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Triangle-free reserving

Methodology and comparison with triangle-based projection methods

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- ✓ Why: the fight against information compression
- ✓ An analogy with pricing
- ✓ How it works: run-through of the methodology
- ✓ Model-free triangle-free reserving

2. Comparison with triangle-based methods

- ✓ How do we compare reserving methods?
- ✓ The generation of control data
- ✓ Experimental set-up
- ✓ Results and conclusion

Executive summary

1. Triangle-free reserving methods are increasingly being proposed for the estimation of future reserves and especially for the estimation of the distribution of outstanding liabilities (IBNR + IBNER + UPR).
2. We will describe a possible implementation of a triangle-free reserving method that mimics the way we do pricing.
3. However, the focus of this presentation is not on advertising a specific triangle-free methodology, but to compare triangle-free and triangle-based approaches using a loss generator.
4. We will show that two popular triangle-based methods and some of their variants are inadequate to model the reserve distribution unless the reserve distribution is Gaussian.
5. We will also show that even a basic model-free version of triangle-free reserving (based on resampling) performs significantly better than those triangle-based methods when measured using a standard statistical distance such as Kolmogorov-Smirnov, Kuiper or Anderson-Darling

I. Triangle-free reserving

Motivation and methodological notes

The reserving problem and its classical solution

The reserving problem

Estimate the distribution of possible outcomes for the total reserves for a portfolio:

Total reserves = (Pure) IBNR + IBNER + UPR

The classical approach

Standard approach: aggregate data into triangles and use triangle development techniques (e.g. chain ladder) to produce the best estimate for the mean.

Regulatory pressure (e.g. Solvency II) to measure the uncertainty around the point estimate and hence the whole distribution of the total reserves

Lossy triangles

5,000 claims over 10 years

… compressed into 10 x 11 / 2 = 55 points

Based on 55 points we extract: (i) a point estimate; (ii) some measure of volatility; (iii) the full reserving distribution!!!

Criticism of the classical approach and alternatives

Problems with the classical approach

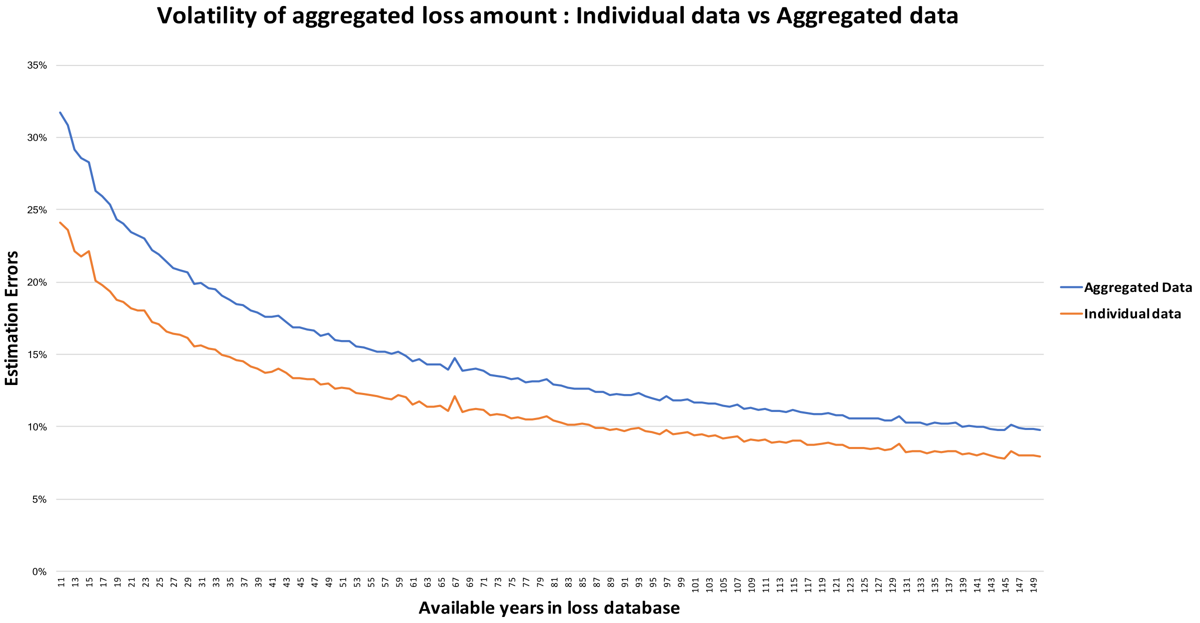
- ✓ Information compression: most of the useful data on individual losses is forgotten
- ✓ Actuaries' expert judgment often translates into tweaking factors with no immediate interpretation in terms of the loss production process
- ✓ There is a misalignment with the pricing, which measures the same risk but is normally more granular

An alternative: triangle-free reserving

Over the years other approaches based on the use of granular rather than aggregate data have been proposed. E.g., Parodi (2014) has proposed a frequency/severity approach based on the collective risk model

Individual vs aggregated data

We observe something analogous in pricing: the estimation error of the volatility (and therefore, of the total loss distribution) will be larger using burning cost (based on aggregate data) than using a frequency/severity method (based on individual data)



Generating model: Poisson/Lognormal

Triangle-free reserving: Model assumptions

1. The underlying loss production process can be described by a collective risk model
   - A common assumption in pricing, and quite general
   - Can be relaxed to account for correlation shocks affecting both frequency and severity
2. The distribution of delays, $\Pr(T \leq t)$, is constant across the years and size-independent
   - Can be relaxed if sufficient data is available to spot changes of the distribution over time and size-dependency
3. The severity distribution depends on the occurrence year and on the reporting year only through a scale factor
   - Can be relaxed if there are enough claims per accident year to produce a different severity distribution per year
   - This assumption is used extensively in pricing

