A Three-Component Model for Adult Mortality

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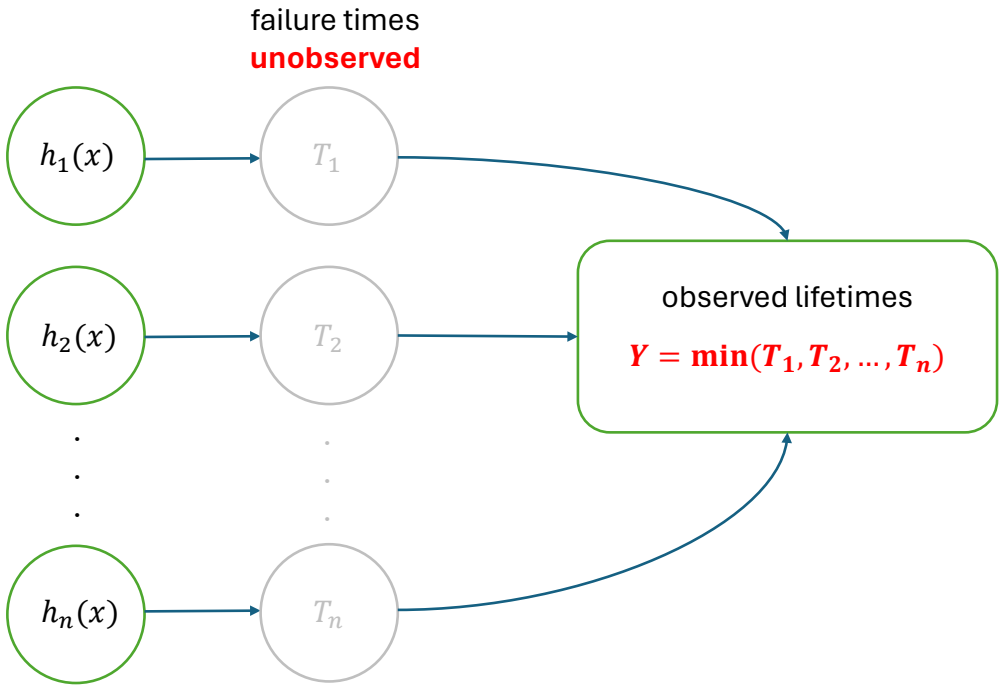
European Population Conference 2024, Edinburgh, 14.06.2024
Session 74 Mortality Modelling

Competing-Risk Models as Mixtures



Competing-Risk Models as Mixtures

r.v.	T_1	T_2	$\ldots$	T_n	– not necessarily independent
hazard	$h_1(x)$	$h_2(x)$	$\ldots$	$h_n(x)$	– in the absence of other
p.d.f.	$f_1(x)$	$f_2(x)$	$\ldots$	$f_n(x)$	competing risks


References

r.v.	$T = \min\{T_1, \ldots, T_n\}$	– actual observed lifetime
hazard	$h(x) = h_1(x) + \ldots + h_n(x)$	– additive risks
p.d.f.	$\boxed{f(x) = \sum_{i=1}^{n} \pi_i \, g_i(x)}$	$g_i(x) \neq f_i(x)$

$g_i(x)$ and π_i are, repspectively, the p.d.f. and prevalence of deaths from i-th risk, $i = 1, \ldots, n$, in the presence of all others.

Additive-Risk Models as Mixtures: Benefits

- All additive-risk models are competing-risk models, whereby the competing risks are not necessarily independent
- Estimating the model, one can compute $g_i(x)$ and π_i to calculate the share of deaths from i-th risk (in the presence of all others) at every age x
- For specific functional forms of $h_1(x), \ldots, h_n(x)$, calculate different mortality indicators such as (remaining) life expectancy, the modal age of death, etc.

Patricio, S.C. and Missov, T.I. (2024). Makeham Mortality Models as Mixtures. *Demographic Research* (forthcoming). Preprint: https://arxiv.org/abs/2304.08920

Parametric Additive-Risk Mortality Models

All Makeham parametric mortality models are additive:

$$h(x) = \mu(x) + c$$

$\mu(x)$	$=$	ae^{bx}	Gompertz
$\mu(x)$	$=$	$\frac{ae^{bx}}{1+\frac{a\gamma}{b}(e^{bx}-1)}$	gamma-Gompertz
$\mu(x)$	$=$	$\frac{ae^{bx}}{1+kae^{bx}}$	Beard
$\mu(x)$	$=$	$\frac{ae^{bx}}{1+ae^{bx}}$	Kannisto
$\mu(x)$	$=$	$a_1e^{-b_1x}+ae^{bx}$	Siler

Death occurs either as a result of biological processes at early or late ages, or due to extrinsic risk c, whatever strikes first

Extending the Makeham Model

Predecessor: κ-Gompertz model (Vaupel and Wisser 2015):

$$\mu(x) = ae^{bx} + ce^{(b-\kappa)x}$$

Our extension:

$$h(x) = h_1(x) + h_2(x) + h_3(x)$$

- senescent: $h_1(x) = \frac{ae^{bx}}{1+\frac{a\gamma}{b}(e^{bx}-1)}$
  aging-related hazard
- behavior-related: $h_2(x) = \eta\, h_1(x)\, \kappa(x)$
  inclination to act risky, captured by $\kappa(x)$, interacting with age-increasing incurring damage
  (η is a scaling factor)
- external: $h_3(x) = c$
  non-aging-related hazard

CPop Assumptions for $\kappa(x)$

- $\kappa(x)$ decreases with age
  valid when fitting the model from an adult starting age

$$\kappa(x) = S(x)\,, \text{i.e.,} \qquad h_2(x) = \eta\, h_1(x)\, S(x)$$

- $\kappa(x)$ first increases and then decreases with age
  valid when fitting the model from a pre-adult starting age

$$\kappa(x) = f(x)\,, \text{i.e.,} \qquad h_2(x) = \eta\, h_1(x)\, f(x)$$

Distributions: exponential, gamma, Rayleigh, log-normal, skew normal

Estimating the Three-Component Model

- Input: age-specific death counts $D(x)$ and exposures $E(x)$
- Assumptions:
  1. $D(x) \sim \text{Poisson}(E(x)\, h(x))$, where

$$h(x) = h_1(x) + h_2(x) + h_3(x)$$

  2. $f(x), S(x) \sim$ exponential/gamma/Rayleigh/log-N/skew N

- Estimation: Bayesian procedure (details in Patricio and Missov 2023)

