tax on process, a 5% subsidy on product.

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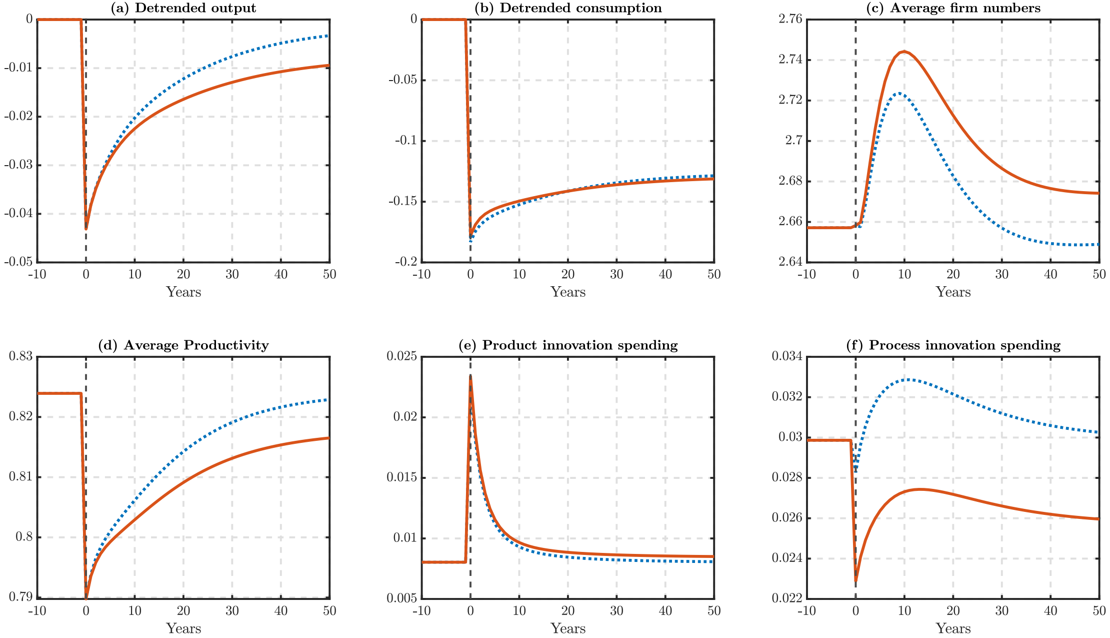
Transition Dynamics under the Optimal Policy



Sources of Welfare Changes

- Welfare changes: Welfare decomposition formula
  - Main force: misallocation of output between consumption and process R&D.
  - However, the life cycle also seems close to optimal (the welfare effects are quantitatively small).

	Baseline	**Robustness** (Delayed Policy)
	(a)	(b)
Product R&D policy rate (ξ^{product})	5.00	5.00
Process R&D policy rate (ξ^{process})	-12.50	-10.00
Consumption share	0.43	0.34
Output, of which ...	-0.40	-0.31
... Misallocation wedge	0.05	0.03
... Love of variety	0.12	0.09
... Average productivity	-0.56	-0.43
CE Welfare	0.04	0.03

Conclusions

- New technological opportunities trigger industry-level life cycles.
  - ICT Revolution led to an increase in entry in affected industries, followed by a shakeout.
  - These dynamics were driven by an increase in process innovations.
- New quantitative macro model to capture these dynamics.
  - Shakeout due to process innovation, early entry predicts survival –as in Klepper (1996, 1997).
  - Quantitative result: in the wake of a revolution, tax process and subsidize product innovation.
- **What's next:**
  1. More on optimal policy $\rightarrow$ Timing, duration, policy mix, ..., following a technological revolution.
  2. Fully endogenous growth? $\rightarrow$ Technology revolutions as a by-product of firm-level innovations.

Thank you!