A bird's eye view of the methodology

A. Estimate the IBNR distribution
   1. Estimate the reporting delay distribution
   2. Use the reporting delay distribution to estimate the IBNR claim count distribution → Output: frequency model
   3. Estimate the severity distribution taking IBNER into account → Output: severity model
   4. Combine the frequency and severity model with e.g. MC simulation to produce an aggregate loss model for IBNR

B. Estimate the IBNER distribution

   Can be done by traditional methods (e.g. Murphy-McLennan), GLM or other machine learning methods

C. Estimate the UPR distribution

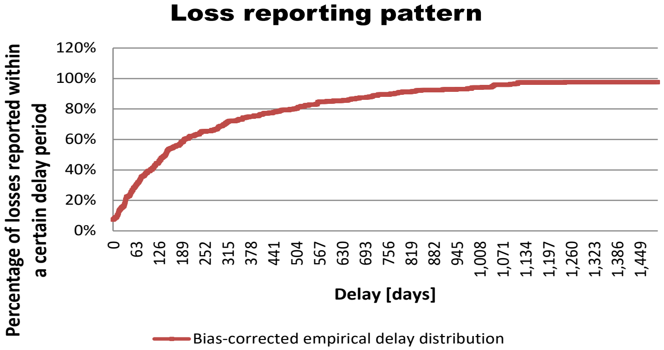
   A pricing exercise!

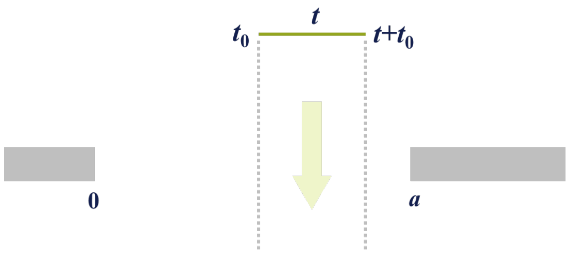
D. Combine IBNR, IBNER and UPR to produce an overall aggregate loss model

   Straightforward (e.g. using the outputs of the MC simulation) if independent

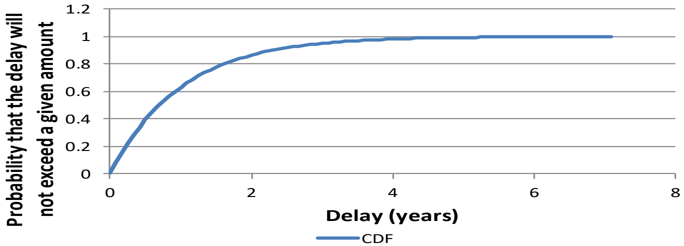
A1. Estimate ultimate claim count for all years

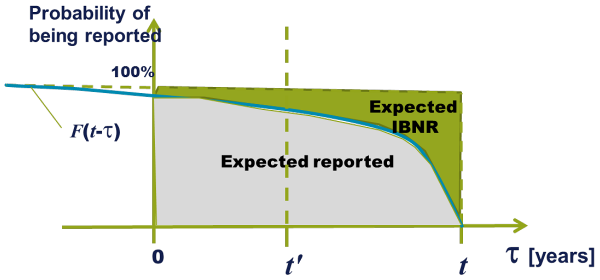
Empirical (biased) delay distribution





Empirical (unbiased) delay distribution + tail





$$\text{ult}(t) = \frac{t}{\int_0^t F(t-u)\,du} \times \text{rep}(t) = \frac{\textbf{Exp IBNR} + \textbf{Exp Reported}}{\textbf{Exp Reported}} \times \text{rep}(t)$$

A2. Build a frequency model

Based on the projections to ultimate, we can estimate the year-on-year volatility and hence decide which frequency model to adopt (e.g. Poisson, NB)

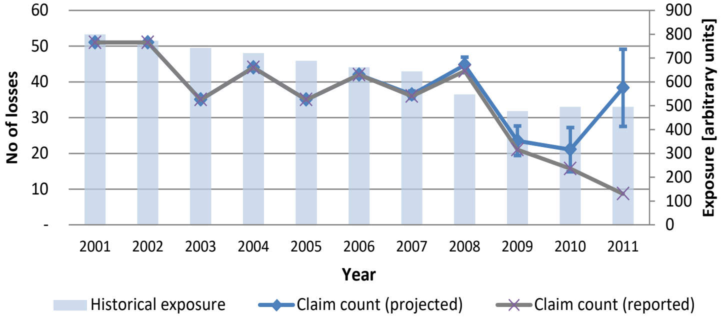
PROJECTION TO ULTIMATE -- ALL YEARS TOGETHER

Period	Days elapsed	Earned days	Factor to ultimate	Exposure	Latest reported	Ultimate losses	Standard error
2001-10	3,652.00	1,967.62	1.86	10,000	352	653.33	31.85