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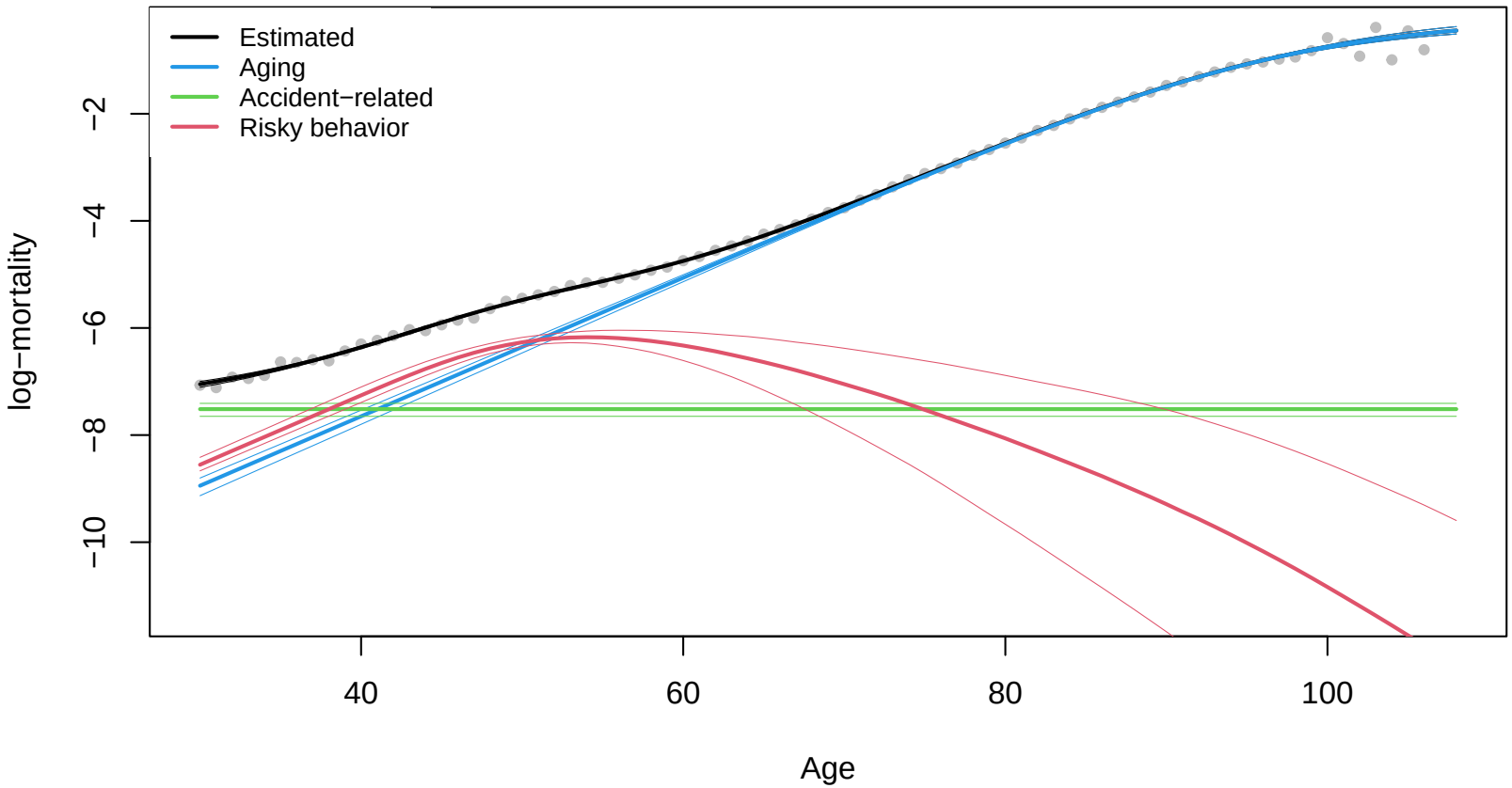
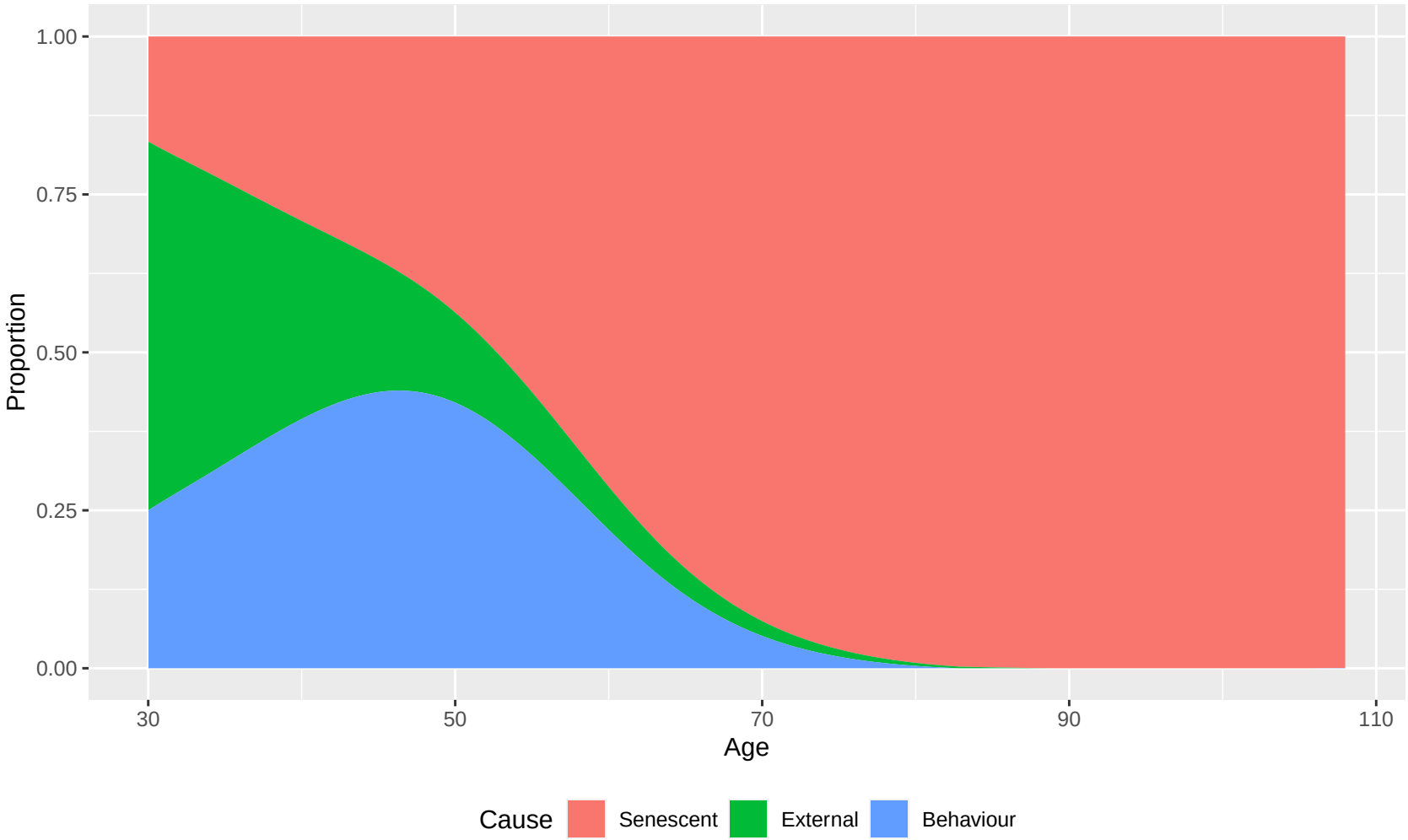
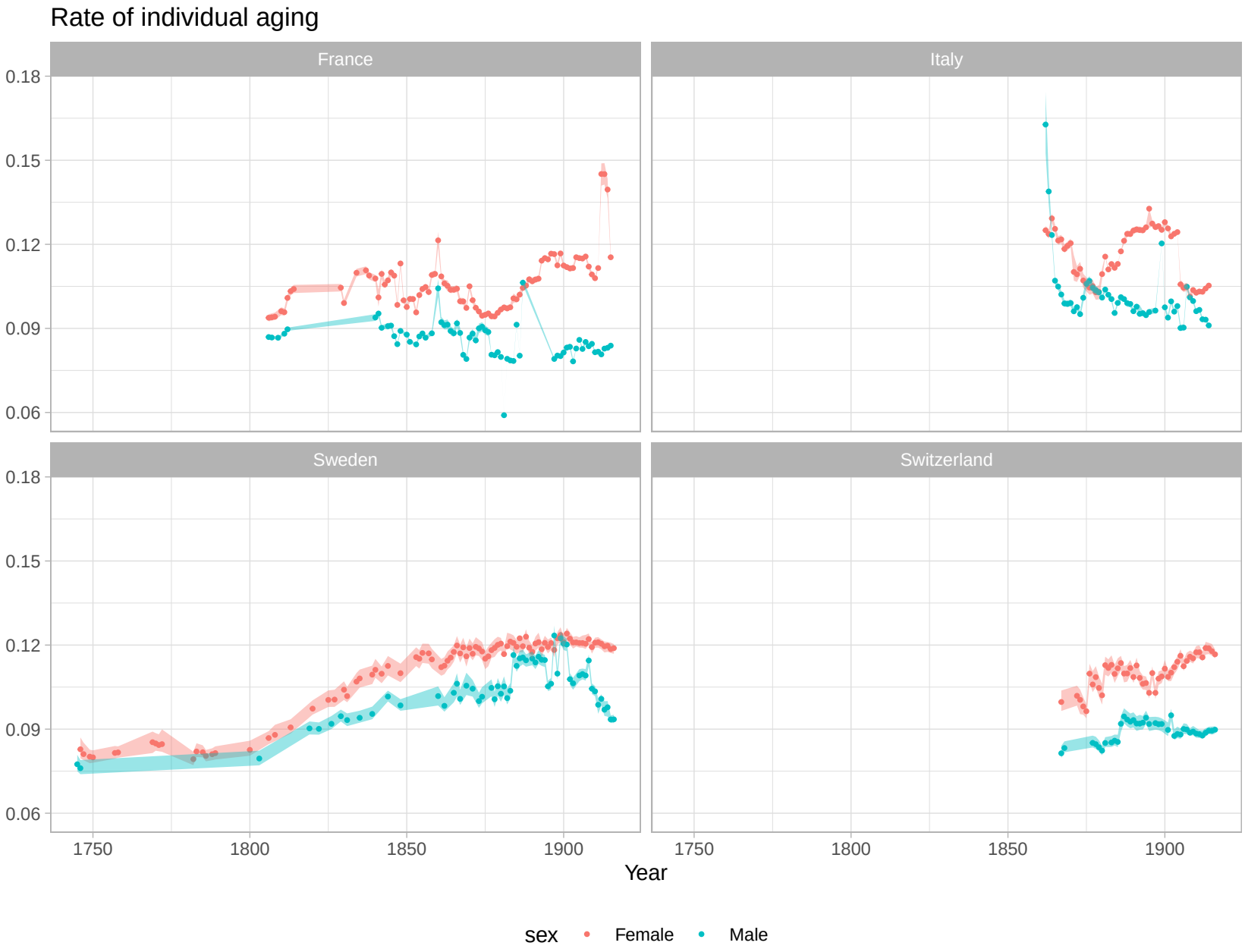
Application 1: Decomposition



Application 1: Decomposition



Application 2: Checking the *b*-Hypothesis



Application 2: Checking the *b*-Hypothesis

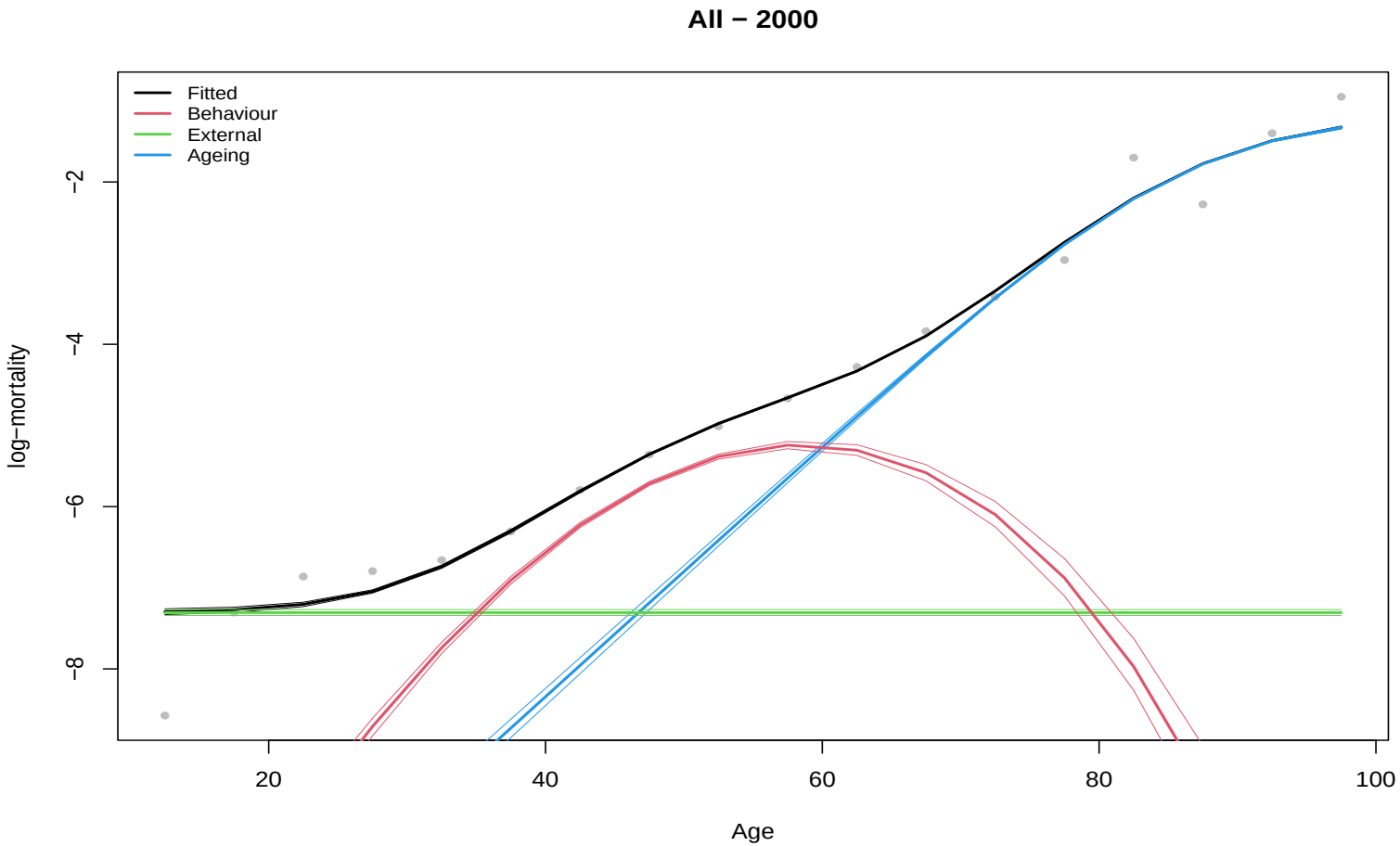
Fitting the Three-Component Model to COD Data

