Stress discounting*

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Abstract

Standard evaluations of public policies involve discounting the flow of expected net benefits at a risk free discount rate. Consequently, they systematically ignore the insurance benefits of policies that hedge the aggregate risk, and the social cost of projects that raise the aggregate risk. Normative asset pricing theory recommends adjusting the discount rate to the project's risk, but few countries have attempted to implement this complex solution. We explore an equivalent approach based on the property that the value of a project under uncertainty equals the expected value of its state-contingent NPV, using the relevant state-contingent risk-free discount rate. Under this "stress discounting" approach, projects are evaluated under two polar risk-free economic scenarios, one business-as-usual scenario, and one low-probability catastrophic scenario, in the spirit of the now well-established banking regulation. Ramsey discounting should be performed in each scenario to estimate the corresponding scenario-contingent NPV, which is a simple and intuitive task. This approach automatically values the insurance benefits of projects whose net benefits are negatively correlated with economic growth. We extend this approach to value carbon mitigation projects, combining the two economic scenarios with two polar climatic scenario. We hope that this simpler and more intuitive method will induce more countries to better integrate the key value of risk when shaping optimal public policies, in particular those with long-lasting consequences.

Keywords: Discounting, carbon pricing, rare disasters, cost-benefit analysis, stochastic discount factor.

JEL codes: G12, H43, Q54.

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Perfection is achieved, not when there is nothing more to add, but when there is nothing left to take away. Antoine de Saint-Exupéry

1 Introduction

Under the standard practice, a project is deemed socially desirable if the discounted value of its flow of expected net social benefits is positive, using a unique public discount rate. For example, the official discount rate is 3.5% in the U.K. (Treasury, 2020). But this procedure ignores the social cost of projects that increase the macroeconomic risk, and the social benefit of projects that reduce it. In other words, it ignores risk and risk aversion. Compare for example an investment in a railways infrastructure (which arguably is most useful in a growing economy) to another investment in a mass vaccination infrastructure (which arguably is most useful during a pandemic that kills the economy). The standard public valuation practice values both projects equally if their expected net benefits are the same. However, risk aversion necessarily implies that a marginal reallocation of capital from railways to mass vaccination infrastructures would reduce the aggregate risk at no cost, thereby increasing welfare.[1]

Modern asset pricing theory such as the consumption-based CAPM (CCAPM), interpreted in a normative way, strongly recommends solving this problem by adjusting the discount rate to the risk profile of each project, and potentially of each of its specific benefit (Bodie and Merton, 2000; Brealey et al., 2017). Under the CCAPM and its extensions, the sufficient statistic of the risk profile is the CCAPM, which is the income-elasticity of the benefit under consideration. In the case of a railway infrastructure for example, this procedure would require to estimate the beta of the demand for transportation, but also the social cost of carbon and of air pollution, or the value of time lost and lives saved. But this perfect solution has failed. The complexity of this procedure is likely to explain its low level of adoption in the public sector around the world.

In this paper, we propose a simpler and more intuitive solution based on the fundamental asset pricing principle (Lucas, 1978):[2] Any state-dependent benefit $B_t = B_{ts} \mid_{s \in S}$ materializing at date t has a Present Value at date 0 equaling

$$PV = E[e^{-r_t t} B_t], \tag{1}$$

where $r_t = r_{ts} \mid_{s \in S}$ is the stochastic discount rate associated to maturity t and E is the expectation operator with respect to the states of nature $s \in S$. The state-dependent discount rate r_{ts} is the rate at which benefits in t should be discounted conditional to state s, i.e., under certainty. The Ramsey rule can be used to determine it. Project analysts just need to perform a sequence of NPV estimations under certainty. This approach is much simpler than the CCAPM approach because it just requires to use the Ramsey discount rate contingent to each state considered, thereby eliminating the complexity of risk-adjusting the discount rate together with the task to estimate the betas. This alternative approach is also more intuitive because the Ramsey rule is by now well understood. Moreover, measuring the value creation of an asset through the expectation of its contingent NPV should also be easily understood

[1] This is only partly accounted for in the U.K. where the public discount rate for health projects is reduced to 1.5% (Treasury, 2020).

[2] This principle is a direct consequence of the double additivity of the Discounted Expected Utility model, with respect to the states and to time. It does not hold under recursive preferences.

by practitioners. Finally, this approach does not give up anything to the basic principles of asset pricing. In fact, the CCAPM is a special case of the Stochastic Discount Factor (SDF) approach when assuming a geometric Brownian motion to consumption growth. More generally, the fact that the contingent discount rate is smaller in bad states of nature, i.e., in low-consumption states, provides the right instrument to deliver a premium to assets that yield more benefits in those states. Finally, our approach provides information about the circumstances in which the project is most valuable, as it measures the conditional PV of the project in each state of nature. This may be useful for the transparency of the decision-making and the public debate.

The complexity of this alternative approach depends upon the number of states of nature (or macroeconomic contingencies) that evaluators will have to consider. To choose the number of scenarios to consider and their characteristics, let us start by recalling that the CCAPM fails to predict interest rates and risk premia in financial markets (Mehra and Prescott, 1985; Weil, 1989). In short, this means that the gaussian volatility of the growth rate of consumption is too small to explain why the interest rate has been so low in the XXth century, and why the aggregate risk premium has been so larger. Rietz (1988) and Barro (2006, 2009) and Weitzman (2007) showed that this failure of the CCAPM can be solved by recognizing that the lower tail of the distribution of economic growth is fatter than assumed in the standard CCAPM. In Barro's work, the equilibrium risk-free rates and risk premia are mainly determined by the probability of a macroeconomic catastrophe, and the gaussian volatility plays only a marginal role on these matters. In this paper, we push this idea to this logical end by proposing to perform the expected NPV valuation with only two states. In the Business-As-Usual (BAU) scenario, consumption grows at a constant positive rate that reflects the likely economic prosperity. But in the low-probability catastrophic scenario, consumption drops immediately by an amount similar to what is suggested by Barro, and then gradually recovers at a growth rate smaller than in the BAU scenario. Calculating the contingent discount rates from the Ramsey rule in this two scenarios is then trivial. The aggregate risk in this model has thus 4 easy-to-understand parameters: the probability of catastrophe, the drop in consumption in that state, and the constant growth rates in the two states. The calibration of these parameters should closely fit the observed asset prices in the economy.

Other sources of risk can easily be added into this framework. To illustrate, in the context of climate change, the debate on the Social Cost of Carbon has explored the role of uncertainty mostly from the point of view of an uncertain climate sensitivity (Dietz et al., 2018; Daniel et al., 2019). Conditional to each macroeconomic scenario, a specific emission path can be considered together with an uncertain climate sensitivity to calibrate climate damages. We show that it is appropriate in that case to consider 2 types of shocks (economic and climatic), and therefore four distinct scenarios, to correctly evaluate cliate mitigation projects. Our objective here is to illustrate the method and to prove its efficiency. We check that the method gives satisfactory results by benchmarking it with more elaborate approaches. This is what we do in section 3 and 4 of this document, before applying these methods to real-based cases.

Thus, in this article, we start from the failure of national governments and key public institutions (World Bank, European Investment Bank,...) to adopt an efficient approach to discounting. There are various reasons why economists failed to improve the public discounting systems in the western world. The obvious argument is that using a single discount rate makes life much easier for the evaluators who are well accustomed with this practice, as estimating CCAPM betas may be technically difficult (Cherbonnier and Gollier (2020)). The second argument is that there is much at stake for many lobbies, in particular in those sectors with large betas. They are the losers of risk-adjusting the standard discounting system. There is a clear agency problem associated to the existence of various asymmetric information problems in this context. More flexibility in choosing project-specific discount rates may favor strong lobbies equipped with a good understanding of how to estimate CCAPM betas. Third, many public economists working in the sphere of public policy evaluations are not experts in asset pricing theory. At the same time, most asset pricing experts in academia continue to ignore public sector finance. Finally, adjusting discount rates to risk is a common practice in financial markets, but is often considered as highly inefficient by public servants in charge of implementing cost-benefit analyses in the public sector. They may be right, in the sense that the risk-adjustments used by private institutions and their stakeholders are inefficient. But this does not help. It is important to disentangle the normative nature of modern asset pricing theory, which provides a strong argument for the risk adjustment, from how market participants do apply it in practice.