Note that the method also provides with a standard error

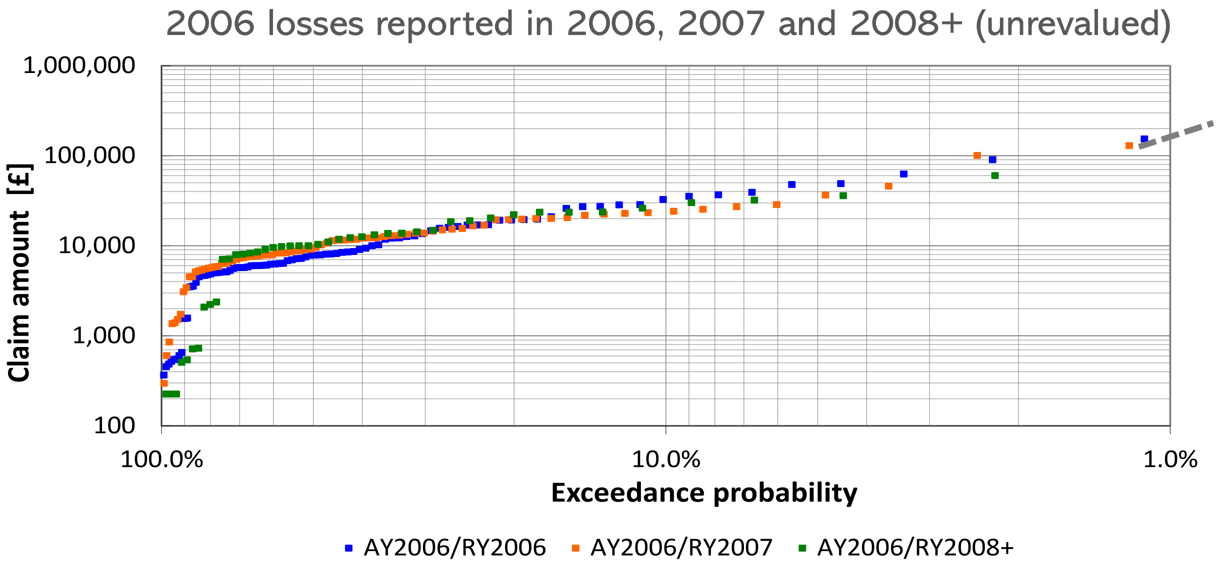
PROJECTION TO ULTIMATE -- YEAR BY YEAR

Year	Days elapsed	Earned days	Factor to ultimate	Exposure	Latest reported	Ultimate losses	Standard error
2001	365	347.94	1.05	1,000	56	58.75	3.16
2002	365	302.13	1.21	1,000	59	71.28	6.06
2003	365	275.29	1.33	1,000	43	57.01	7.24
2004	366	254.25	1.44	1,000	41	59.02	8.07
2005	365	220.78	1.65	1,000	42	69.44	9.18
2006	365	190.62	1.91	1,000	41	78.51	10.10
2007	365	158.90	2.30	1,000	23	52.83	10.98
2008	366	119.16	3.07	1,000	30	92.15	12.00
2009	365	71.35	5.12	1,000	14	71.62	13.10
2010	365	27.19	13.42	1,000	3	40.27	14.05
					Mean	65.09	
					Variance-to-mean ratio	3.28	



A3. Build a severity model

i. Adjust for IBNER, e.g. using the Murphy-McLennan methodology*
ii. Revalue losses
iii. Build kernel severity model (KSM) at year t_0, $X \sim F_X(x)$
iv. Adjust KSM for the different years: e.g., $X(\text{occ}@t, \text{rep}@\tau) \sim F_X\left(\frac{(1+r)^{t-t_0}}{(1+s)^{\tau-t}} x\right)$



In many cases, the severity distribution can be shown not to depend on the reporting date

KS(2006,2007)=1.22 (p=10%)
KS(2006,2008+)=1.36 (p=5%),
KS(2007,2008+)=0.74 (p>>10%)

* May contain triangles!

A4. Create an aggregate IBNR model by simulation

Simulation design requires some care, especially for NB processes (see Parodi, 2012)

The output will look like a standard simulation output:

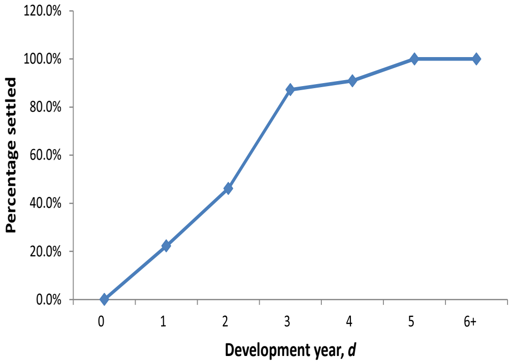
Percentile	Number of IBNR/UPR losses	IBNR and UPR
50%	39	1,767,666
75%	44	2,421,773
80%	45	2,615,037
90%	49	3,438,802
95%	51	4,646,643
98.0%	55	7,495,076
99.0%	57	10,083,991
99.5%	59	14,758,372
99.8%	61	20,596,802
99.9%	63	31,941,402
Mean	39.7	2,217,791
Std Dev	7.0	2,080,169

B. IBNER component

The IBNER component can be calculated much in the same way as the IBNER component of the IBNR claims, using e.g. Murphy-McLennan's method

By sampling possible values of the IBNER factors it is possible to obtain an empirical distribution around the central value of IBNER

Step 1 – Settlement pattern



Step 2 – Simulating IBNER-adjusted losses

Loss ID	*Development year (fixed)*	*O/S percentage (fixed)*	*Sampled settlement year*	*Sampled IBNER factor*	*IBNER-adjusted loss*
Loss_1	2	81.9%	3	1.000	87,086
Loss_2	4	28.6%	5	1.070	32,207
Loss_3	4	20.0%	5	0.029	4,001
Loss_4	4	100.0%	5	1.314	5,904
Loss_5	1	49.1%	2	0.857	41,622
Loss_6	3	94.4%	5	1.000	52,843
Loss_7	3	42.3%	5	0.734	13,159
Loss_8	3	100.0%	5	0.032	1,452
Loss_9	1	95.6%	2	0.801	61,984
Loss_10	0	100.0%	3	2.732	50,376
Loss_11	2	83.8%	3	1.000	48,121
Loss_12	3	80.6%	5	0.807	48,455
Loss_13	2	100.0%	3	1.199	41,320
Loss_14	1	100.0%	2	0.983	13,422
Loss_15	3	100.0%	5	1.000	20,929
...	...	...	...	...	...
Loss_68	0	100.0%	3	0.232	6,960
All losses					*2,237,926*