- France, females, years 2000–2015
- Human Cause-of-Death Database

▸ All ▸ COD

Overall Mortality: Composition by Age

All COD

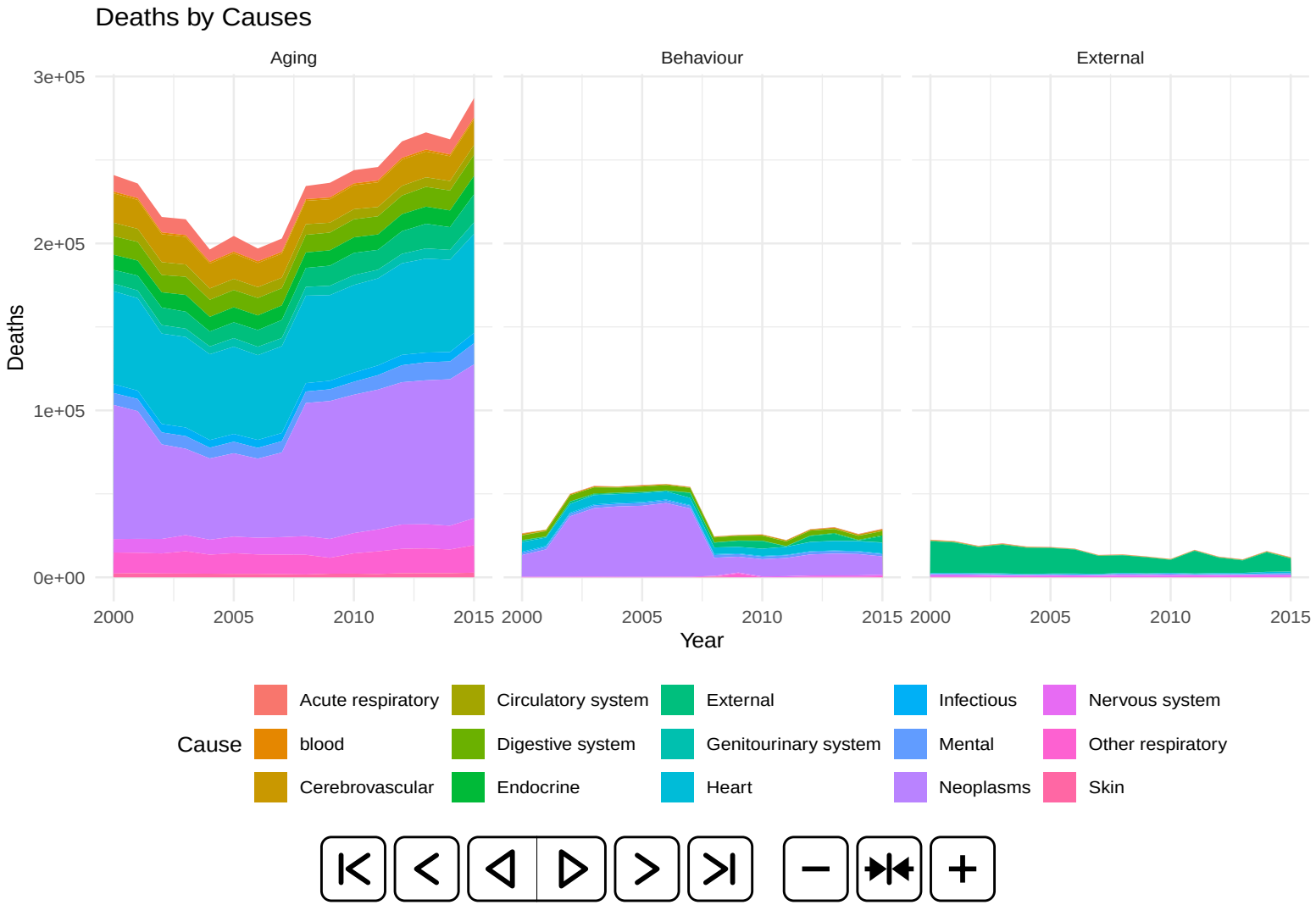


COD-specific Mortality: Convergence of MCMC

All	1.0000	Cerebrovascular	1.0000
Infectious	1.0000	Circulatory system	1.0000
Neoplasms	1.0000	Acute respiratory	1.0000
Blood	1.0000	Other respiratory	0.9375
Endocrine	0.9375	Digestive system	1.0000
Mental	1.0000	Skin	0.8750
Nervous system	1.0000	Genitourinary system	0.8750
Heart	1.0000	External	0.8125

COD-share by Components at Different Ages

COD-deaths by Components at Different Ages



Component-share by COD at Different Ages

Further Steps and Concluding Remarks

- Competing-risk models are additive-risk models that can be represented as mixtures
- One can characterize the distribution of deaths for all subpopulations stratified by competing risks
- Estmating a three-component additive-risk model aids identifying age-specific regularities for COD
- Forecasting COD may be carried out component-wise in a CoDA setting.

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References: Competing Dependent Risks

Gail, M. (1975). A review and critique of some models used in competing risk analysis. *Biometrics* 31(1): 209–222.

Elandt-Johnson, R.C. (1976). Conditional failure time distributions under competing risk theory with dependent failure times and proportional hazard rates. *Scandinavian Actuarial Journal* 1976(1): 37–51.

Hakulinen, T. and Rahiala, M. (1977). An example on the risk dependence and additivity of intensities in the theory of competing risks. *Biometrics* 33(3): 557–559.

▸ Competing Risks as Mixtures

Acute Respiratory: Composition by Age

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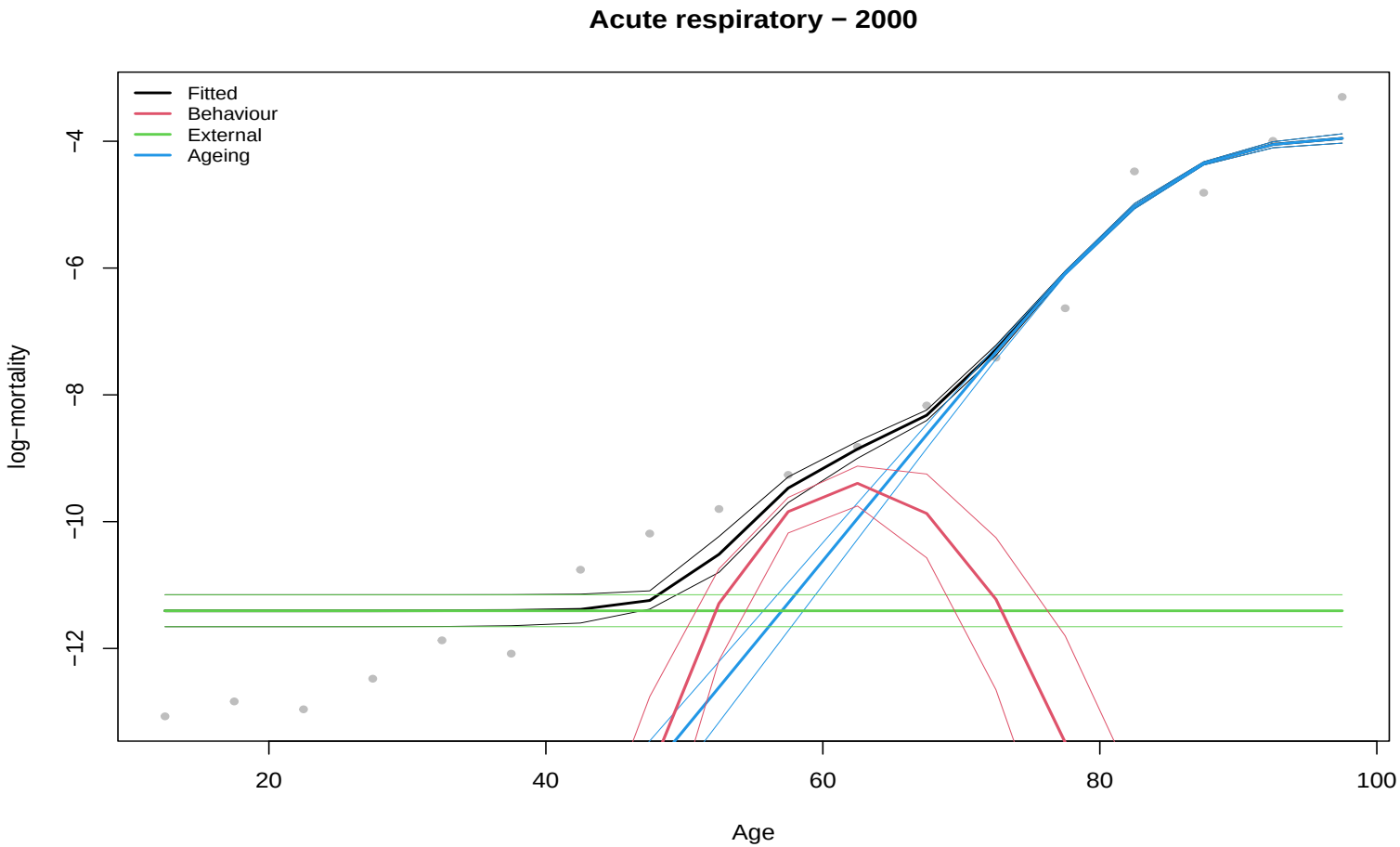