Fifty years of developments in asset pricing theory has clearly demonstrated the key role of covariance with aggregate consumption when measuring the welfare impact of an asset. These developments justifying adjusting the discount rate to the risk profile of the projects had a very limited impact on the public practice of discounting. We are aware of only three countries which have attempted to adapt their evaluation practices in that direction. In 2020, the Dutch government (Rijksoverheid (2020)) has implemented a discounting system that contains three risk-adjusted discount rates. The Norwegian government had also adopted in 1997 a discounting system with three discount rates allocated to three risk classes defined by the projects' contribution to the aggregate uncertainty, but this system has been abandoned in 2012 as choosing the suitable risk class for a project was deemed too arbitrary[3] Since then, Norway uses a single discount rate of 4%. Finally, France has introduced a CCAPM rule since 2013 with a risk-free discount rate of r_f=2.5% and an aggregate risk premium of Π=2% (Quinet (2013)). Evaluators must estimate the beta of their project to determine the rate $r_f + \beta\Pi$ at which the expected net benefit must be discounted. The experience shows that very few evaluators have tried to estimate the beta of their projects, even for those costing tens of billion euros in public funds. They rather used a default beta of one, yielding an implicit single discount rate of 4.5%. In Gollier (2021), the welfare cost of ignoring the risk-adjustment in the discount rate has been estimated to be large, equivalent to 15% of permanent consumption at least. It is time to propose an operational method of discounting that has two properties: (1) approximate the efficient solution in a robust way, and (2) have a better chance to be adopted by the public sector, i.e., be simple, transparent and intuitive.

The paper is organized as follows. We recall in section 2 the principles of the SDF approach for public investment valuation. The basic stress discounting method with two scenarios is

[3]See Hagen et al. (2012) page 77: "*Experience from previous practice with several risk classes suggests that many project analysts have been uncertain about the technical criteria for choosing the risk class, and that such choices may therefore at times seem somewhat arbitrary. These circumstances suggest that it may be preferable to recommend simple and transparent rules that capture the most important aspects of the matter, without being too complex to understand or to apply.*"

explained, calibrated and illustrated in section 3. This latter section also provides some benchmarking exercises by comparing the results and implications of this method with those of the cutting-edge analytical methods (that depart from the classical Gaussian CCAPM in order to explain the asset pricing puzzles). In section 4, we expand the approach to allow for four scenarios, which is especially relevant for projects with a climate dimension. As an illustration, we estimate the discounted social cost of carbon, i.e. the discounted expected value of avoided damage when emitting one ton of CO2 less at a given horizon. Benchmarking exercises are then provided using a numerical approach based on the DICE model. Section 5 presents an application on a French nuclear waste project.

2 Two equivalent valuation methods

In this section, we first summarize the SDF approach to asset pricing. We then recall the methodology of the standard approach to value public investment and policies, stressing its operational complexities.

2.1 Expected and contingent present values

There is a representative agent in the economy whose discrete flow of consumption is given by the stochastic process $(C_0, C_1, ..., C_t, ...)$. This agent extracts utility $u(C_t)$ from consuming C_t at date t. Social welfare at date 0 is measured by the discounted sum of temporal expected utility, using a rate of pure preference for the present δ. Let's consider an investment project that generates a flow of net benefits $(B_0, B_1, ..., B_t, ...)$ that are potentially correlated to consumption.[4]

The Present Value (PV) of the project is defined as the sure monetary benefit received today that has the same impact on social welfare as a marginal investment in that project. In other words, PV equals

$$PV = \frac{1}{u'(C_0)} \left. \frac{\partial}{\partial \varepsilon} \right|_{\varepsilon=0} \sum_{t=0}^{+\infty} e^{-\delta t} E u(C_t + \varepsilon B_t). \tag{2}$$

By definition, it is socially desirable to invest in the project if and only if PV is positive. This condition can be rewritten as follows:

$$PV = E\left[\sum_{t=0}^{+\infty} b_t(C_t) e^{-r_t(C_t)t}\right], \tag{3}$$

where $b_t(C_t)$ is the expected benefit at date t conditional to C_t, and where the state-contingent discount rate $r_t(C_t)$ is defined as

$$r_t(C_t) = \delta - \frac{1}{t} \log\left(\frac{u'(C_t)}{u'(C_0)}\right). \tag{4}$$

This means that the value creation of an investment project is the expectation of the contingent present values $\sum b_t \exp(-r_t t)$, using a stochastic discount factor $\exp(-r_t t)$. We hereafter

[4]As is well-known, risks that are not correlated to aggregate consumption should not be priced. Therefore, B_t should be interpreted as the expected net benefit at date t conditional to C_t.

assume that the utility function u exhibits constant relative risk aversion γ. Let g_t denote the annualized growth rate of consumption:

$$g_t(C_t) = \frac{1}{t} \log \left(\frac{C_t}{C_0} \right) \tag{5}$$

Combining this definition with equation (4) yields the Ramsey rule:

$$r_t = \delta + \gamma g_t(C_t). \tag{6}$$

The pair of equations (3) and (6) fully describes an evaluation procedure in which the project analyst must perform three different tasks:

1. Characterize the flow of net expected benefits $(b_0, b_1, ...)$ conditional to each growth scenario;

2. Compute the contingent PV of this flow in each scenario, using the associated discount rate;

3. Compute the expectation of the contingent PVs to obtain the PV of the project.

Each of these tasks is intuitive and simple. The complexity of the procedure may emerge however if the number of scenarios to consider is large.

A special case is worthy examining in more details at this stage. Consider a risk-free project, or a project whose net benefits are independent of economic growth. In that case, equation (3) simplifies to

$$PV = \sum_{t=0}^{+\infty} B_t e^{-r_{ft} t}, \tag{7}$$

where the risk-free discount rate r_{ft} is defined as follows:

$$r_{ft} = -\frac{1}{t} \log \left(E e^{-r_t t} \right). \tag{8}$$

From this benchmark, one can observe that when projects are risky, their PV will be larger or smaller than the risk-neutral PV depending upon whether their net benefits are negatively or positively statistically linked to economic growth. More precisely, equation (3) directly implies that

$$\begin{aligned} PV &= \sum_{t=0}^{+\infty} E\left[B_t e^{-r_t t}\right] = \sum_{t=0}^{+\infty} \left(E\left[B_t\right] E\left[e^{-r_t t}\right] + cov(B_t, e^{-r_t t}) \right) \\ &\geq \sum_{t=0}^{+\infty} e^{-r_{ft} t} E\left[B_t\right] \end{aligned} \tag{9}$$

whenever the net benefit of the project and the stochastic discount factor covary positively. From the Ramsey rule (6), this is the case when the net benefit and consumption growth vary in opposite direction, i.e., are anti-comonotone. Inequality (9) states that the project has a negative risk premium in that case, i.e., its value creation is larger than if one would assume independence between its net benefit and aggregate consumption. The opposite result holds when they are comonotone.