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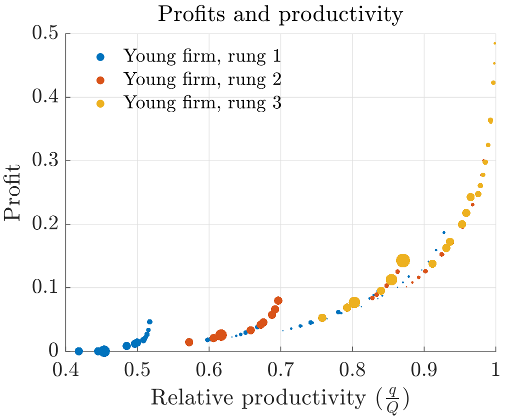
Appendix

Appendix: Profits and Relative Productivity: An Illustration

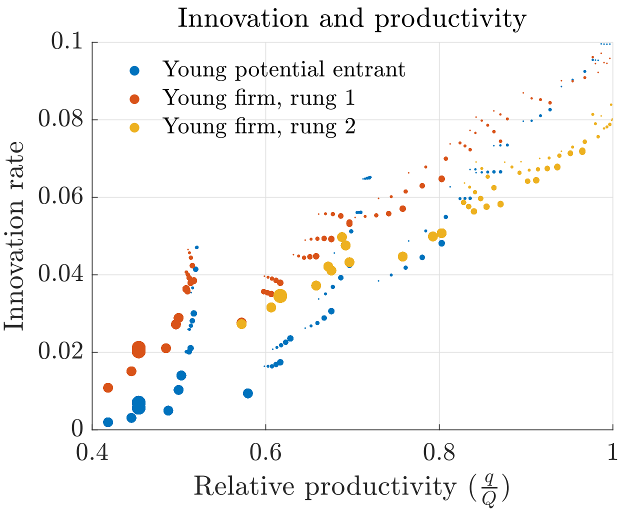
Back



- Profits are generally increasing in relative productivity.
- However, the productivity distribution matters:
  - For example, one good and one bad competitor is preferable to two average competitors...
  - ...even though industry productivity is higher in the former case.

Appendix: Innovation and Relative Productivity: An Illustration

Back



- Innovation is generally increasing in relative productivity.
  - Discouragement effect when other firms innovate first.

Appendix: Data Sources

Back

- Data sources:

  1. **US Census Business Dynamics Statistics** (BDS, 1978-2019).
     - Number of firms, entry and exit rates for NAICS 4-digit industries.
  2. **USPTO PatentsView** (1976-2019):
     - Universe of patents ultimately granted by the USPTO.
     - Set of listed Cooperative Patent Classification (CPC) technology classes (CPC classes classify patents by the technological component of the invention).

- Merging:
  - We match CPCs to 4-digit NAICS industries, using crosswalk by Lybbert and Zolas (2014).

Appendix: Set of ICT Industries

Back

Definition of ICT industry comes from Goldschlag and Miranda (2020):

- NAICS industry is identified as ICT if its share of Science, Technology, Engineering and Math (STEM) occupational employment is higher than 5 times the national average for most years.

NAICS	**Name**
3341	Computer and Peripheral Equipment Manufacturing
3342	Communications Equipment Manufacturing
3344	Semiconductor and Other Electronic Component Manufacturing
3345	Navigational, Measuring, Electromedical, and Control Instruments Manufacturing
5112	Software Publishers
5171	Wired Telecommunications Carriers
5179	Other Telecommunications
5182	Data Processing, Hosting, and Related Services
5191	Other Information Services
5415	Computer Systems Design and Related Services

Figure: Set of 4-digit NAICS industries identified as ICT-related in Braguinsky, Choi, Ding, Jo and Kim (2023), Table A2.