Blood: Composition by Age

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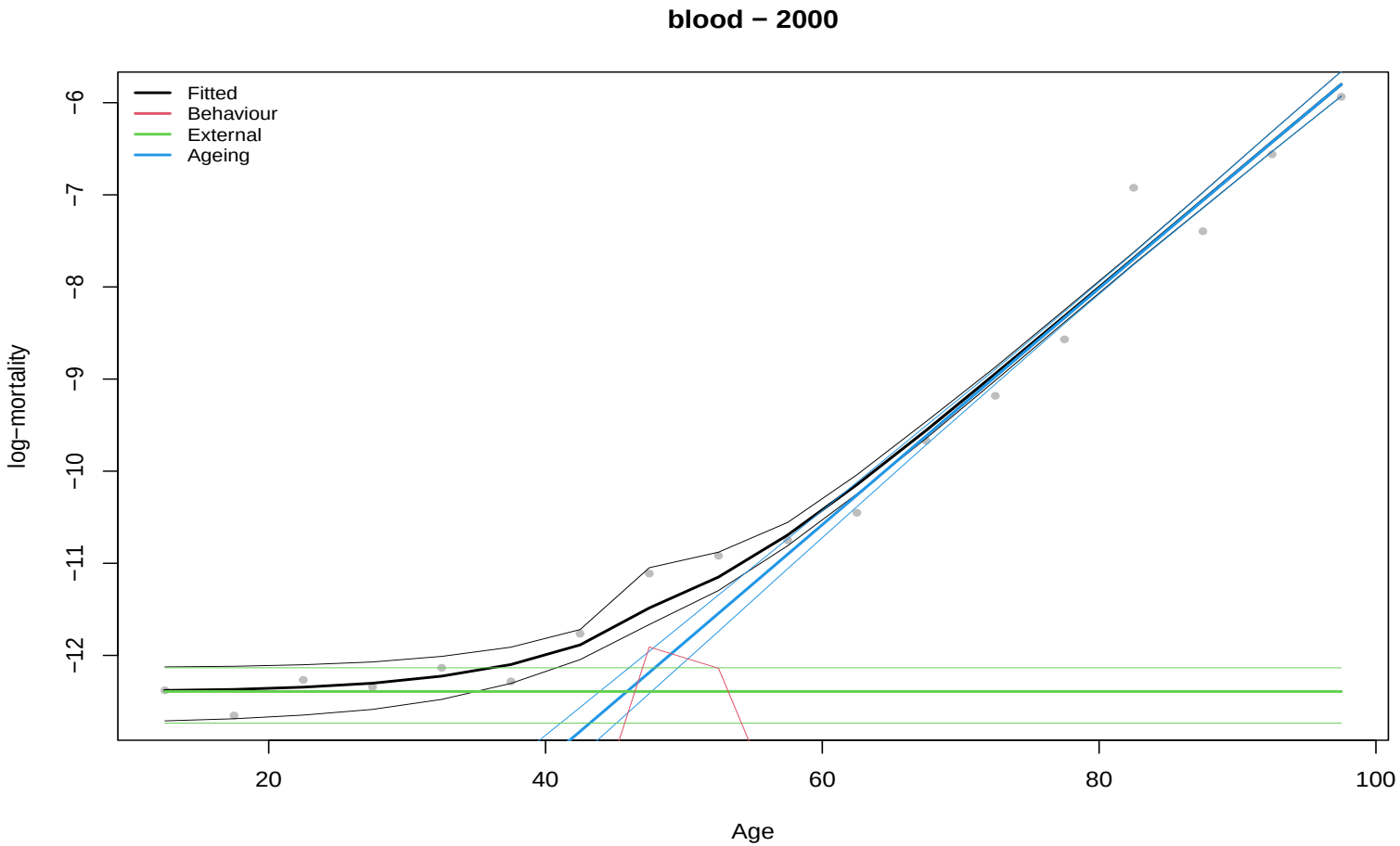

Cerebrovascular: Composition by Age

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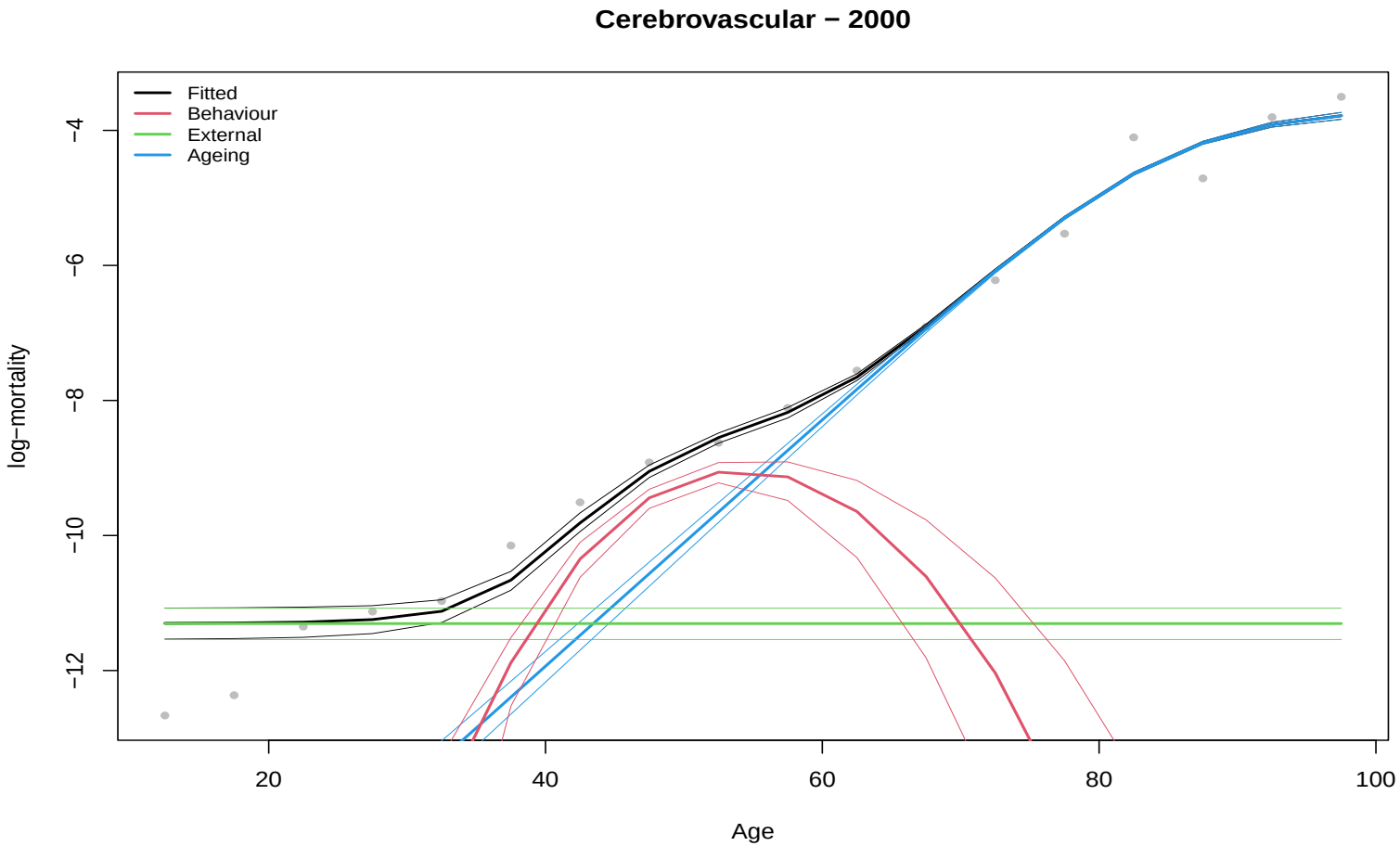

Circulatory system: Composition by Age

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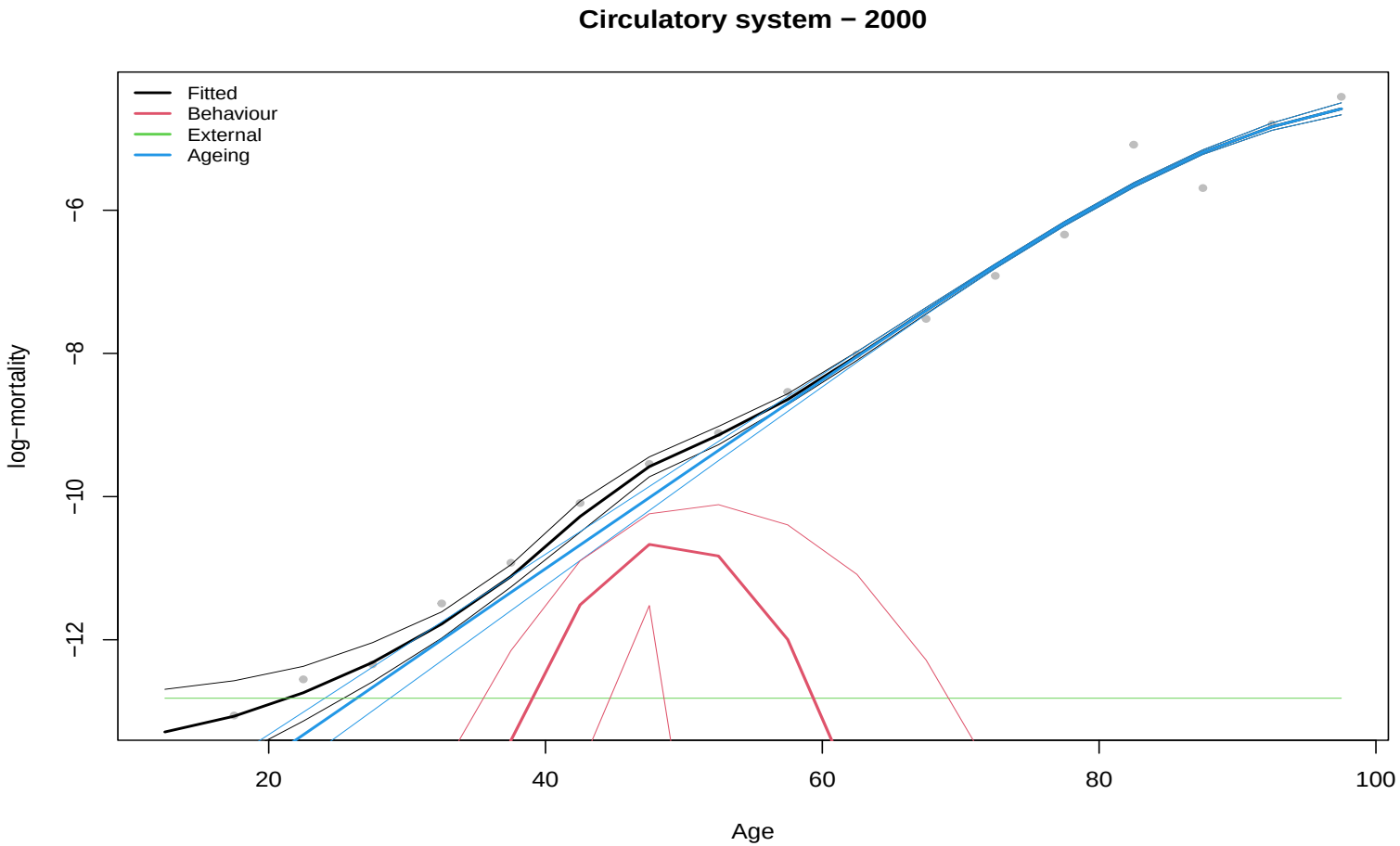