2.2 Lessons from the standard approach

The tradition in the asset pricing literature is to value an asset as the discounted sum of its flow of expected benefits using a risk-adjusted discount rate:

$$PV = \sum_{t=0}^{+\infty} E\left[B_t\right] e^{-\rho_t t} \tag{10}$$

This approach is compatible with the SDF approach described in the previous section if and only the risk-adjusted discount rate ρ_t is defined as

$$\rho_t = -\frac{1}{t} \log \left(E\left[\frac{B_t}{E[B_t]} e^{-r_t t} \right] \right) \tag{11}$$

Further simplifications can be obtained by making two additional assumptions. First suppose that the net benefit of the project is linked to aggregate consumption through the following functional form:

$$\log\left(\frac{B_t}{B_0}\right) = a_t + \beta_t \log\left(\frac{C_t}{C_0}\right) + \epsilon_t, \tag{12}$$

where we assume exogeneity, so that $E\left[\epsilon_t \mid C_t\right] = 0$ for all C_t. Observe that we can interpret the project-specific β_t as the income-elasticity of its net benefit (at date t). Second, suppose that aggregate consumption follows a discrete version of a geometric brownian motion so that $\log(C_t/C_0)$ is normally distributed with mean μt and variance $\sigma^2 t$. In that case, it is well-known that the risk-adjusted discount rate equals

$$\rho_t = r_f + \beta_t \Pi, \tag{13}$$

with risk-free discount rate $r_f = \delta + \gamma\mu - 0.5\gamma^2\sigma^2$ and aggregate risk premium $\Pi = \gamma\sigma^2$. This is the standard CCAPM approach to discounting. Under that approach, the project analyst has an a priori simple task to perform. The analysis requires estimating the income-elasticity β_t of the project to determine the risk-adjusted rate to discount the flow of expected benefits.

In spite of its apparent simplicity, this standard approach faces serious operational difficulties. The most immediate one arises from the estimation of the beta. The flow of benefits may include several distinct components (for example a financial one and some externalities) each of them exhibiting an income-elasticity that requires estimation. The beta of the project is then the sum of the of the income-elasticity of these components. In addition, the income-elasticities need to be evaluated for each project. A difficulty that arises is due to the asset pricing puzzles that the CCAPM generates. With a growth process calibrated with growth rate $\mu = 2\%$ and volatility $\sigma = 3\%$ and with a CRRA $\gamma = 2$, we obtain a risk-free discount rate net of the rate of impatience of 3.82% and an aggregate risk premium 0.18%. In particular this risk premium is so low that it makes sense in practice to discount all project at the mean discount rate of 4%. Financial markets reveal much smaller interest rate on average (risk-free rate puzzle, Weil (1989), and much larger risk premia (equity premium puzzle, Mehra and Prescott (1985)). Over the last two decades, the asset pricing literature has solved these puzzles by considering growth stochastic processes that fatten the tails of the distribution of future consumption.[5] Bansal and Yaron (2004) has pioneered a new literature on "long run risks" in which trend of growth reverses to the mean and in which growth volatility is itself stochastic.[6] Rietz (1988) and Barro (2006, 2009) showed that the inclusion of rare disasters in the growth process can also solve these puzzles.

These strategies to solve the puzzles of the standard CCAPM make it much more complex to operationalize for project analysts. This is because the CCAPM formula (13) needs to be revised when exiting the Gaussian world. Indeed, as shown by Martin (2013), the efficient risk-adjusted discount rate becomes a polynomial function of the beta of the project, where the coefficient associated to the nth power of β is proportional to the $n+1$ cumulant of the annual change in log consumption. This raises some complexity to the analysis. Another source of complexity comes from the observation that the coherence of the calculation requires the project analyst to estimate the flow of expected benefits EB_t by using the complex growth stochastic processes that have been used to estimate the risk-free rate and the aggregate risk premium. In practice, this is infeasible without allowing some shortcuts that have not been provided in the literature up to now.

3 Basic project evaluation: stress discounting with two states of the economy

In the face of these complexities, should we go back to using a single discount rate? We believe not, because the cost of ignoring the social cost of risk in the economy is potentially large (Gollier (2021)). In this section, we develop an evaluation procedure based on the SDF methodology presented in Section 2.1. This alternative procedure aims at two objectives. First, we want it to be simple, intuitive and easy to operationalize. Second, we want it to generate valuations that approximate well the true value of assets and investment projects. Because these objectives goes in opposite directions, one should leave the precise procedure to implement to national circumstances, based on the willingness of project analysts and their principals to use a more complex procedure in exchange for more accurate results. The calibration of the procedure that we use in this paper should be interpreted as an illustration.

Inspired by Barro (2006, 2009), we propose a procedure based on the SDF approach using only two states or scenarios, a BAU scenario and a stressed scenario.

3.1 Projects maturing in one year

In this section, we limit our analysis to the evaluation of short projects, i.e., projects maturing within one period (here, one year). For a one-period horizon, our two-state uncertainty is characterized by three parameters: the growth rate g^b in the Business-As-Usual (BAU), the growth rate $g^s < 0$ in the catastrophic state, and the probability π of the catastrophe. We want to match three moments:

[5] Another path to solve the financial puzzle is to generalize the discounted expected utility framework into its Epstein-Zin-Weil extension. This approach raises two issues. First, Epstein et al. (2014) have shown that the calibration of EZW preferences necessary to solve the asset pricing puzzles generates a new puzzle, which is related to the implausibly large value of an early resolution of uncertainty. Second, under the veil of ignorance, risk aversion and the aversion to consumption fluctuations should be equivalent from a normative viewpoint.

[6] See Gollier (2018) for a discussion on the link between stochastic volatility and the fourth moment of the distribution of consumption.

δ	0	rate of pure preference for the present
γ	4	degree of relative risk aversion
r_f	1.0%	risk-free rate
Π	2.0%	aggregate risk premium
g^b	2.0%	growth rate of consumption in the BAU scenario
g^s	-33.3%	growth rate of consumption in the stress scenario
π	2.33%	probability of the stress scenario
G	1.3%	growth rate of expected consumption
r^b	8.0%	contingent discount rate in the BAU scenario
r^s	-133.4%	contingent discount rate in the stress scenario

Table 1: Benchmark calibration of the evaluation model.

- The growth rate g^b in the BAU. We assume a growth rate of $g^b = 2\%$ per year on the basis of the trend of growth in the western world over the last century.
- The risk-free rate r_f. We assume a risk-free rate of $r_f = 1\%$ corresponding to the average real rate of return of Bills in the western world over period 1880-2005 (Barro, 2009).
- The expected rate of return of a claim on aggregate consumption ($\beta = 1$). (Barro, 2009) documents an average real return on equity of 7.5%, whereas Bansal and Yaron (2004) assume a CCAPM beta of equity equaling $\beta = 3$. This is compatible with an aggregate risk premium Π around 2.17%. We calibrate our model to produce an aggregate risk premium of $\Pi = 2\%$. This means that the risk-adjusted discount rate for a share on aggregate consumption is 3% under our calibration.

We follow Barro (2009) by assuming a degree of relative risk aversion equaling $\gamma = 4$, which is an upper bound of what we recognize as a standard attitude toward risk. Finally, for moral reasons, we assume no discrimination across generations in the welfare function ($\delta = 0$). There is no ethical argument to penalize individuals on the basis of the generation to which they belong. It is in line with what was retained in Stern and Stern (2007) - a rate very close to zero, equal to 0.1% -, but the French and British authorities have for their part retained a rate equal to 1% and 1.5% respectively - cf. Quinet et al. (2013) and Treasury (2020).

Using the framework presented in the previous section, it is easy to verify that matching the above-mentioned moments univocally determines the remaining two parameters of our two-state model: The probability of catastrophe must be equal to $\pi = 2.33\%$, and the rate of growth in the catastrophic state must be equal to $g^s = -33.3\%$. Observe that the values of these two stress parameters are similar to those obtained by Barro (2009), as he estimated from international data a probability of catastrophe of 1.7% and an expected drop in GDP in that case of 29%. The efficient discounting system and its underlying economic context that supports it is summarized in Table 1. The expected consumption grow at rate 1.3%.