Step 3 – Output

Percentile	Number of RBNS losses	RBNS total amount
50%	68	2,196,863
75%	68	2,533,844
80%	68	2,643,906
90%	68	3,052,900
95%	68	3,551,902
98.0%	68	4,717,209
99.0%	68	5,630,207
99.5%	68	6,013,556
99.8%	68	7,692,927
99.9%	68	9,315,854
Mean	68.0	2,377,816
Std Dev	68.0	734,858

C. UPR component

Basically a pricing exercise

Can be incorporated in the calculation of the IBNR component, so that in practice we only have two components to calculate: IBNR/UPR and IBNER

D. Combine IBNR/UPR and IBNER to obtain an overall model for O/S liabilities

Overall reserves = (Pure) IBNR + UPR + IBNER

If IBNR, IBNER and UPR can be considered independent, it is straightforward to find the overall reserve distribution (e.g. by Monte Carlo or FFT methods)

Percentile	Number of IBNR/UPR losses	IBNR and UPR	Number of RBNS losses	RBNS total amount	Overall number of losses	Overall losses
50%	39	1,767,666	68	2,196,863	107	4,115,957
75%	44	2,421,773	68	2,533,844	112	4,895,375
80%	45	2,615,037	68	2,643,906	113	5,171,516
90%	49	3,438,802	68	3,052,900	117	6,220,342
95%	51	4,646,643	68	3,551,902	119	7,537,303
98.0%	55	7,495,076	68	4,717,209	123	10,329,807
99.0%	57	10,083,991	68	5,630,207	125	13,140,436
99.5%	59	14,758,372	68	6,013,556	127	16,884,020
99.8%	61	20,596,802	68	7,692,927	129	23,828,525
99.9%	63	31,941,402	68	9,315,854	131	34,267,323
Mean	39.7	2,217,791	68.0	2,377,816	107.7	4,595,607
Std Dev	7.0	2,080,169	68.0	734,858	7.0	2,218,176

Model-free triangle-free reserving

Basic loss data adjustment (e.g., inflation adjustment)

Use empirical severity distribution

Use empirical delay distribution

Use IBNER correction based on empirical IBNER factors

Use Poisson/NB for claim count distribution, depending on past experience [this is because there are only a few years of experience and the empirical claim count distribution would be meaningless]

No actuarial judgment

This is not a suggested approach, but **a dumbed-down version of triangle-free reserving** that is suitable for automation and objective testing

The big question

Can we prove the inadequacy of triangle-based methods?

Parodi (2014): under some weak assumptions on the loss production process, can we prove that specific triangle-based methods do not produce the right distribution?

Yes – with the use of control data

Control (artificial) data cannot prove *the adequacy* of a specific method, but can show the inadequacy of it

Also, we can show that a large class of TFR methods is bound to perform better under those assumptions

II. Comparing triangle-free and triangle-based methods using control data

A large-scale experiment

Overview of the proposed testing approach

Validating reserving models using portfolio data requires – in theory – checking the models' predictions against the observed claims development. This is tricky even for the projected mean, but is overwhelmingly difficult when we try to assess the full distribution of outstanding liabilities, as only one of many possible future trajectories is observed.

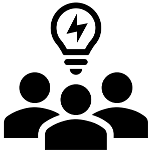
However, it is possible to simulate those future trajectories within a probabilistic framework which allows us to produce a large number of artificial conditional future loss development scenarios

Criticisms

- Loss generators are not adequate to represent real-world data as the latter are too complex
- The loss generator can be "hacked" to obtain the desired results

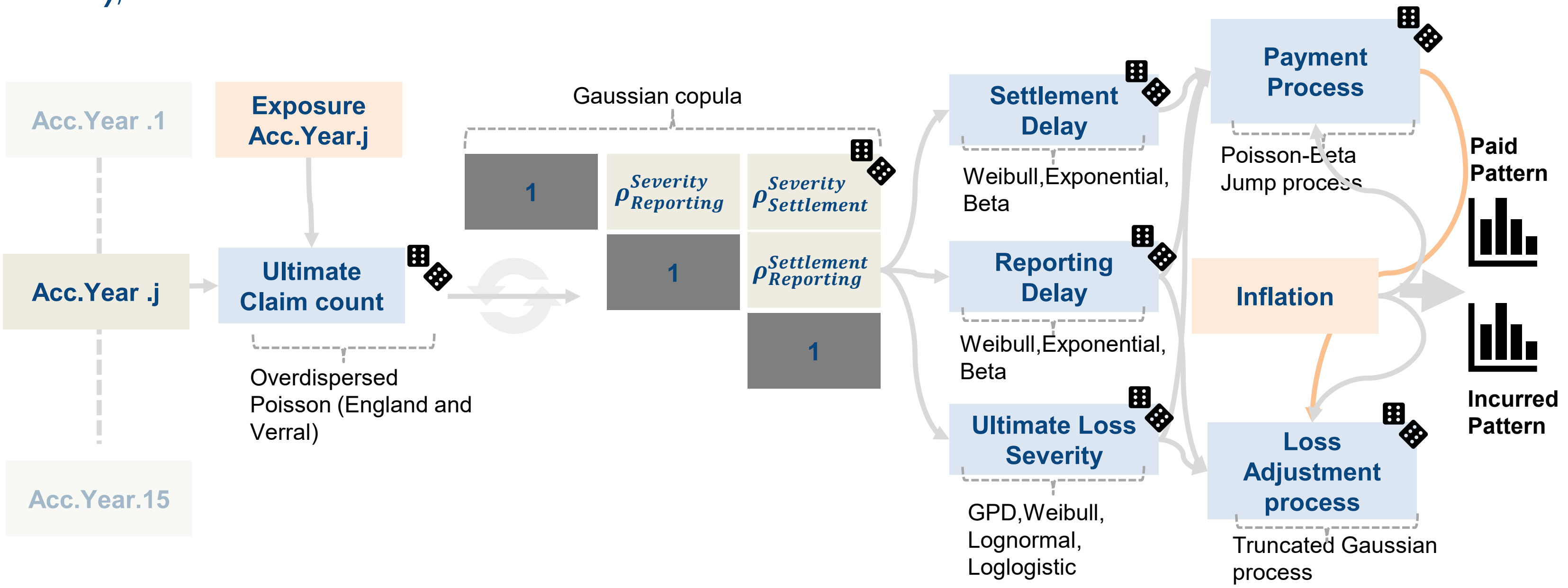
These are legitimate criticisms but:

- Rejection of loss generators should not be an excuse for never testing models!
- Good loss generators are effective at refuting bad reserving models



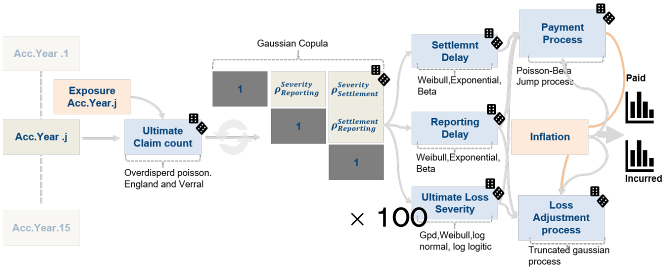
Stochastic Insurance claims simulator

Our validation model, used to produce artificial historical data sets and related conditional future developments, was inspired by the CAS Loss Simulator 2.0 (Alvarado et al.), and has the architecture below:



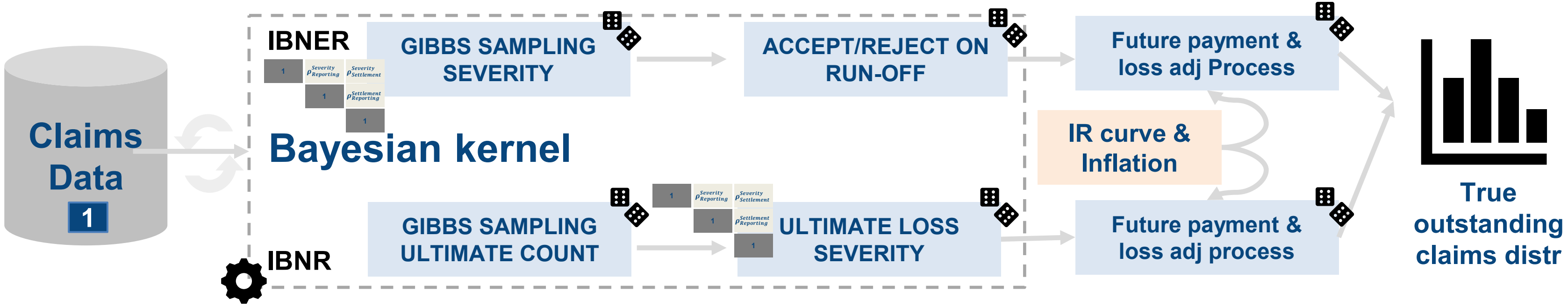
Experimental protocol

1 Sample a **single**, incomplete portfolio run-off (upper triangles) and store the data



	dev1	dev2	dev3	dev4	dev5	dev6	dev7	dev8	dev9	dev10	dev11	dev12	dev13	dev14	dev15
orgin1	22,526,489	19,050,577	15,059,542	9,332,375	4,825,121	3,979,245	2,665,601	1,859,959	1,304,643	780,519	444,017	69,037	0	87,261	80,470
orgin2	24,347,974	24,252,457	14,499,914	9,152,776	6,066,425	3,000,797	2,111,180	950,313	577,449	357,051	366,343	29,591	6,694	12,481	0
orgin3	24,739,826	21,554,693	15,657,846	7,387,460	7,093,570	3,387,593	1,468,194	1,481,780	403,974	410,616	895,098	240,928	221,333	0	0
orgin4	33,559,511	25,718,302	16,880,335	14,615,938	6,931,716	4,218,933	3,742,286	1,071,703	824,035	968,770	611,997	344,475	0	0	0
orgin5	22,410,392	17,326,154	11,798,764	8,817,234	5,341,164	2,799,265	1,568,484	1,790,274	704,511	534,083	197,948	0	0	0	0
orgin6	25,170,636	23,411,492	14,203,023	10,057,666	6,355,804	5,633,926	2,285,475	2,643,244	200,435	265,826	0	0	0	0	0
orgin7	25,988,044	22,359,213	14,740,133	6,769,081	5,308,880	3,808,368	2,887,016	565,106	603,288	0	0	0	0	0	0
orgin8	21,812,782	23,745,803	15,982,879	9,212,523	9,109,015	2,323,474	1,925,551	1,114,621	0	0	0	0	0	0	0
orgin9	22,411,020	20,047,536	15,293,792	11,596,073	5,895,769	2,907,835	2,168,408	0	0	0	0	0	0	0	0
orgin10	25,701,217	20,282,528	16,441,950	7,874,285	6,729,317	5,061,832	0	0	0	0	0	0	0	0	0
orgin11	24,498,912	17,762,322	13,881,183	9,268,824	8,825,919	0	0	0	0	0	0	0	0	0	0
orgin12	28,227,934	28,015,467	17,738,443	12,934,195	0	0	0	0	0	0	0	0	0	0	0
orgin13	22,999,453	22,733,204	12,485,420	0	0	0	0	0	0	0	0	0	0	0	0
orgin14	24,527,042	24,551,804	0	0	0	0	0	0	0	0	0	0	0	0	0
orgin15	26,520,697	0	0	0	0	0	0	0	0	0	0	0	0	0	0