Digestive system: Composition by Age

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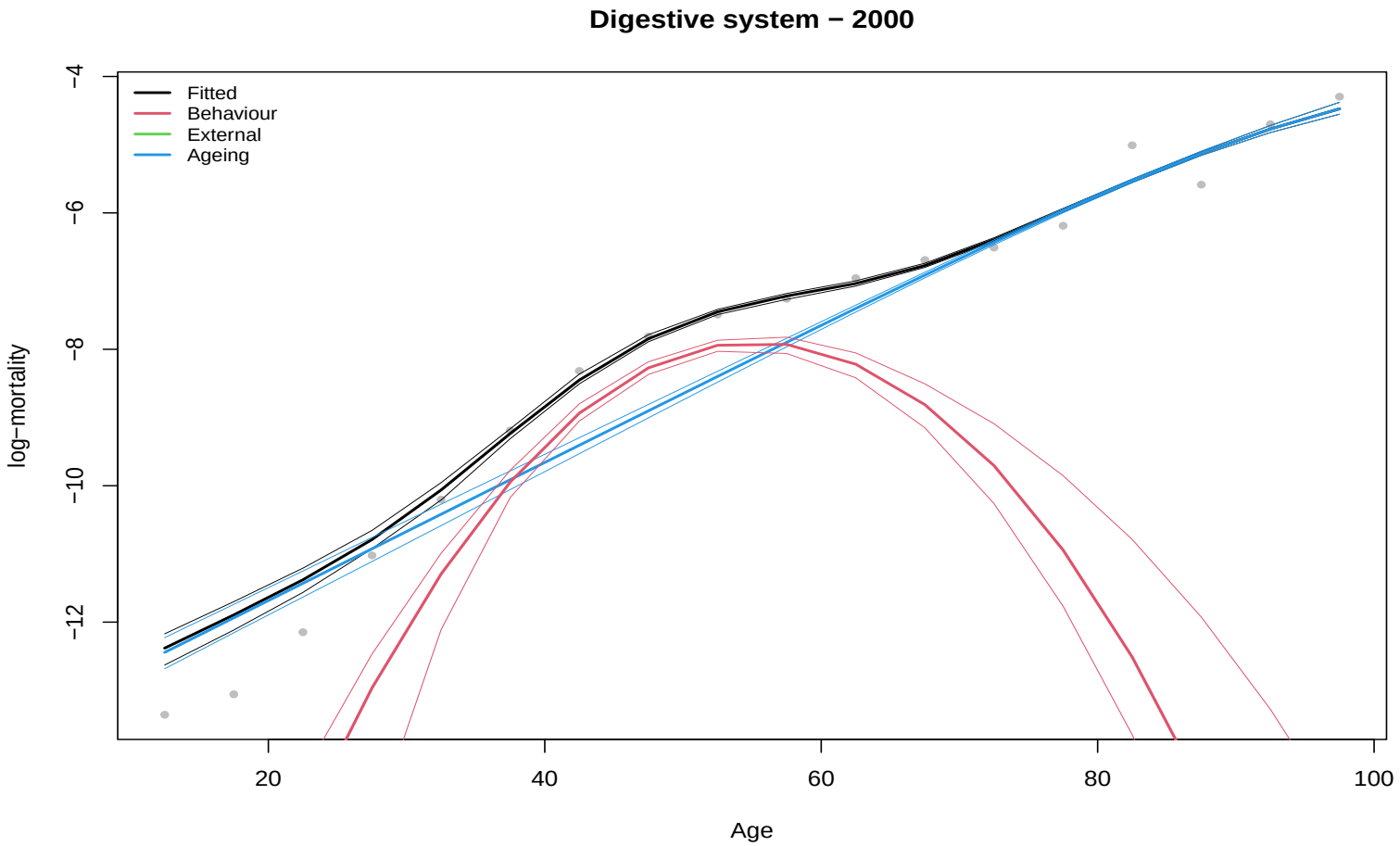

Endocrine: Composition by Age

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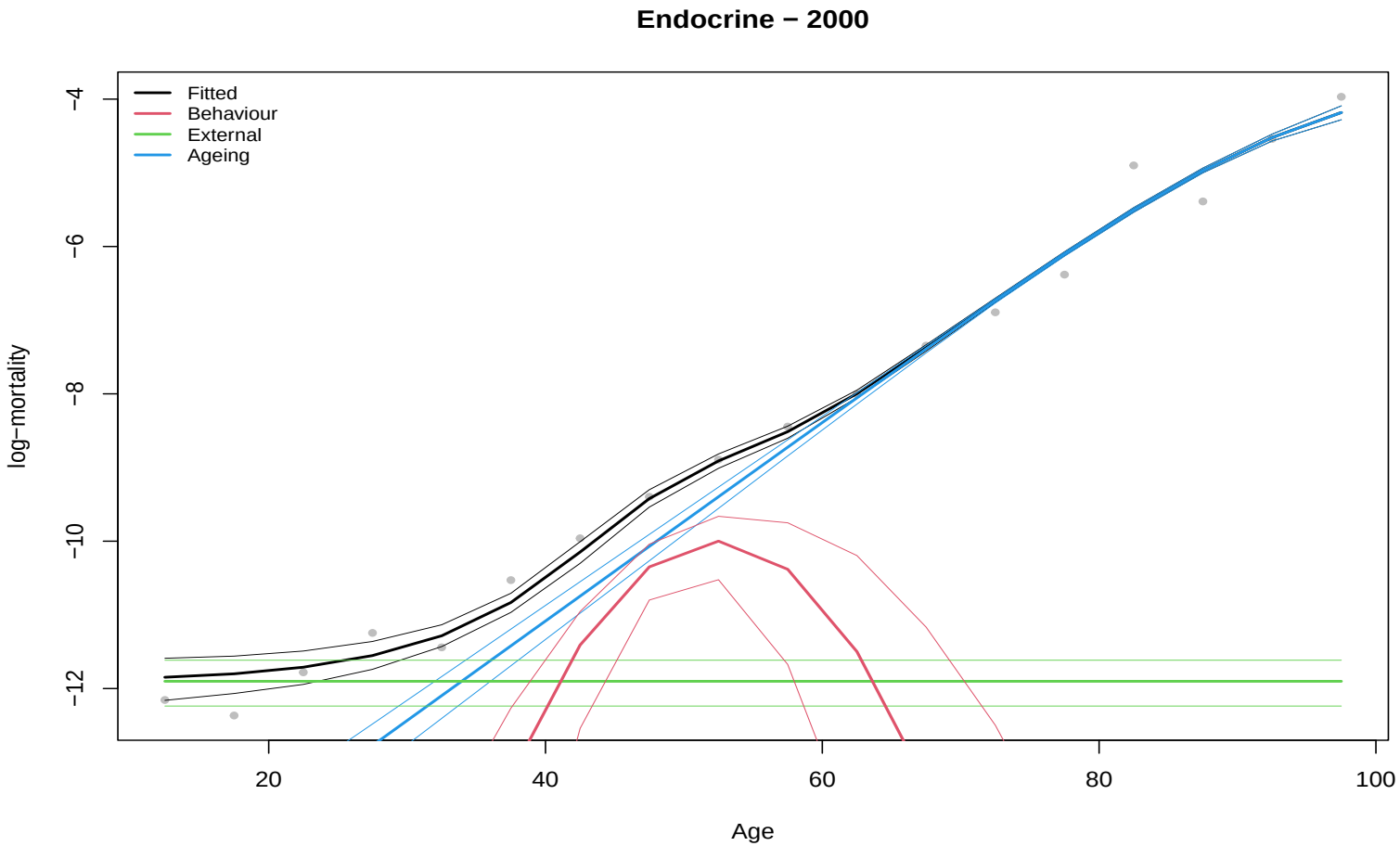

External: Composition by Age

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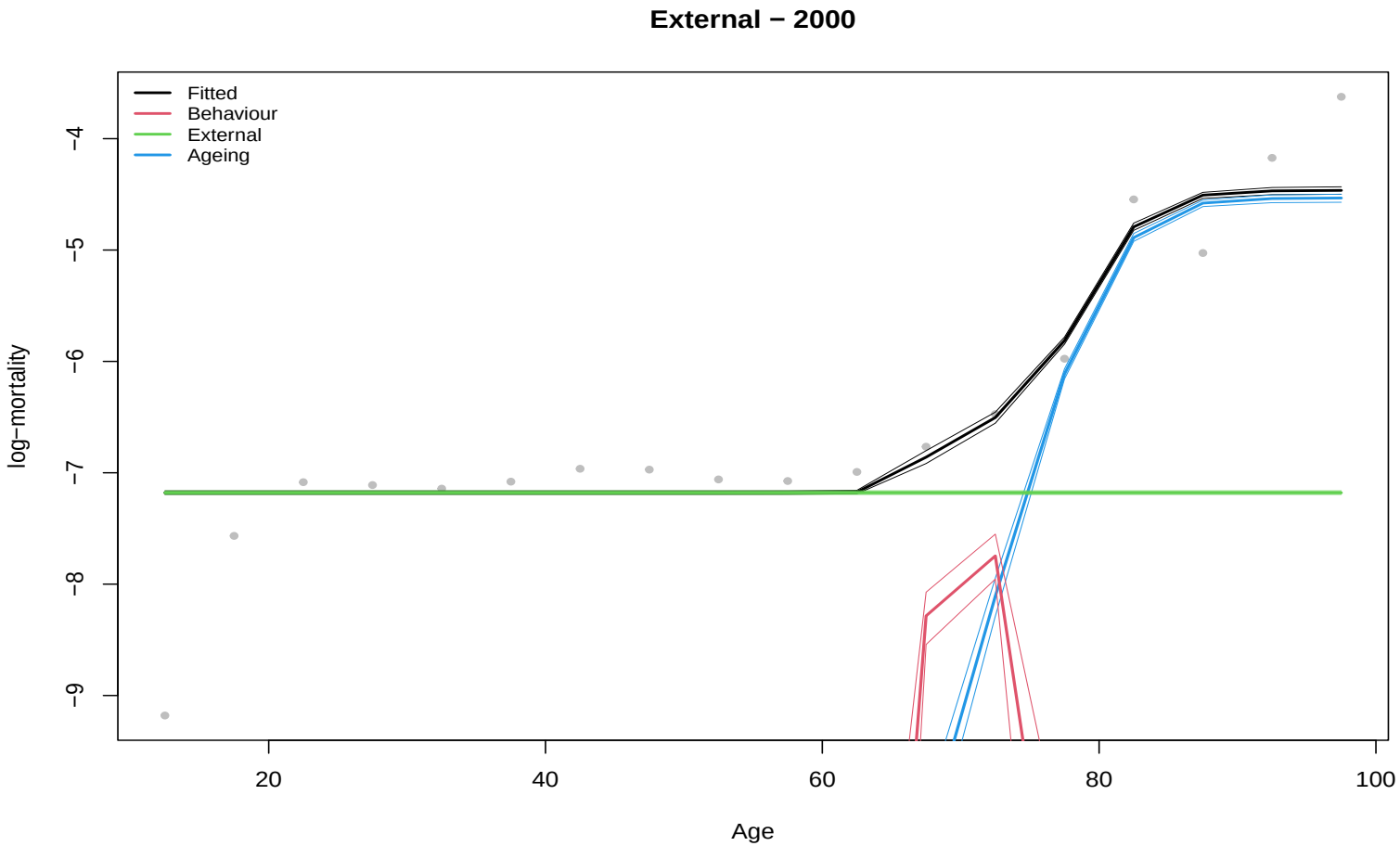