We now show that adding some gaussian noise to the growth process does not add much to the resolution of the asset pricing puzzle. Compared to our calibration for the 1-year

	risk free rate	aggregate risk premium
certainty	5.2%	0.0%
benchmark	1.0%	2.0%
benchmark with gaussian noise	0.7%	2.1%

Table 2: Effect of risk on asset prices. The "certainty case" corresponds to an economy growing at the sure rate $G = 1.3\%$. The benchmark case corresponds to the two-state growth model described in Table 1. In the "benchmark with gaussian noise", I add to the benchmark a gaussian noise in the BAU scenario of the benchmark model, with a standard deviation of 2%.

maturity, Barro (2009) has the additional ingredient of a gaussian noise around $g^b = 2\%$ in the BAU scenario. In his paper, the gaussian noise has a standard deviation of 2%. In Table 2, I compare the risk free rate and the aggregate risk premium under three economic growth process. The reference is an economy with no uncertainty at all, growing at the sure rate $G = 1.3\%$. In such an economy, the risk free rate equals 5.2%. We see that the two-state uncertainty examined in our benchmark dramatically reduces this interest rate to 1%. The addition of the gaussian noise with a volatility of 2% in the BAU scenario generates an additional reduction of the risk free rate to 0.7%. Concerning the aggregate risk premium, the two-state risk increases it to 2.0%, from 0 in the case of certainty. The addition of the gaussian noise has the marginal effect to increase it to 2.1%. This illustrates the fact that our two-state benchmark risk captures most of the asset pricing impact of the uncertainty of the Barro's model. In other words, removing the gaussian noise from the model has a marginal impact in the evaluation of projects.

Going back to our benchmark model, we know that there are two approach to the evaluation of projects. We support the simple SDF approach. Consider a 1-year project generating an expected benefit b_1^b conditional to the BAU scenario, and an expected benefit b_1^s conditional to the stress scenario. Using the exact Ramsey rule, we obtain the state-dependent discount rates $r^b = \gamma gs = 8.0\%$ and $r^s = \gamma\theta = -133.4\%$, Using the associated SDF e^{-r}, this yields the simple pricing formula

$$PV = (1-\pi)(0.92 b_1^b) + \pi(3.80 b_1^s) = 0.902 b_1^b + 0.088 b_1^s. \tag{14}$$

The strong asymmetry between the two state-dependent discount factors illustrates the valuation bonus for projects able to generate benefits in the stress scenario.

Let us compare this very simple valuation formula to what would be required in the CCAPM approach consisting in discounting the expected benefit at a risk-adjusted rate. The evaluator should first estimate the beta of the project. An estimation of this income-elasticity of the benefit is given by the following equation:

$$\beta = \frac{Log(b_1^b) - Log(b_1^s)}{g^b - g^s}. \tag{15}$$

This allows the evaluator to estimate in turn the risk-adjusted discount rate $(1 + 2\beta)\%$. Finally, the evaluator must discount the expected benefit $(1-\pi)b_1^b + \pi b_1^s$ at this rate. This is a long and obscure detour to produce a value for projects.

SDF Approach	
Present value	PV=$0.902 \times 2 + 0.088 \times 4 = 2.157$
Discount rate (eq. (11))	$\rho_1 = -\log(PV/Eb) = -5.25\%$
CCAPM approach	
beta	$\beta = (\log(b_1^b) - \log(b_1^s))/(g^b - g^s) = -1.96$
Discount rate (eq. (13))	$\rho_1 = (1+2\beta)\% = -2.92\%$
Present value	$=e^{-R}(0.9767 \times 2 + 0.0233 \times 4) = 2.107$

Table 3: Two approaches to the estimation of the value of a one-year project with state-dependent benefits ($b_1^b = 2, b_1^s = 4$) in the benchmark calibration of the two-state model of Table 1.

Let me illustrate the SDF and CCAPM approaches with a simple example under our benchmark calibration described in Table 1. We assume $b_1^b = 2$ and $b_1^s = 4$, which describes a negative-beta project. The expected benefit of the project equals 2.047. Following the SDF valuation rule (14), we immediately obtain a project value equaling 2.15. This is compatible with a risk-adjusted discount rate of -5.25%. Under the CCAPM approach, we first use equation (15) to obtain an estimation of the beta of the project. This gives $\beta = -1.96$, and thus a risk-adjusted discount rate of -2.92%. This yields a present value of 2.11. Beyond the unnecessary complexity of the CCAPM approach, notice the large discrepancy between the two estimations in terms of the (implicit) discount rate. this CCAPM approach does not generate an exact solution. Indeed, remember that the linear CCAPM discounting rule $r_f + \beta\Pi$ is exact only when the growth rate of consumption is gaussian, which is not the case as soon as one recognizes the existence of fat tails or catastrophic events.

We have already explained that outside the gaussian world, the efficient risk-adjusted discount rate is a polynomial function of the beta of the asset. Martin (2013) gives the following exact rule:

$$\rho_1 = \delta + \sum_{n=1}^{+\infty} \frac{\kappa_n}{n!} \left(\beta^n - (\beta - \gamma)^n\right), \tag{16}$$

where κ_n is the n-th cumulant of log consumption growth in the first period. The first cumulants are familiar: κ_1 is the mean, $\kappa_2 = \sigma^2$ the variance, $\kappa_3/Sigma^3$ the skewness, and κ_4/σ^4 is the excess kurtosis of $\log(C_1/C_0)$. In the gaussian case, only the first term in the summation operator of equation (16) is non-zero, which makes it equivalent to equation (13). In all other cases, higher-order cumulants are non-trivial and the risk-adjusted discount rate becomes a non-linear function of β. In our two-state benchmark, the log consumption growth is heavily negatively skewed, whereas $\beta^3 - (\beta - \gamma)^3$ is convex in β, so that this term introduces a concavity in the $(_1, \beta)$ relationship. In Figure 1, we represented the exact risk-adjusted discount rate and its CCAPM linear approximation. The special case described in Table 3 is illustrated in the left side of this figure.

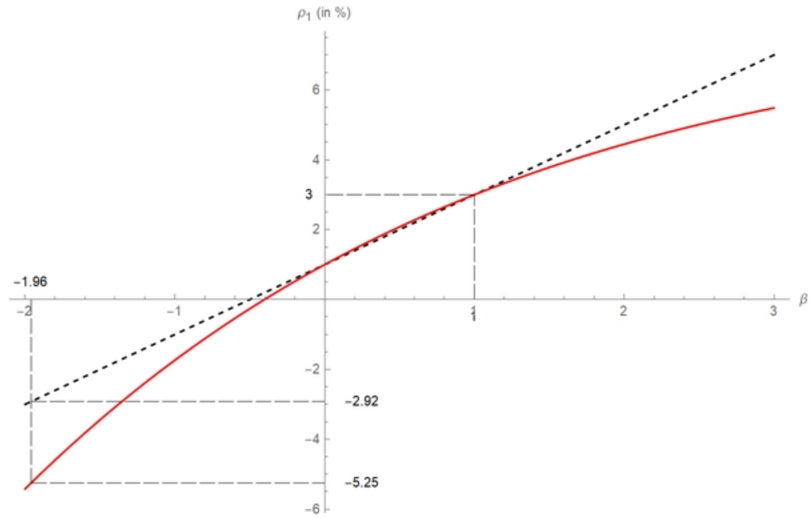

Figure 1: Risk-adjusted discount rate for a project with net benefit $b_1 = C_1^{\beta}$ as a function of β in the benchmark two-state model calibrated in Table 1. The red curve is the exact rate from equation (11), whereas the dashed line corresponds to its CCAPM approximation from equation (13).