2 Sample 10 000 future loss developments conditioning on upper triangle in 1



Experimental protocol (cont'd)

3 Fit & simulate TFR and triangle-based stochastic models

Triangle-free method

- «Distribution-free» version based on empirical estimators
- When empirical estimators cannot be constructed (e.g. frequency), distributions different from that of the generator have been used

Triangle-based methods

- «Distribution-free» version based on residuals resampling (except for Mack/Lognormal).
- Estimation error component switched off (to avoid the comparison to be unfair to triangle-based methods)

4 Analyse results

Each empirical distribution is compared to the one at Step 2 (the true one) and deviance measures are computed

5 Repeat Steps 1 to 4 100 times

More details on the modelling strategy

Triangle-free*

- Non-parametric estimation of reporting delay, settlement delay, payment pattern,severity and case reserve uncertainty.
- Negative binomial for ultimate frequency (moment matching parameters fitting)
- Inflation adjustments on the kernel severity (empirical CDF)
- Linear model for IBNER factors on individual data
- No provision for correlation: full indepence assumed

Triangle-based**

- Inflation adjustements based on Taylor's method
- Exclusion of outlier residuals
- Heterogeneity adjustments on dispersion parameters
- Hat-matrix-based residual bias adjustments
- Exposure adjustments
- For the incurred-based methods, the payment pattern is obtained with bootstrap on paid
- When models were seriously non-fitting, the results for LogNormal-Mack were used instead

* P.Parodi (2013)
** Bootstrap Modeling: Beyond the Basics Mark R. Shapland, Jessica (Weng Kah) Leong (2010)

Experiments strategy

Two reserving scenarios **(A**,**B)** are considered:

A. **Stable portfolio**. This is the typical direct short tail business portfolio where because of large volumes and homogeneity and the Central Limit Theorem the outstanding claims reserves follow a normal distribution.

B. **Volatile portfolio.** This can be, e.g., an excess portfolio with low frequencies and high severities or more in general a portfolio with an “unstable” triangle where the outstanding claims reserve distribution is expected to be significantly skewed

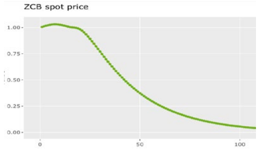
Scenario A (stable portfolio)

	Exposure	Avg. Frequency	V/m	ULR
AccYear 1	105 mln £	7539	2	74 %
AccYear 2	111 mln £	7902	2	74 %
Acc Year 3	114 mln £	7971	2	74 %
Acc Year 4	144 mln £	9946	2	74 %
Acc Year 5	98 mln £	6620	2	74 %
Acc Year 6	123 mln £	8234	2	74 %
AccYear 7	109 mln £	7218	2	74 %
AccYear 8	115 mln £	7475	2	74 %
AccYear 9	110 mln £	7011	2	74 %
AccYear 10	118 mln £	7461	2	74 %

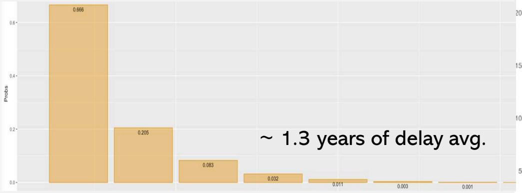
Ultimate frequency distribution assumptions

Overdispersed Poisson with flat rate and λ linked to exposure .Frequency related only to claim above the policies deductibles

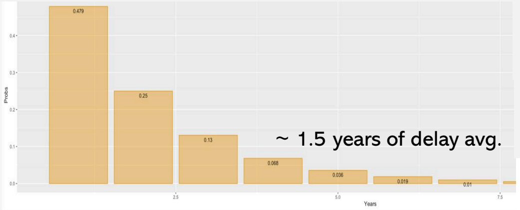
Risk free rate curve (Europe)



Empirical Delays distribution assumptions

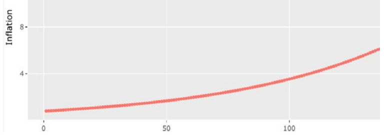


Reporting : *Beta derived* (0.7 , 7)

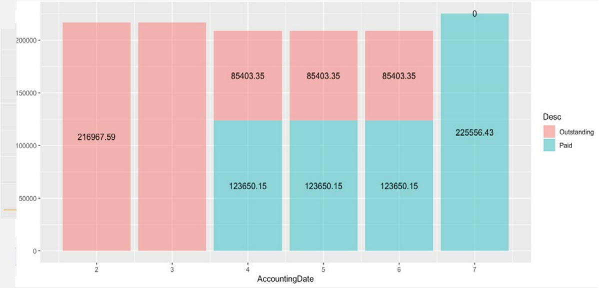


Settlement : *Exponential derived* (0.65)

Inflation : deterministic, yearly compounded rate = 1.5%



Loss adjustment process assumptions



Payment Process : *Poisson Beta Jump Process*
λ = 0.33 and *Beta* (2,2)

Loss Adjustment Process : *Normal Process*
μ= Outstanding and σ = 0.05* Outstanding