Genitourinary system: Composition by Age

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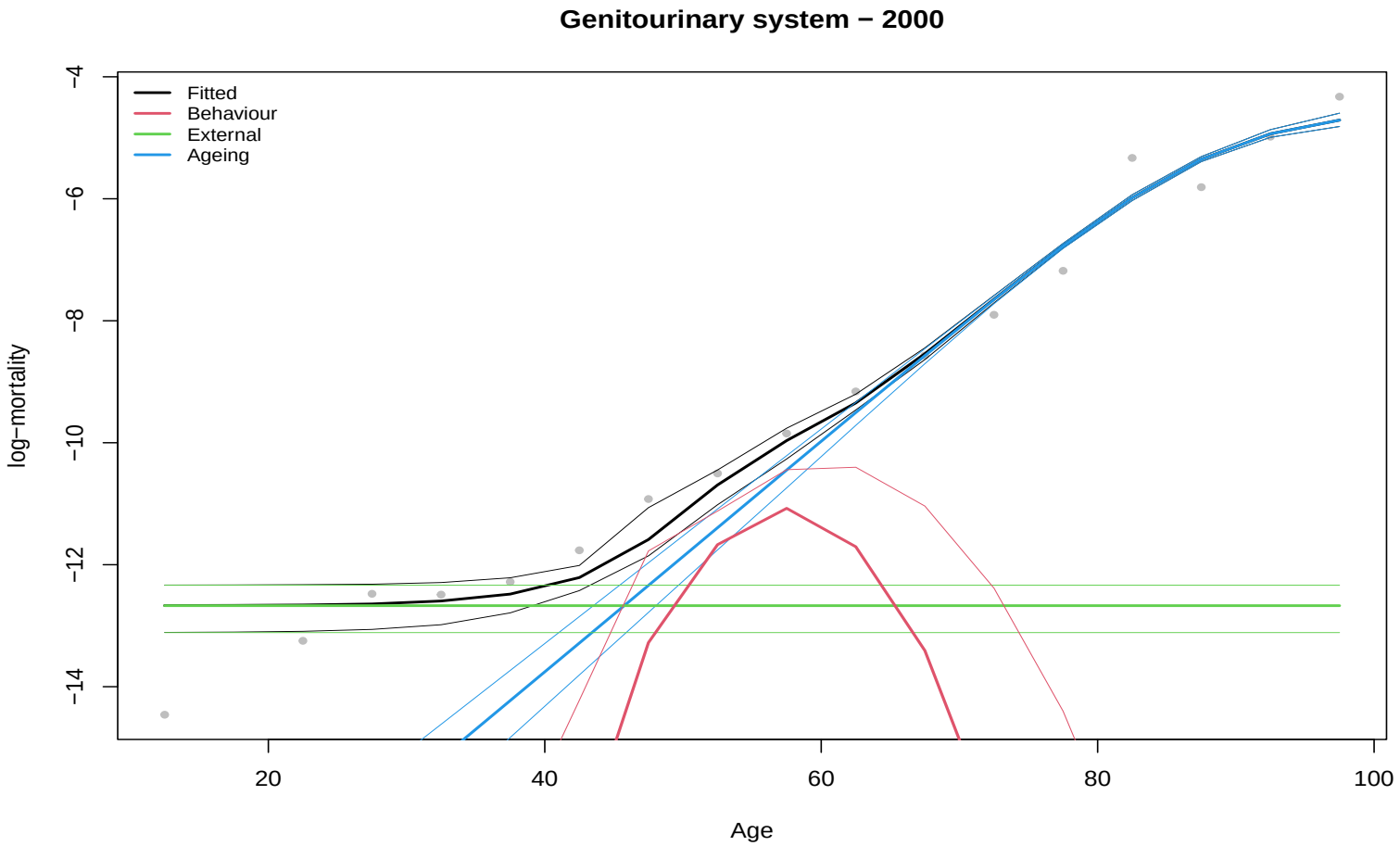

Heart: Composition by Age

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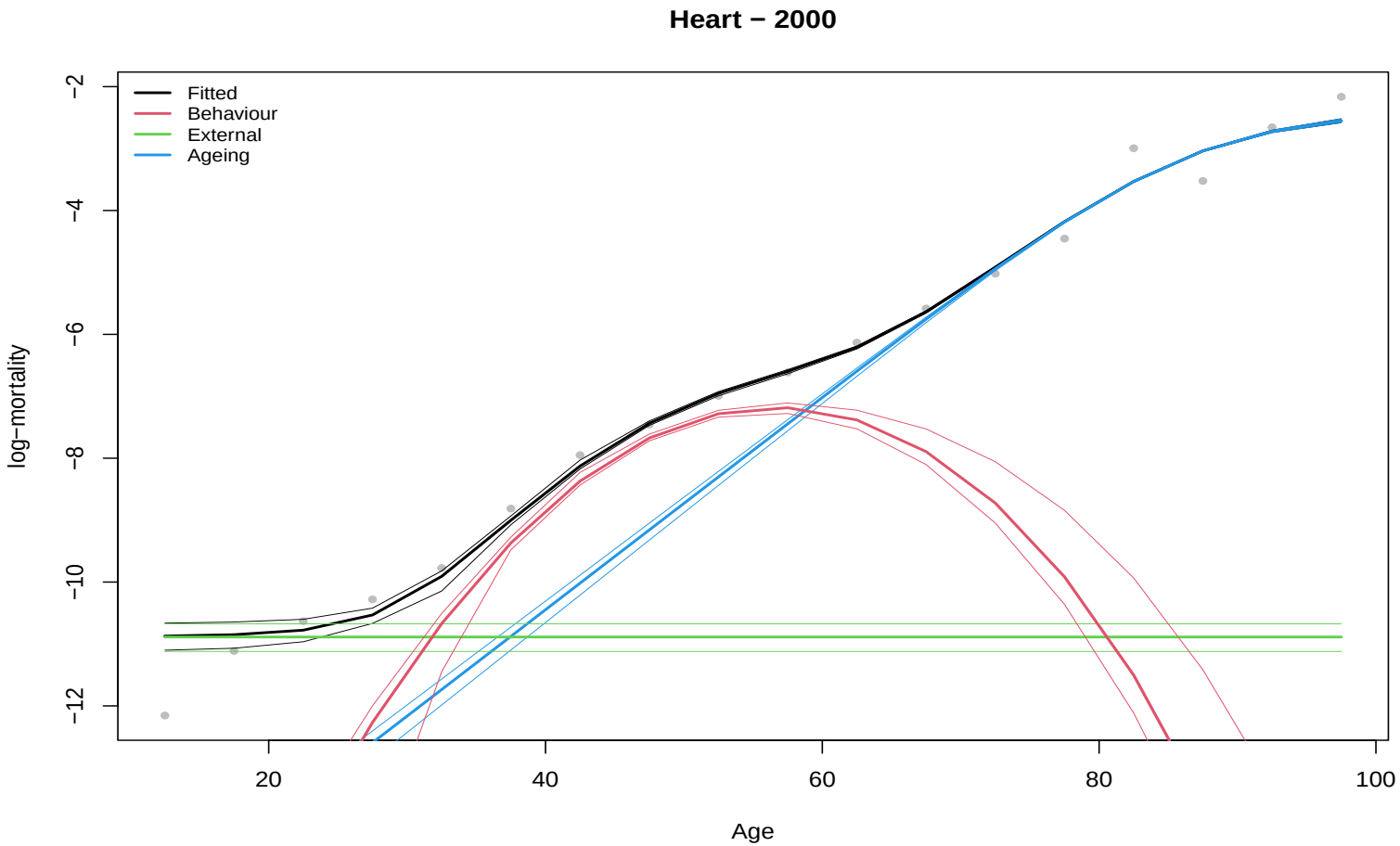