3.2 Multiple-year projects

In this section, we generalize our model to time horizons larger than one year. In Barro's model, the growth process is i.i.d. over time, so that a catastrophe can occur every year. One advantage of this assumption is that the term structure of discount rates is flat, i.e., ρ_t is independent of t. The disadvantage is that evaluators face a myriad of scenarios to examine when considering long-lived projects. To keep our model as simple as possible, we continue to assume that there are only two possible scenarios, with a stress scenario occurring with probability $\pi = 2.33\%$, and a BAU scenario occurring with probability $1-\pi$. $\{C_t^s\}\mid_{t\in\mathbb{N}}$ and $\{C_t^b\}\mid_{t\in\mathbb{N}}$ describe the deterministic growth process of consumption in the stress and BAU scenarios respectively. The state-dependent annualized growth rate of consumption is denoted $g_t^i = (1/t)\log(C_t^i/C_0)$, $i \in \{s,b\}$. In this benchmark, we propose to select these two state-dependent growth processes in such a way that the term structures of risk-free rates and aggregate risk premia be flat, respectively at $r_f = 1\%$ and $\Pi = 2\%$ as in the one-year case. These two sets of conditions

$$r_f = \delta - \frac{1}{t}\log\left(\pi\exp(-\gamma g_t^s t) + (1-\pi)\exp(-\gamma g_t^b t)\right) \tag{17}$$

$$r_f + \Pi = \delta - \frac{1}{t}\log\left(\pi\exp((1-\gamma)g_t^s t) + (1-\pi)\exp((1-\gamma)g_t^b t)\right) + \frac{1}{t}\log\left(\pi\exp(g_t^s t) + (1-\pi)\exp(g_t^b t)\right), \tag{18}$$

for all $t \in \mathbb{N}/0$ univocally determine our two-state growth process characterized by $\{(g_t^s, g_t^b)\}\mid_{t\in\mathbb{N}/0}$. Of course, for $t=1$, the solution is as described in the previous section, with $g_1^s = -33.3\%$ and $g_1^b = 2.0\%$. Consumption levels and annualized gowth rates in the two scenarios are described in Table 4 and in Figure 2. In the stress scenario, the annualized growth rate increases

t	g_t^s	g_t^b	C_t^s	C_t^b	r_t^s	r_t^b
1	-33.40	2.00	0.72	1.02	-133.00	-133.00
11	-6.07	1.27	0.51	1.15	-24.30	-24.30
21	-3.53	1.20	0.48	1.29	-14.10	-14.10
31	-2.49	1.18	0.46	1.44	-9.94	-9.94
41	-1.90	1.19	0.46	1.63	-7.59	-7.59
51	-1.52	1.20	0.46	1.85	-6.08	-6.08
61	-1.25	1.23	0.47	2.12	-5.01	-5.01
71	-1.05	1.26	0.47	2.45	-4.22	-4.22
81	-0.90	1.31	0.48	2.88	-3.60	-3.60
91	-0.78	1.35	0.49	3.43	-3.11	-3.11
101	-0.68	1.41	0.50	4.14	-2.71	-2.71
111	-0.60	1.46	0.52	5.05	-2.38	-2.38
121	-0.53	1.51	0.53	6.22	-2.11	-2.11
131	-0.47	1.56	0.54	7.72	-1.87	-1.87
141	-0.42	1.60	0.56	9.61	-1.67	-1.67
151	-0.37	1.65	0.57	12.00	-1.49	-1.49
161	-0.33	1.68	0.58	15.00	-1.34	-1.34
171	-0.30	1.71	0.60	18.80	-1.20	-1.20
181	-0.27	1.74	0.61	23.50	-1.08	-1.08
191	-0.24	1.77	0.63	29.40	-0.97	-0.97

Table 4: Growth process and state-dependent discount rates in our two-state benchmark model. The state-dependent annualized growth rates g_t^i and discount rates r_t^i are expressed in % per year. We assume $\gamma = 4$ and $\delta = 0$. This yields a constant risk-free interest rate of $r_f = 1\%$ and a constant aggregate risk premium of $\Pi = 2\%$, for all maturities.

over time. Consumption is at a minimum 30-50 years after the crash, a period during wich consumption is 54% smaller than today. Two centuries after the crash, consumption remains 37% smaller than today. In the BAU scenario, consumption has an exponential trajectory, but the annualized growth rate of consumption is hump-shaped, with a minimum annualized growth rate at 1.2% around a time horizon of 20-50 years.

The corresponding efficient state-contingent discount rates are immediately derived from the Ramsey rule:

$$r_t^i = \delta + \gamma g_t^i, \tag{19}$$

for all t and for $i \in \{s, b\}$. Table 4 and Figure 2 describe the term structures of these two state-dependent discount rates. The stress-specific discount rates has an increasing term structure, but it remains negative for all maturities under consideration (200 years). The term structure of the BAU discount rates is hump-shaped in parallel to the BAU annualized growth rates.

The discounting system described in Table 4 provides an easy workplace for the evaluator, who must estimate the flow of expected benefits $\{b_t^s, b_t^b\} \mid_{t \in \mathbb{N}/0}$ of the project under scrutiny

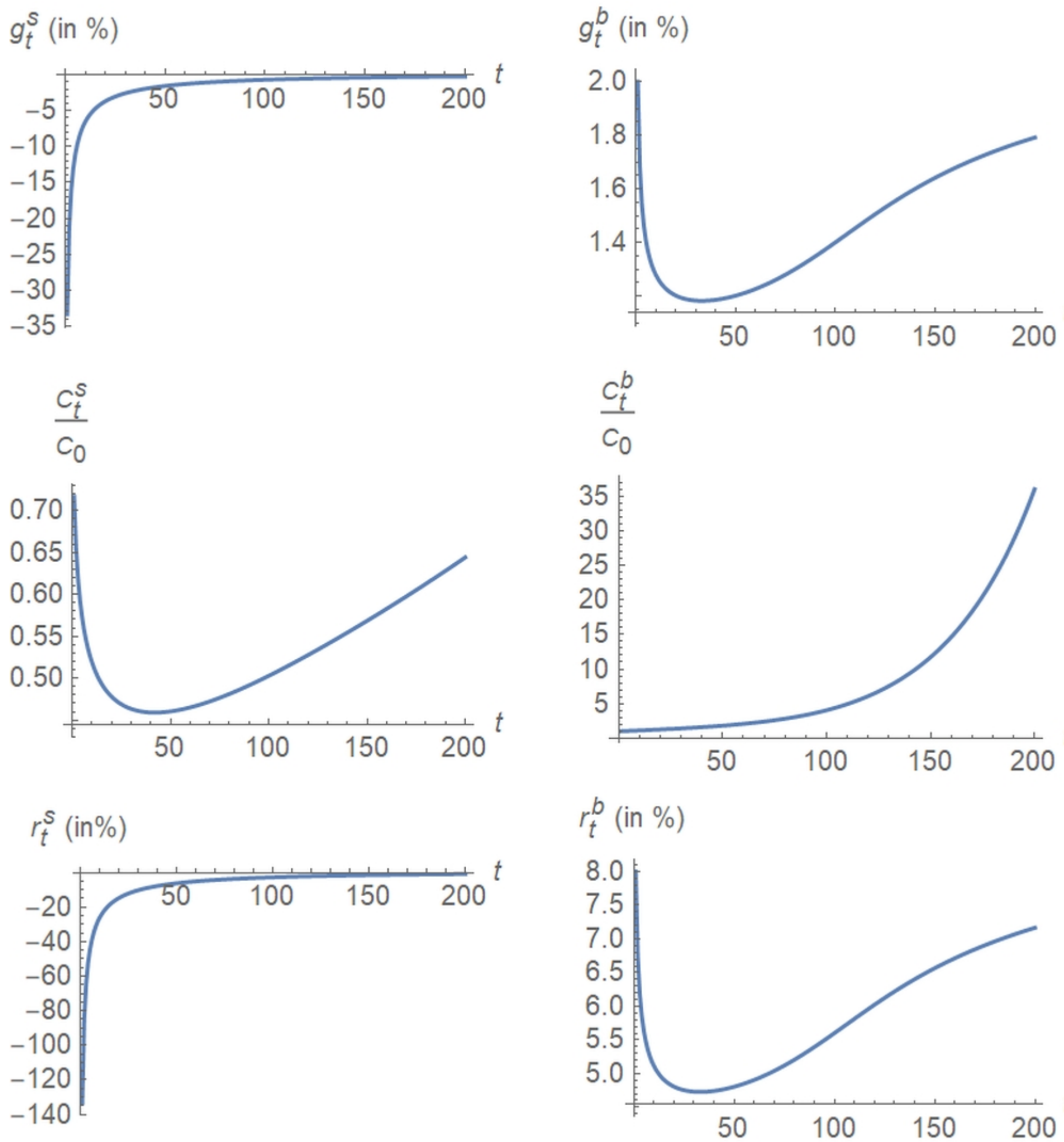

Figure 2: State-contingent annualized growth rates (line 1), consumption levels (line 2) and discount rates (line 3) in our benchmark model. This is a graphical representation of Table 4.

under the two scenarios. This allows the evaluator to compute the contingent PVs

$$PV^i = \sum_{t=0} b_t^i \exp(-r_t^i t), \tag{20}$$

and eventually the value of the project which is the expected PV $\pi PV^s + (1-\pi)PV^b$.