Appendix: Aggregates

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- **Aggregate productivity:** (where $h_t(s) \equiv$ Share of industries in state $s \in \{1, \ldots, \overline{S}\}$ at time t)

$$Q_t \equiv \exp\left(\sum_{s=1}^{\overline{S}} h_t(s) \ln(Q_t(s))\right), \qquad \text{where} \quad Q(s) \equiv \left(\sum_{n \in \mathcal{N}(s)} (q_n(s))^{\varepsilon - 1}\right)^{\frac{1}{\varepsilon - 1}}$$

- **Aggregate markup:**

$$\mathcal{M}_t \equiv \left[\sum_{s=1}^{\overline{S}} h_t(s) \left(\sum_{n \in \mathcal{N}_t(s)} \sigma_n(s)(m_n(s))^{-1}\right)\right]^{-1}$$

- **Aggregate output:**

$$Y_t = \underbrace{\mathcal{M}_t M_t^{-1}}_{\substack{\text{Misallocation} \\ \text{term } \leq 1}} Q_t L, \qquad \text{where} \quad M_t \equiv \exp\left(\sum_{s=1}^{\overline{S}} h_t(s) \ln(M_t(s))\right)$$

  - Average markup M_t drives a wedge between wages and productivity $\rightarrow \frac{w_t}{Q_t} = M_t^{-1}$.
  - Aggregate markup $\mathcal{M}_t$ equals the inverse of the labor share $\rightarrow \frac{w_t L}{Y_t} = \mathcal{M}_t^{-1}$

Appendix: Welfare Decomposition

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- Change in welfare between any two BGPs:

$$\Delta \text{Welfare} = \frac{1}{\rho}\left[\Delta \ln \begin{pmatrix} \text{Consumption} \\ \text{share of output} \end{pmatrix} + \underbrace{\Delta \ln \begin{pmatrix} \text{Love-for-} \\ \text{variety} \end{pmatrix} + \Delta \ln \begin{pmatrix} \text{Average} \\ \text{productivity} \end{pmatrix} + \quad}_{= \Delta \ln(\text{Aggregate Output})} \right]$$

where

1. **Consumption share:** $\frac{C}{Y} = 1 - \sum_{s=1}^{\overline{S}} h(s) \Big[\overbrace{\phi\, N(s)}^{\text{Fixed costs}} + \overbrace{X(s)}^{\text{Innovation costs}} \Big]$
2. **Love-for-variety**: $N = \exp\left(\left(\frac{1}{\varepsilon - 1}\right) \sum_{s=1}^{\overline{S}} h(s) \ln(N(s)) \right)$
3. **Average productivity**: $Q = \exp\left(\sum_{s=1}^{\overline{S}} h(s) \ln \left(\frac{1}{N(s)} \sum_{n \in \mathcal{N}(s)} (q_n(s))^{\varepsilon - 1} \right)^{\frac{1}{\varepsilon - 1}} \right)$
4. **Misallocation wedge:** $\mathcal{W} = \left[\sum_{s=1}^{\overline{S}} h(s) \left(\sum_{n \in \mathcal{N}(s)} \sigma_n(s) \frac{M(s)}{m_n(s)} \right) \right]^{-1}$

Appendix: Welfare Decomposition

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$$\Delta\text{Welfare} = \frac{1}{\rho}\left[\Delta\ln\left(\begin{array}{c}\text{Consumption}\\ \text{share of output}\end{array}\right) + \underbrace{\Delta\ln\left(\begin{array}{c}\text{Love-for-}\\ \text{variety}\end{array}\right) + \Delta\ln\left(\begin{array}{c}\text{Average}\\ \text{productivity}\end{array}\right) + \Delta\ln\left(\begin{array}{c}\text{Misallocation}\\ \text{wedge}\end{array}\right)}_{=\Delta\ln(\text{Aggregate Output})}\right]$$

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2. **Love-for-variety:** $$N = \exp\left(\left(\frac{1}{\varepsilon-1}\right)\sum_{s=1}^{\bar{S}} h(s)\ln(N(s))\right)$$

3. **Average productivity:** $$Q = \exp\left(\sum_{s=1}^{\bar{S}} h(s)\ln\left(\frac{1}{N(s)}\sum_{n\in\mathcal{N}(s)}(q_n(s))^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}\right)$$

4. **Misallocation wedge:** $$\mathcal{W} = \left[\sum_{s=1}^{\bar{S}} h(s)\left(\sum_{n\in\mathcal{N}(s)}\sigma_n(s)\frac{M(s)}{m_n(s)}\right)\right]^{-1}$$

Appendix: Welfare Decomposition

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Appendix: Welfare Decomposition

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