Infectious: Composition by Age

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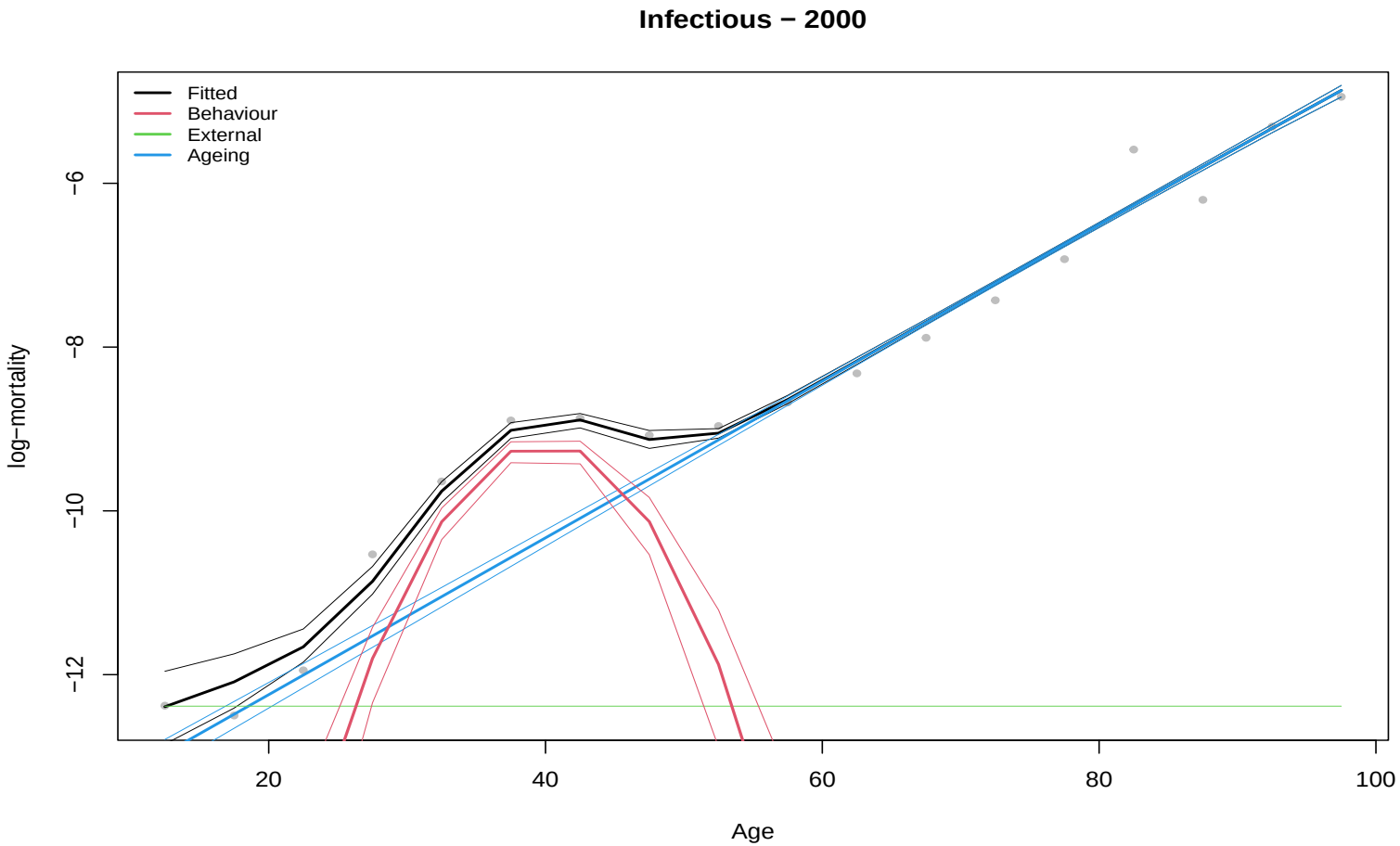

Mental: Composition by Age

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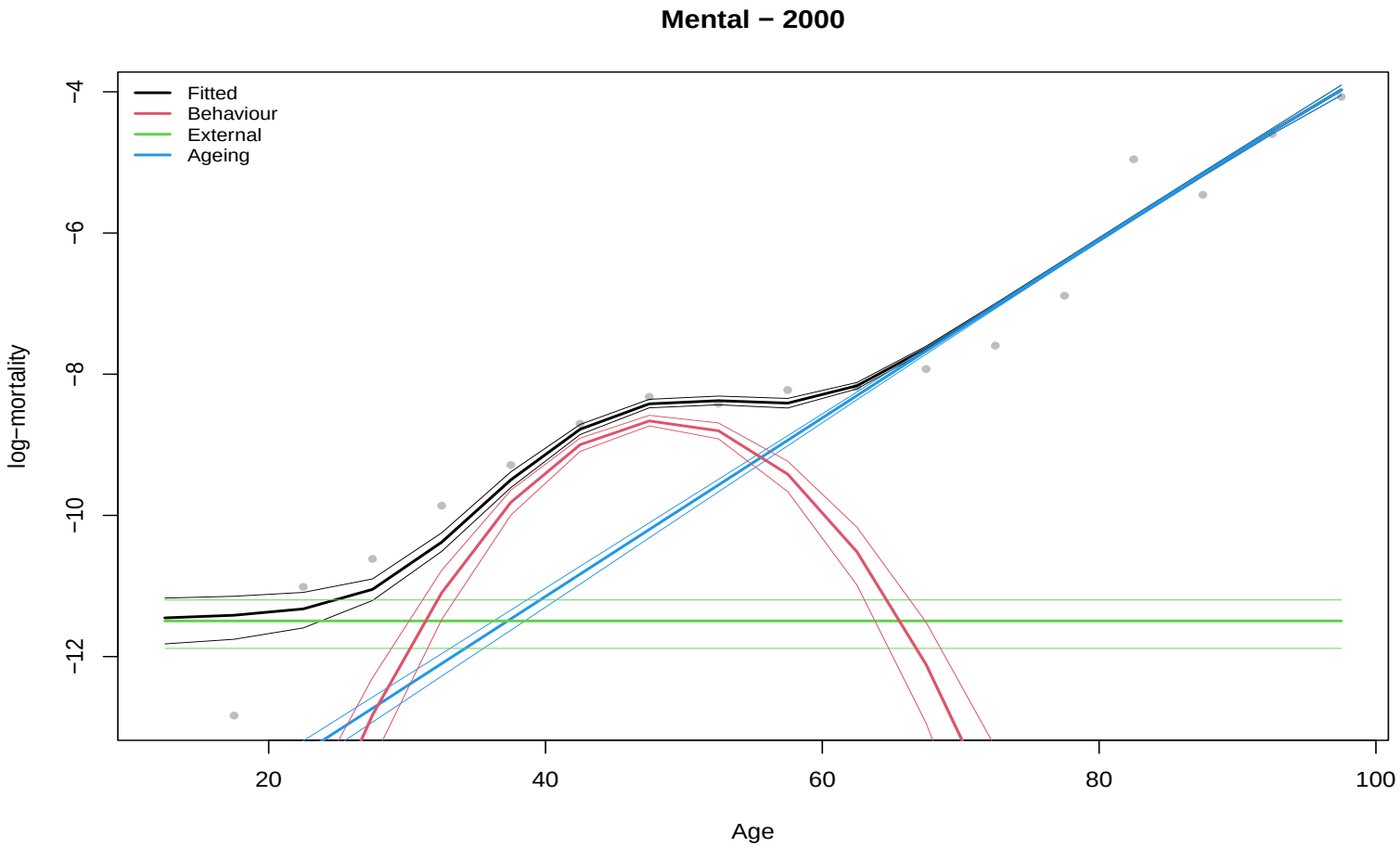

Neoplasms: Composition by Age

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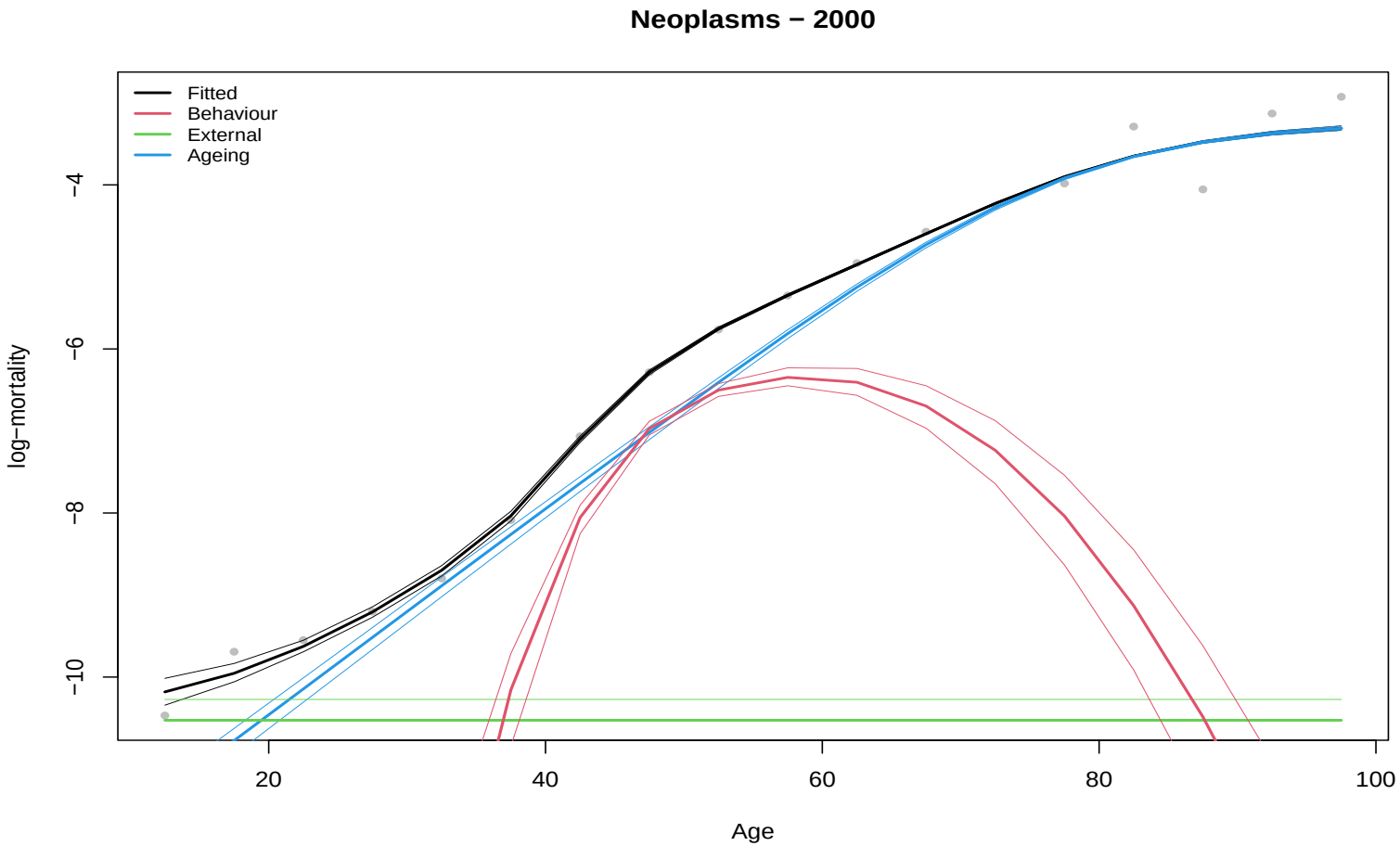