4 When climate matters: Stress discounting with four scenarios

In the context of climate change, future aggregate consumption will mostly depend upon two sources of uncertainty: the growth of Total Factor Productivity (TFP), and the climate sensitivity. A large climate sensitivity will raise climate damages, and will therefore adversely affect aggregate consumption. As discussed by Dietz et al. (2018), Cai and Lontzek (2019) and Lemoine (2021), these two sources of risk imply opposite correlations between consumption and the benefit of mitigation. If TFP uncertainty predominates, consumption and mitigation benefits will be positively correlated, as a larger growth yields more emissions and a larger marginal benefit of mitigation, assuming a convex damage function. If the uncertainty affecting the climate sensitivity dominates, consumption and mitigation benefits will be negatively correlated. Indeed, a larger climate sensitivity implies at the same time a larger mitigation benefit and a smaller future aggregate consumption. This means that the risk-adjusted discount rate to value mitigation efforts, i.e., to estimate the Social Cost of Carbon (SCC), is highly sensitive to the way these two sources of risk. According to Dietz et al. (2018), the so-called “climate beta”, is smaller tan 1, and has a decreasing term structure. Indeed, the risk of climate damage is increasing over time while negatively correlated with economic growth. Consequently, the beta is decreasing over time, and could take low or even negative values on the long term. This means that climate mitigation projects might exhibit an insurance value over the long run.

In this section, we propose to re-examine these issue by considering two risks with two possible outcomes each, i.e., a four-state structure of risk.

4.1 Methodology

Applying the stress discounting procedure to value an investment project requires more than two scenarios to take into account both economic and climatic uncertainties. We follow Dietz et al. (2018) who analyzes 10 sources of uncertainty and shows that the two prevailing ones are shocks on TFP and climate sensitivity. Consequently, we consider two binary risks, that of an economic disaster as before, and that of a climatic catastrophe corresponding to a very high climate sensitivity. We combine these two sources of uncertainty, giving a total of four possible scenarios.

One of the key outputs is the expected discounted value of carbon benefit. In practice, the public authorities identify a tutelary value for carbon, that is a social cost of carbon path $\overline{SCC}_t$, equal to the consumption-equivalent at time t of the damage induced by emitting one more ton of CO_2 at the same period. What matters here is the present social value of a project avoiding carbon emissions. The evaluator can compute these values by discounting the social cost of carbon, using the official discount rate r defined by the public authorities.

This is what is shown in the left part of the following equation. In reality, the social cost of carbon SCC_t is not the same along the different states of nature (depending on growth g and climate sensitivity S), and the exact calculation should compute the expected weighted value, as shown in the right part of this equation:

$$e^{-rt}\overline{SCC}_t \simeq \frac{E[u'(C_t(S,g))SCC_t(S,g) \mid S,g]}{u'(C_0)}$$

As explained in the previous section, when the avoided carbon emissions are proportional to the consumption raised to the β power (as it would be the case for instance if they are linked to a volume of passenger transport), the expected value of environmental externality is then

$$e^{-rt}\overline{SCC}_t C_t^{\beta} \simeq \frac{E[u'(C_t(S,g))SCC_t(S,g)C_t^{\beta}(S,g) \mid S,g]}{u'(C_0)}$$

where the right-hand side corresponds to the approximate method currently used and the left-hand side gives the exact value when considering all the states of nature. The stress-discounting method amounts here to do this calculation by considering only four states of nature. It requires to provide the evaluator for each of the four scenarios with a social cost of carbon path and a basic assumption on economic growth (that translates into a constant discount rate). We will apply this method in section 5 but need first to calibrate then benchmark it. This is done in the two following sections.

4.2 Calibration

Before anything else, general assumptions are needed on risks and on climate damage. Regarding environment-economy modelling, we use the DICE 2016R2, with slightly modified parameters to allow easy comparisons with the analysis done in section 3.[7] We also assume that the environmental policy is set at a low level of ambition (which amount to stop all emissions in 2080), and is not modified to take into account any surprises on growth or climate. This amounts to saying that we do not become aware until too late if the sensitivity to the climate is much higher than expected (Pindyck (2021)) or if we enter in a period of secular stagnation with very low economic growth.[8]

As previously mentioned, we also follow Dietz et al. (2018) who show that two main sources of uncertainty matter, respectively on economic growth and on climate sensitivity, and use their calibration. More precisely, they assume that the growth rate of the total factor productivity (TFP) is driven by a fist-order autoregressive process with an uncertain trend growth rate g_0 that follows a normal law with mean 1.6% and standard deviation 0.9%. Regarding climate sensitivity, Dietz et al. (2018) consider as in Dietz and Stern (2015) that it follows a log-logistic function with a mean at $2.9C^{\circ}$, a standard deviation equal to $1.4C^{\circ}$, truncated from below at 0.75°.

As mentioned before, all these choices are only indicative - in particular, it is up to the public authorities to choose these assumptions, and possibly use a more recent Integrated

[7]We choose as previously a pure rate of preference equal to zero, instead of 1.5% as originally in DICE, and assume no deceleration of TFP, i.e. using the notations used in this model $dela = 0$ instead of 0.5%.

[8]In practice, this means that we maintain in these simulations the emission control rate (MIU parameter in DICE) at its optimal level as calibrated in the base scenario (we assumed here that emissions cannot become negative, following Anderson and Peters (2016)).

Assessment Model. Similarly, regarding the scenarios, we arbitrarily choose to retain disaster scenarios with a probability of occurring equal to 8%. Regarding climate, under the assumptions defined in the previous paragraph, there is a probability equal to 92% that the climate sensitivity is under 4.5° with a mean at 2.6°, and otherwise above this threshold with a mean at 5.7°. We analyze growth distribution using Monte Carlo simulations, and check that the average growth in the 8% worst case corresponds to a stressed scenario with a sudden drop of 43% and a subsequent yearly negative TFP growth equal to -0.2%. The average growth in the 92% remaining cases correspond to a TFP growth equal to 2.3%. We check that the consumption growth is relatively independent of the climate sensitivity, and equals respectively to 3.6% and -0.1% in the BAU and the stressed economic scenarios. We thus obtain the following four scenarios :

- Good climate and BAU (probability 84.64%) : $S = 2.6$ and $g = 3.6\%$;
- Bad climate and BAU (probability 7.36%) : $S = 5.7$ and $g = 3.6\%$;
- Good climate and Stress (probability 7.36%) : $S = 2.6$ and $g = -0.1\%$ after 43% drop of consumption;
- Bad climate and Stress (probability 0.64%) : $S = 5.7$ and $g = -0.1\%$ after 43% drop of consumption.

We can then estimate the paths of temperatures and the social cost of carbon for the four corresponding scenarios, as show in figure 3. As expected, in the scenario with an economic shock, the SCC is higher at the beginning (before the sudden drop in consumption) and lower later on ~~after~~ because it corresponds to the monetary equivalent of damage while future generations are poor in those stressed scenarios.