Correlation assumptions

Gaussian Copula

Severity	1		
Reporting	0	1	
Settlement	0.5	0.35	1

Severity distribution assumptions

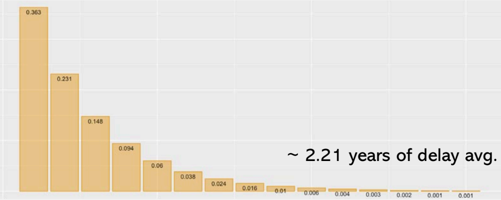
	LogNormal
μ	8.903
σ	1.013
Limit	2.5 mln $

Scenario B (volatile portfolio)

	Exposure (£)	Avg. Frequency	V/m	ULR
AccYear 1	65 mln £	265	5	74 %
AccYear 2	41 mln £	165	5	74 %
Acc Year 3	30 mln £	119	5	74 %
Acc Year 4	35 mln £	136	5	74 %
Acc Year 5	37 mln £	142	5	74 %
Acc Year 6	38 mln £	144	5	74 %
AccYear 7	42 mln £	157	5	74 %
AccYear 8	45 mln £	165	5	74 %
AccYear 9	49 mln £	177	5	74 %
AccYear 10	53 mln £	189	5	74 %
AccYear 11	51 mln £	181	5	74 %
AccYear 12	48 mln £	167	5	74 %
AccYear 13	47 mln £	161	5	74 %
AccYear 14	46 mln £	157	5	74 %
AccYear 15	45 mln £	150	5	74 %

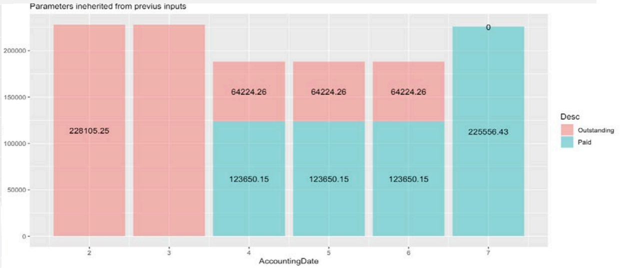
Delays distribution assumptions

Reporting : *Beta*(3 , 28)



Settlement : *Exponential*(0.45)

Loss adjustment process assumptions



Payment Process : *Poisson Beta Jump Process*
λ = 0.33 and *Beta* (2,2)

Loss Adjustment Process : *Normal Process*
μ = 1.03*Outstanding and σ = 0.15*Outstanding

Correlation assumptions

Gaussian Copula

Severity	1		
Reporting	0	1	
Settlement	0.5	0.35	1

Severity distribution assumptions

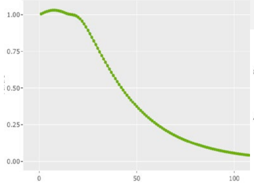
	GPD
β	29286
ξ	1.042
Limit	15.0 mln £

Ultimate frequency distribution assumptions

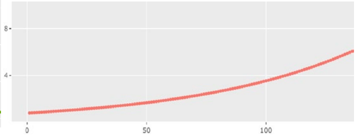
Overdispersed Poisson with flat rate and λ linked to exposure

Frequency related only to claim above the policies deductibles

Risk free rate curve : Europe



Inflation : deterministic yearly compunded rate = 1.5%



Performance metrics

	Name	Formula	Description
(a)	RMSP (Mean)	$\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\text{Reserve}_{\text{predicted},i} - \text{Reserve}_{\text{true},i}\right)^2}$	Prediction error on the expected value of the unpaid.
(b)	RMSP (VaR @ 99.5%)	$\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\text{VaR@99.5}_{\text{predicted},i} - \text{VaR@99.5}_{\text{true},i}\right)^2}$	Prediction error on the Value at Risk at the 99.5th percentile.
(c)	KS	$D = \sqrt{\frac{n_1 n_2}{n_1 + n_2}} sup\left\lvert\widehat{F_x^n} - F_x\right\rvert$	Normalised Kolmogorov-Smirnov distance
(d)	KUIPER	$D = D^+ + D^-$ $D^- = \sqrt{\frac{n_1 n_2}{n_1 + n_2}} sup\left(\widehat{F_x^n} - F_x\right)$ $D^+ = \sqrt{\frac{n_1 n_2}{n_1 + n_2}} sup\left(F_x - \widehat{F_x^n}\right)$	Normalised Kuiper distance
(e)	AD	$AD = \sqrt{\frac{n_1 n_2}{n_1 + n_2}} sup\left\lvert\frac{\widehat{F_x^n} - F_x}{\sqrt{F_x(1-F_x)}}\right\rvert$	Normalised Anderson-Darling distance

Note

The normalised distance measures are based on the empirical distributions centred around zero for all the 100 samples and averaged over all samples. This choice was made better to capture the shape of the distributions and also because, in practice, the stochastic methods are often scaled to different mean (e.g. to match balance sheet figures) so only the deviations around the average are retained.