Nervous system: Composition by Age

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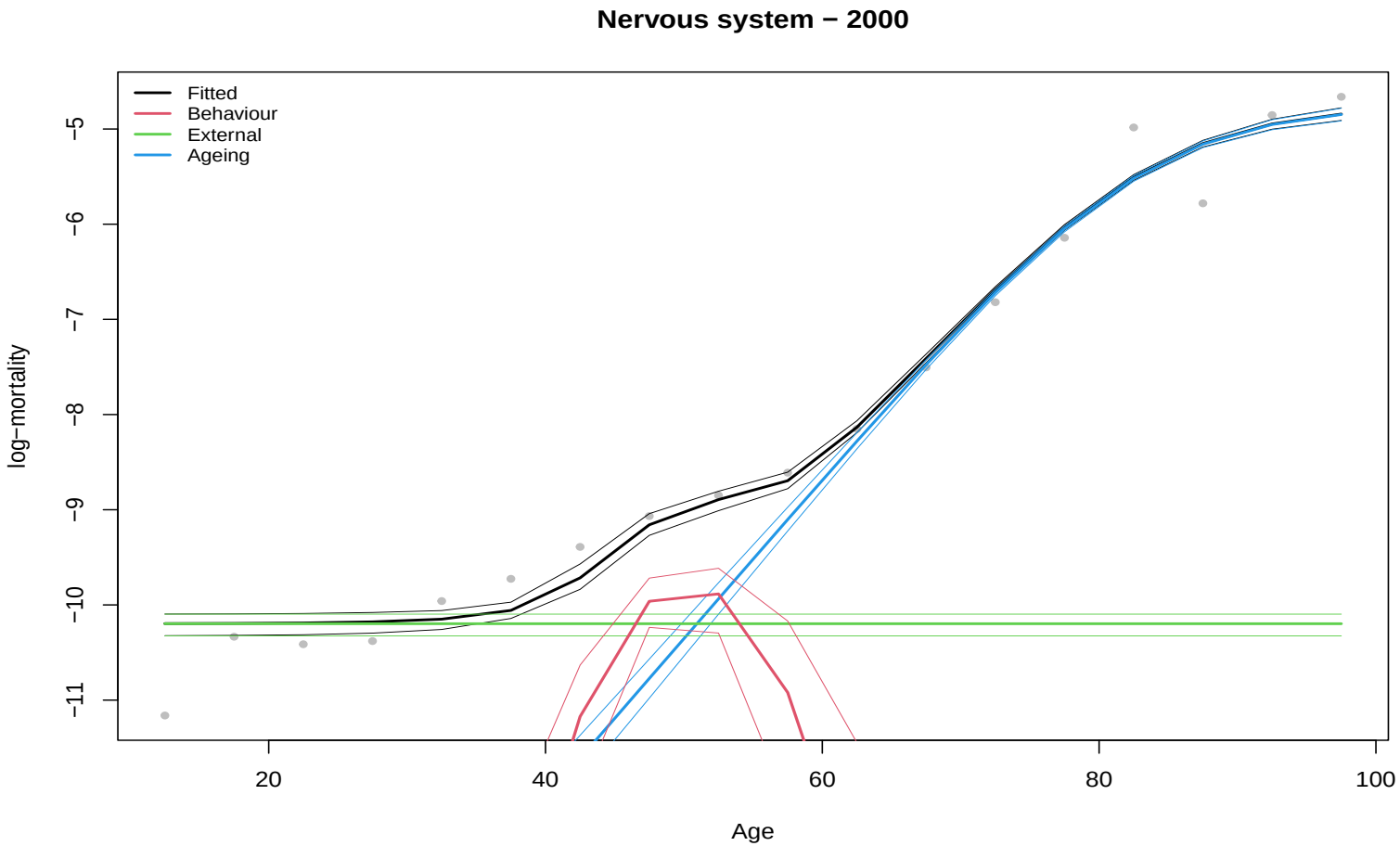

Other respiratory: Composition by Age

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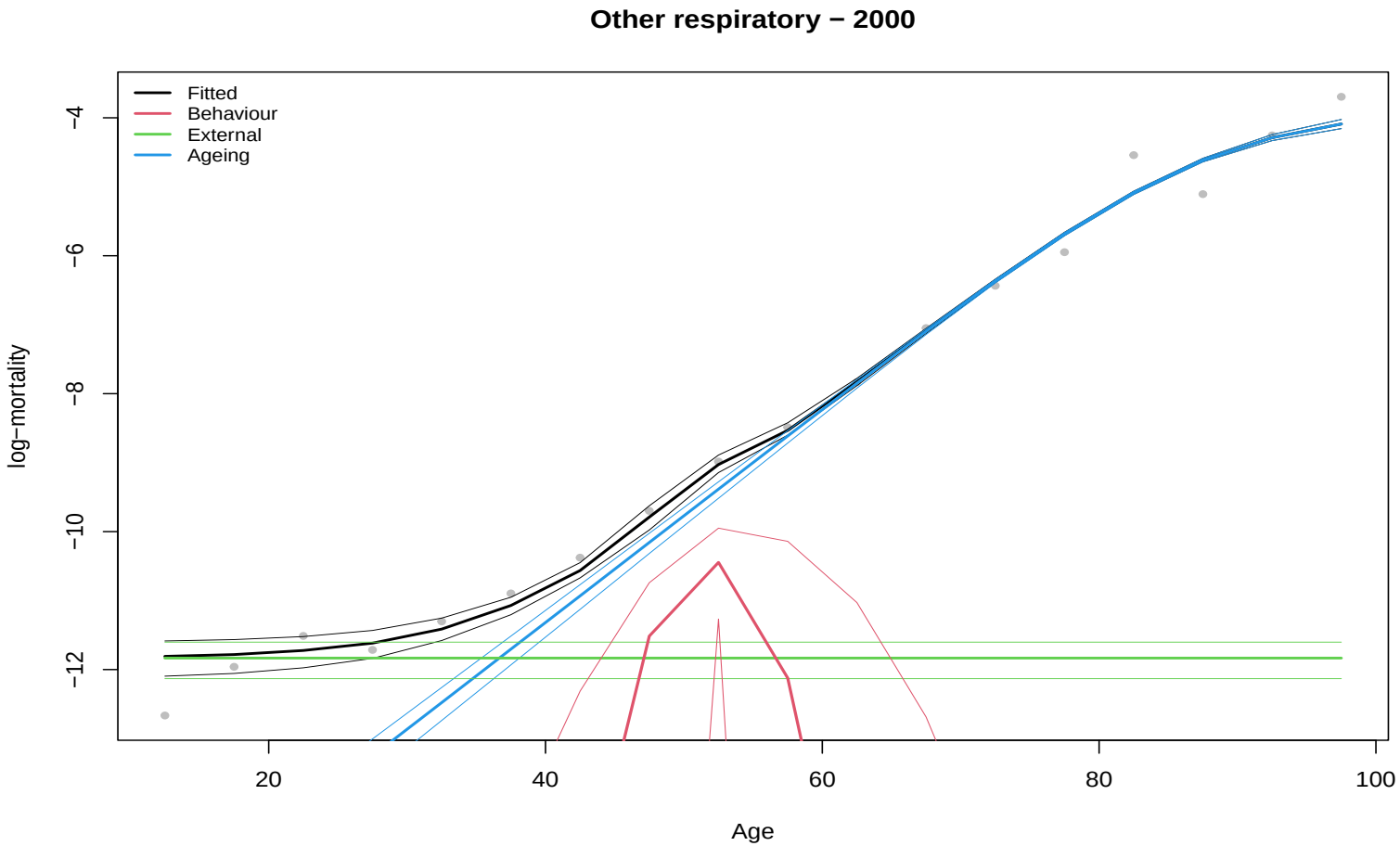

Skin: Composition by Age

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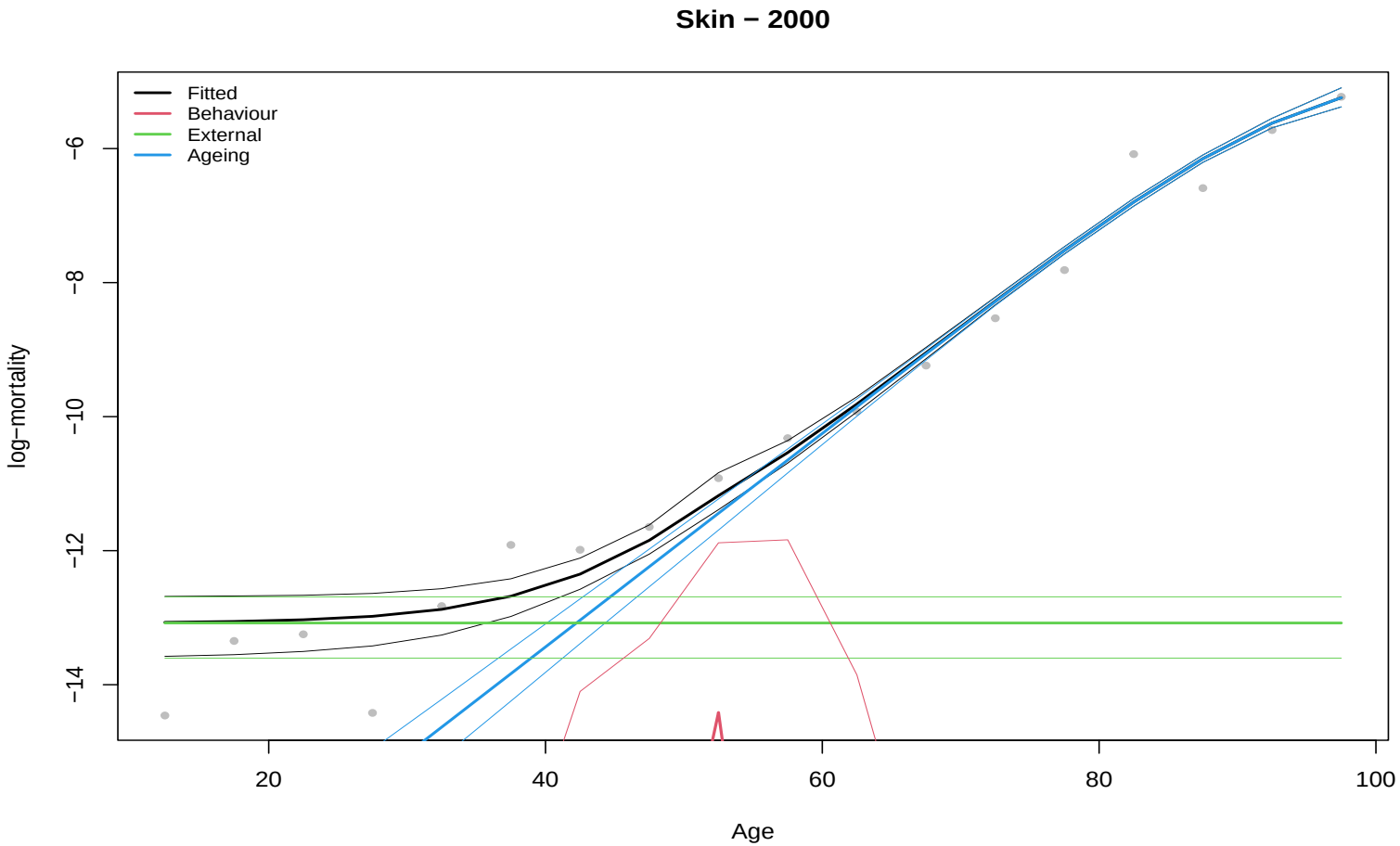

COD-Overall Mortality: Composition by Age

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Overall Mortality: Composition by Age

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