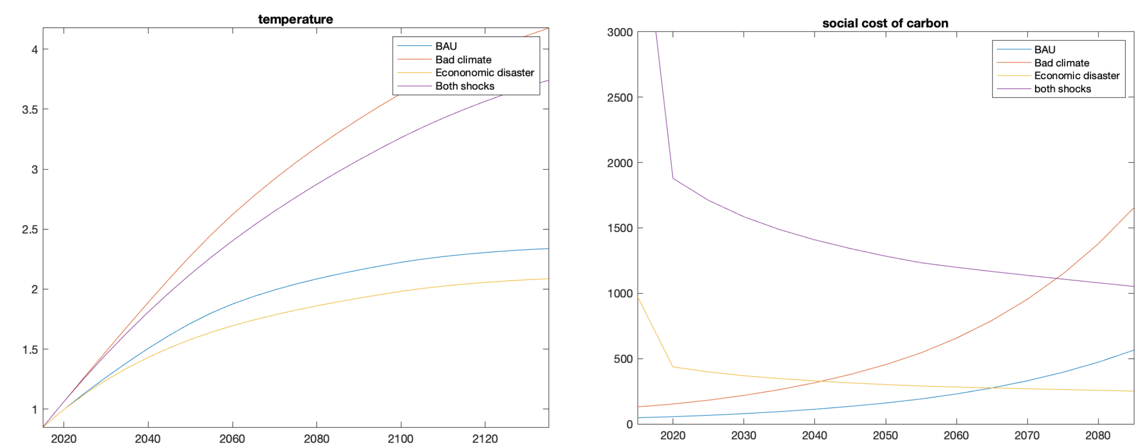

Figure 3: Temperature and social cost of carbon for the four scenarios.

4.3 Benchmark comparison

We can then compute the expected discounted value of carbon benefit, as explained in previous section, and benchmark it to the exact value obtained by running 5000 draws of a

Monte Carlo simulation. We also benchmark it to what would be obtained by using a single discount rate and a single social cost of carbon path, corresponding to the average scenario. The latter scenario is obtained by assuming a mean climate sensitivity equal to 2.9° and taking the mean TFP obtained from the Monte Carlo simulation. This is close to assuming that annual consumption growth is 2.7%, which corresponds to a non-risk adjusted social discount rate equal to 3.915%. The result is shown in the left part of the figure 4. We see that the value obtained using the stress-discounting method is very close to the exact value, whereas the method based on a single growth scenario and a single discount rate provides a value more than 50% too low. This shows that taking risk into account greatly raises the carbon value (this point is discussed in more detail on an example in the next section) and that limiting ourselves to four schematic scenarios is a good approximation. The graph on the right hand-side provides the risk-free rates according to those three approaches, and shows that the stress-discounting method provides, as seen in section 4, a declining term structure close to the real value.

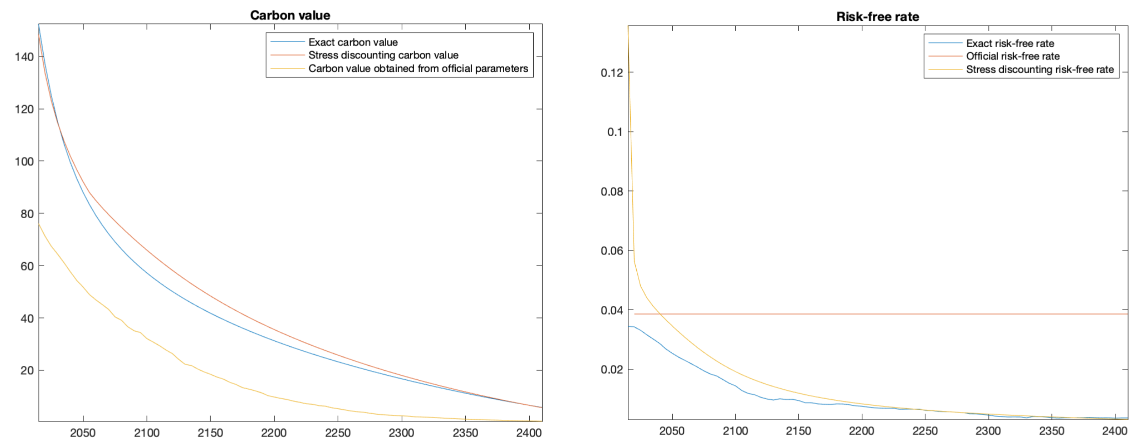

Figure 4: Discount rates and expected present carbon value obtained from a numerical estimation (with 5000 Monte Carlo simulations), from the stress-discounting method and from an approximation based on a single discount rate and a single carbon path (mimicking the method currently followed by public administration).

Another benchmark can be provided through the illustrative example of a project that avoid emitting 1 ton of CO2 each year during 50 years starting from now. The expected present value of such a project is presented in figure 5.

STRESS DISCOUNTING			
	probability	NPV	*contribution*
No negative shocks	*85%*	335	*283*
climate shock	*7%*	932	*69*
eco shock	*7%*	7322	*539*
both shocks	*1%*	31361	*201*
stress		1091	
BENCHMARK			
Exact value		1080	
base		600	

Figure 5: Valuation of a project avoiding 1tCO_2 per year

We observe that

- The stress discounting method gives a very good approximation of the exact value, whereas the official method is very wrong:
- The fourth scenario, despite his very low probability of occurrence, has a significant impact. When both shocks are present, the discounted carbon value is very high because of the high climate sensitivity and because future generations will be relatively poor.

5 Applications

5.1 Nuclear wastes

In France, the second generation of nuclear power plans (1970-2050) will produce a total of 83,000 m^3 of nuclear wastes of high activity or medium activity/long life. The current policy project is to build a geological repository at a depth of 500 meters in the French Ardennes. The site will take 10 years to be built, and the wastes will progressively be transferred in the repository over the next century, for a final irreversible closure around 2150. The flow of gross costs associated to this project is described in Figure 6, with a non-discounted sum of 25.5 billion euros. The code name of this project is Cigéo.[9] Its management has been delegated to ANDRA, the national agency in charge of nuclear waste management. There exist various alternative solutions to Cigéo. Many are either not technologically mature or prohibitively more expensive (Bouttes et al. (2021)). Let us examine the credible alternative of a Permanent Surface Storage (PSS). The PSS strategy consists in periodically repackaging the nuclear wastes to be stored on surface or subsurface, as currently practiced in all countries producing nuclear electricity. For France, the annual gross costs of PSS strategy are estimated at 100 million euros. Gross costs do not take account of the elasticity of these costs (of labour, cement, land use, capital,...) to changes in GDP. Following (Bouttes et al. (2021)), we assume an income-elasticity of these costs equal to 0.8. Notice that this implies that costs will be larger in the BAU scenario than in the stress scenario, and that Cigéo has no macro-hedging benefit if we limit the analysis to the income-elasticity of these costs.

[9] For "Centre Industriel de stockage GEOlogique" in French.

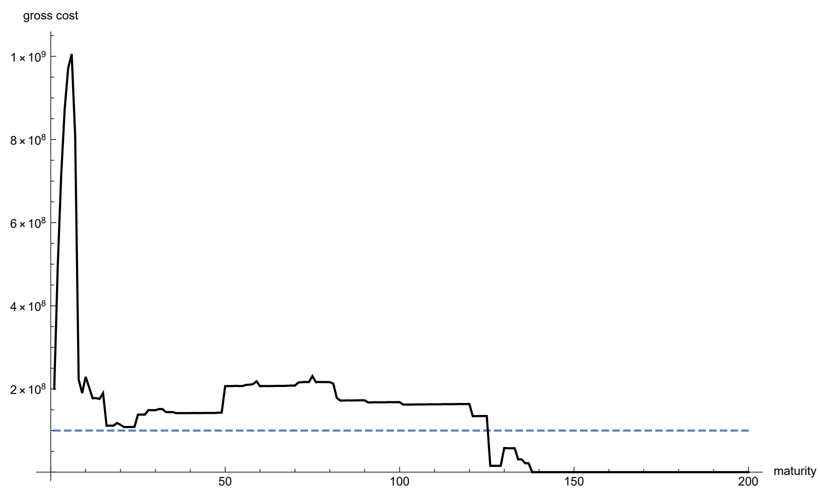

Figure 6: Flow (to be extrapolated to infinity) of the gross costs (in euros per year) of Cigéo (geological repository, plain curve) and PSS (surface storage, dashed curve). Source: ANDRA.

	contingent value		expected
	BAU	stress	value
PV Cigéo	10.61	47.00	12.43
PV PSS	4.68	163.69	12.63
PV PSS with damages	4.68	240.72	16.48

Table 5: Valuation (in billion euros) of Cigéo and PSS costs. In the last line, we add a permanent flow of health and environmental damages $x = 50$ million euros/year materializing in the stress scenario if the PSS option is implemented ex ante.