Results : Scenario A (large retail portfolio)

Models ID	AD**	KUIPER**	KS**	RMSP (VaR 99.5%)	RMSP (mean)
NAIVE_TFR	1.64	0.85	0.47	6.1%	1.4%
CL_Normal_Incurred*	3.45	1.98	1.07	20.3%	1.6%
CL_ODP_Incurred*	3.58	2.16	1.14	23.0%	1.6%
CL_Gamma_Incurred*	3.76	2.29	1.21	24.7%	1.6%
MACK_Incurred	9.51	8.09	4.15	55.4%	1.6%
CL_ODP_Paid*	28.82	4.66	2.40	72.8%	3.0%
CL_Inverse Gausssian_Incurred*	44.36	4.33	3.35	688.0%	1.6%
CL_Normal_Paid*	27.45	5.33	2.72	58.3%	3.0%
MACK_Paid	54.37	7.45	3.80	88.2%	3.0%
LOG_MACK_Paid	19.10	18.67	9.46	92.2%	3.0%
LOG_MACK_Incurred	22.39	22.09	11.19	99.6%	1.6%
CL_Gamma_Paid*	170.25	12.49	6.39	352.3%	3.0%
CL_Inverse Gaussian_Paid*	348.97	18.70	9.58	1549.0%	3.0%

Findings

- ✓ Prediction for the expected values are accurate regardless the model choice
- ✓ TFR performs on average significantly better in capturing the shape of the outstanding claims distribution. KS is well below the critical value @ 5% (1.36)
- ✓ Paid triangle-based method are generally less accurate than Incurred based
- ✓ Inverse gaussian residuals are strongly inadequate to capture uncertainty

* Variants of chain ladder with bootstrap
** Anderson-Darling (upper tail), Kuiper, Kolmogorov-Smirnov [Chernobay, Rachev, Fabozzi (2005)]

Results : Scenario B (small specialty portfolio)

Models ID	AD**	KUIPER**	KS**	RMSP (VaR 99.5%)	RMSP (mean)
NAIVE_TFR	11.14	3.88	2.62	27.2%	18.3%
CL_ODP_Incurred*	29.55	5.86	3.74	49.8%	21.0%
CL_Gamma_Incurred*	34.94	6.49	4.14	112.8%	21.0%
CL_Normal_Incurred*	52.94	5.79	3.82	175.7%	21.0%
MACK_Incurred	34.37	8.40	5.71	149.6%	21.0%
CL_Normal_Paid*	58.05	5.23	3.07	59.3%	55.3%
CL_ODP_Paid*	21.01	7.81	4.66	59.4%	55.3%
CL_Inverse Gausssian_Incurred*	103.44	7.85	5.52	446.0%	21.0%
LOG_MACK_Paid	86.06	6.07	4.20	298.6%	55.3%
LOG_MACK_Incurred	87.74	8.51	5.73	513.7%	21.0%
CL_Gamma_Paid*	130.41	8.22	4.85	170.9%	55.3%
MACK_Paid	177.34	10.26	6.37	219.5%	55.3%
CL_Inverse Gausssian_Paid*	145.40	9.30	5.64	710.2%	55.3%

*   Variants of chain ladder with bootstrap
** Anderson-Darling (upper tail), Kuiper, Kolmogorov-Smirnov [Chernobay, Rachev, Fabozzi (2005)]

Findings

- ✓ In general all the models show less predictive power for both uncertainty and mean, and KS is above the critical value for all cases
- ✓ However TFR is almost twice as accurate as the best triangle based method in capturing the uncertainty and shape of the distribution.
- ✓ Paid triangle-based methods are extremely inaccurate
- ✓ Inverse Gaussian residuals are strongly inadequate to capture uncertainty also in this case.

Conclusions

- ✓ The triangle-based methods that we have investigated are inadequate to capture the loss distribution of reserves when losses are produced according to a reasonably general loss generator. For large volumes, however, triangle-based methods for estimating the reserves give good practical approximations of the overall distribution.

- ✓ Even a naïve model-free triangle-free reserving approach performs significantly better than triangle-based methods. It is expected that sound modelling (including that carried out using, e.g., machine learning techniques for IBNER reserves) will outperform the model-free triangle-free approach.

- ✓ The experiments do not prove that triangle-free reserving will be successful in general situations with real data, but that it does not fail a necessary condition for adequacy (test with control data), and so is a good candidate for dealing with real-world situation

Limitations and future research

- ✓ The comparison should be extended to other, non-canonical methods, such as the double chain ladder and other regression structures. The inclusion of loss-ratio-based methods such as Clark's Cape Cod method could also be considered, although the presence of «expert judgment» is a challenge
- ✓ The generation of ultimate losses could be refined by introducing the concept of portfolio profile and heterogeneous exposures (different limits, sums insured etc.), stochastic inflation using econometrics models, a stochastic interest rate curve
- ✓ The tool used for the analysis could be streamlined to produce a stand-alone loss generator

Previous experimental work done/used by the authors

Parodi (2012; 2014a)

Heuristic comparison which gave promising results but had an unspecified built-in bias as it used for the analysis the same distributions that it had used in the production of control data

Results: KS distance (triangle-free) <~ 40% of KS distance (triangle-based)

Glionna's initial results – See Parodi (2016)

Preliminary results with an improved experimental set-up, but still not completely model-free (a Poisson assumption was used for the frequency component during both production and analysis).

Results: RMSEP (mean,TFR) < 50% of RMSEP (mean, T-based)

Glionna & Parodi (2018)

Improved, bias-free and model-free (naïve TFR) experimental set-up. Limitations in the loss generation process (no consideration given to IBNER).

Results: KS distance (triangle-free) 12-52% of KS distance (triangle-based)

Alvarado et al. (2018)

A more sophisticated loss generation process, which inspired our current work

Main references (directly used in this work)

The original paper

Parodi, P. (2012; 2014a), Triangle-free reserving - A non-traditional framework for estimating reserves and reserve uncertainty, British Actuarial Journal, March 2014. (Also on GIRO proceedings, 2012.)

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Glionna, A. & Parodi, P. (2018), Triangle-free reserving vs triangle-based methods: An Empirical comparison based on control data, International Conference of Actuaries, Berlin, June 2018

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England, P.D., Verrall, R.J. (2002), Stochastic claims reserving in general insurance, British Actuarial Journal 8/3, 443-518, 2002.

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Thank you