In Table 5, we summarize the stress discounting procedure to evaluate the competing options Cigéo and PSS. In the BAU scenario, the contingent PV of PSS costs is much smaller than the contingent PV of Cigéo costs. This is due to the observation that with a discount rate as high as $r^b = 3.6\%$, the PSS option is very attractive given the postponment of most expenditures. If one were sure that the BAU scenario would prevail, Cigéo should not be implemented. But the opposite conclusion should be made contingent to the stress scenario, because of the much smaller discount rate $r^s = 0.2\%$ combined with the large penalty $\Delta = 3.08$ to be used when evaluating costs in that scenario. When taking the expected value of these two pairs of contingent PV costs using the stress probability $\pi = 5\%$, we obtain basically the same PV of the costs, around 12.5 billion euros, with a small advantage for Cigéo.

An important piece of the story is missing in this comparison of the two options to manage nuclear wastes in the long run. The geological repository is used as a passive natural barrier to radionuclides. On the contrary, the surface storage of nuclear wastes requires an active maintenance to guarantee its safety. Cigéo has thus an important safety benefit compared to PSS, in particular in the stress scenario. Such a scenario is likely to be associated with a degradation of our democratic institutions and their ability to maintain the right level of supervision, protection and maintenance of the surface storage, as well as cope with the potential adverse consequences of a nuclear incident. To model this idea, let us assume that in the stress scenario, a permanent flow $\bar{B}_s$ of health and environmental damages is incurred by the local population if the PSS option had been selected ex ante. The contingent PV of these damages equals $\bar{B}_s\Delta/(1-\exp(-r^s))$. Multiplying this contingent PV by the probability π of the stress scenario yields the additional PV of costs of the PSS option. Suppose for example a flow $\bar{B}_s = 50$ million euros of annual damages. The contingent PV of this flow in the stress scenario is 77 billion euros. Consistent with equation (**??**), this adds 3.85 billion euros to the expected PV costs, thereby inducing a strong preference for Cigéo. The safety issue associated to the PSS option means that Cigéo is an insurance for future generations, and Table 5 makes that apparent. Although the risk of a chaotic evolution of our society is small, Cigéo should be implemented because of its insurance benefit against the large health and environmental risk of the alternative option.

Imagine how the project analyst would have proceeded if asked to use the standard CCAPM approach to evaluate Cigéo, using PSS as the default option. This analyst should first estimate the CCAPM-beta of the net benefit of Cigéo. It would include the positive income-elasticity of the costs and the negative beta associated to the safety issue examined above. The global CCAPM-beta of Cigéo has a term structure, that should be used to determine the maturity-specific Cigéo risk-adjusted discount rates using the term structures of the risk-free rate and aggregate risk premia examined in Section 3. Then, the analyst should estimate the flow of expected net benefits of Cigéo, using the information about the distribution of future incomes and the income-elasticity of the costs that include health and environmental damages. Finally, this flow should be discounted using the maturity-specific discount rates. It will generate a positive NPV for Cigéo. However, this standard approach is complex and does not provide the essence of the argument for why Cigéo should be implemented.

5.2 Climate mitigation project ?

transportation project with strong climate mitigation impact, such as grand paris express ?

6 Conclusion

[TBD]

References

ANDERSON, K. AND G. PETERS (2016): "The trouble with negative emissions," Science, 354, 182–183.

BANSAL, R. AND A. YARON (2004): "Risks For the Long Run: A Potential Resolution of Asset Pricing Puzzles," Journal of Finance, 59, 1481–1509.

Barro, R. (2006): “Rare Disasters and Asset Markets in the Twentieth Century,” Journal of Finance, 121, 823–866.

——— (2009): “Rare disasters, asset prices, and welfare costs,” Amercican Economic Review, 99, 243–264.

Bodie, Z. and R. Merton (2000): Finance, Pearson.

Bouttes, J.-P., C. Gollier, A.-L. M. Allemand, A. Pommeret, and E. Preud’homme (2021): “Contre-expertise de l’évaluation socioéconomique du projet de CIGEO,” Tech. rep., Secrétariat Général pour l’Investissement.

Brealey, R., S. Myers, and F. Allen (2017): Principles of corporate finance, 12th edition, McGraw Hill Education.

Cai, Y. and T. S. Lontzek (2019): “The social cost of carbon with economic and climate risks,” Journal of Political Economy, 127, 2684–2734.

Cherbonnier, F. and C. Gollier (2020): “Risk-adjusted social discount rates,” TSE DP.

Daniel, K. W., R. B. Litterman, and G. Wagner (2019): “Declining CO_2 price paths,” Proceedings of the National Academy of Science, 116, 20886–20891.

Dietz, S., C. Gollier, and L. Kessler (2018): “The climate beta,” Journal of Environmental Economics and Management, 87, 258–274.

Dietz, S. and N. Stern (2015): “Endogenous growth, convexity of damage and climate risk: how Nordhaus’ framework supports deep cuts in carbon emissions,” The Economic Journal, 125, 574–620.

Epstein, L., E. Farhi, and T. Strzalecki (2014): “How Much Would You Pay to Resolve Long-Run Risk?” American Economic Review, 104, 2680–2697.

Gollier, C. (2018): “Stochastic volatility implies fourth-degree risk dominance: Applications to asset pricing,” Journal of Economic Dynamics and Control, 95, 155–171.

——— (2021): “The welfare cost of ignoring the beta,” TSE DP.

Hagen, K., S. Bernstein, B. Bye, L. Hltkrantz, K. Nyborg, K. Pedersen, M. Sandsmark, G. Volden, and G. Avistand (2012): “Cost-benefit analysis,” Tech. rep., Official Norwegian Reports NOU 2012:16.

Lemoine, D. (2021): “The climate risk premium: how uncertainty affects the social cost of carbon,” Journal of the Association of Environmental and Resource Economists, 8, 27–57.

Lucas, R. (1978): “Asset prices in an exchange economy,” Econometrica, 46, 1429–46.

Martin, I. (2013): “Consumption-based asset pricing with higher cumulants,” Review of Economic Studies, 80, 745–773.

Mehra, R. and E. C. Prescott (1985): “The Equity Premium: A Puzzle,” Journal of Monetary Economics, 15, 145–161.

Pindyck, R. S. (2021): "What we know and don't know about climate change, and implications for policy," Environmental and Energy Policy and the Economy, 2, 4–43.

Quinet, E. (2013): "L'évaluation socioéconomique des investissements publics," Tech. rep., Commissariat Général à la Stratégie et à la Prospective.

Quinet, E. et al. (2013): "L'évaluation socioéconomique des investissements publics—Rapport final," Paris: Commissariat général à la stratégie et à la prospective (CGSP).

Rietz, T. (1988): "The equity risk premium: A solution," Journal of Monetary Economics, 22, 117–131.

Rijksoverheid (2020): "Rapport Werkgroep discontovoet 2020," Tech. rep., Dutch Ministerie van Financien.

Stern, N. and N. H. Stern (2007): The economics of climate change: the Stern review, cambridge University press.

Treasury (2020): "Green book: Central government guidance on appraisal and evaluation," Tech. rep., U.K. Treasury.

Weil, P. (1989): "The equity premium puzzle and the risk-free rate puzzle," Journal of Monetary Economics, 24, 401–421.

Weitzman, M. (2007): "Subjective expectations and asset-return puzzle," American Economic Review, 97, 1102–1130.