Coherent Cause-Specific Mortality Forecasting via Constrained Penalized Regression Models

Giancarlo Camarda
Institut National d'Etudes Démographiques, Aubervilliers, France

INSTITUT NATIONAL D'ÉTUDES DÉMOGRAPHIQUES

in collaboration with María Durbán (Universidad Carlos III de Madrid)

SCOR Chair on mortality research
Workshop 1

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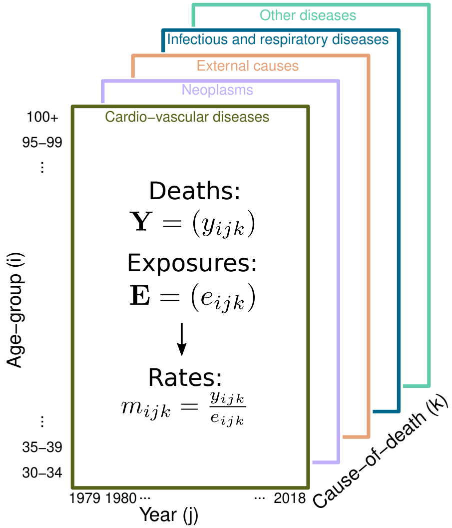
- For long time it has been argued that all-cause mortality projections based on cause-specific mortality present serious drawbacks (Wilmoth, MPS, 1995)
- Recent developments:
  - Lee-Carter model for specific cause (Kjærgaard et al., JRSS-C, 2019)
  - Bayesian hierarchical model for cause-specific death rates in geographic subunits (Foreman et al., JRSS-C, 2017)



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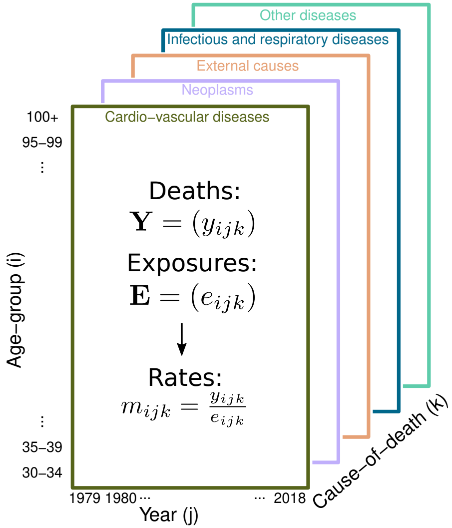
Data structure



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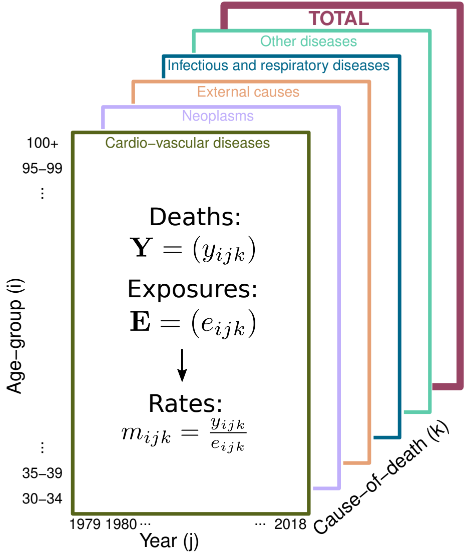

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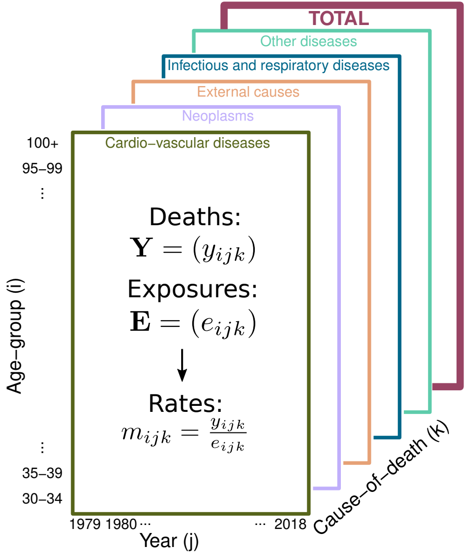

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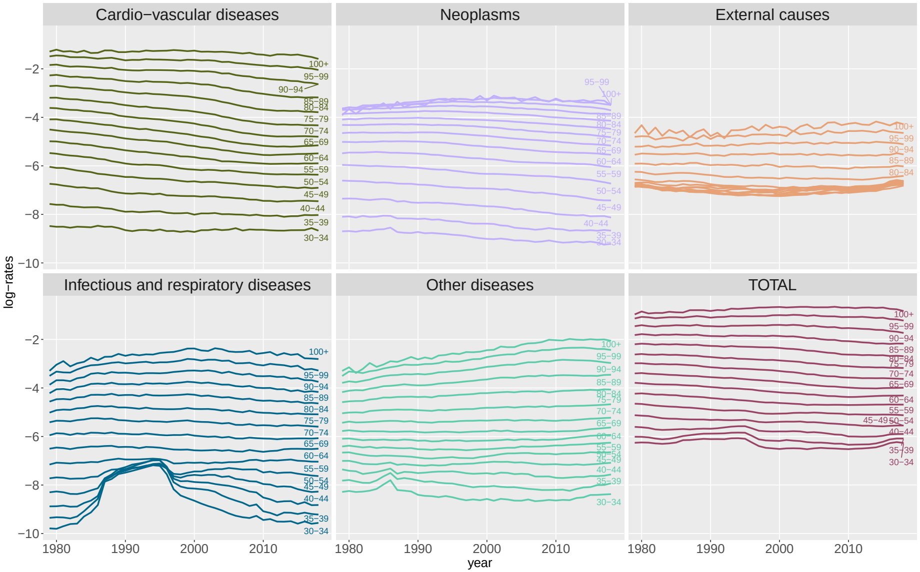
- Each layer in $\boldsymbol{E}$ includes the same age-year matrix

Example: USA mortality, males. Log-rates



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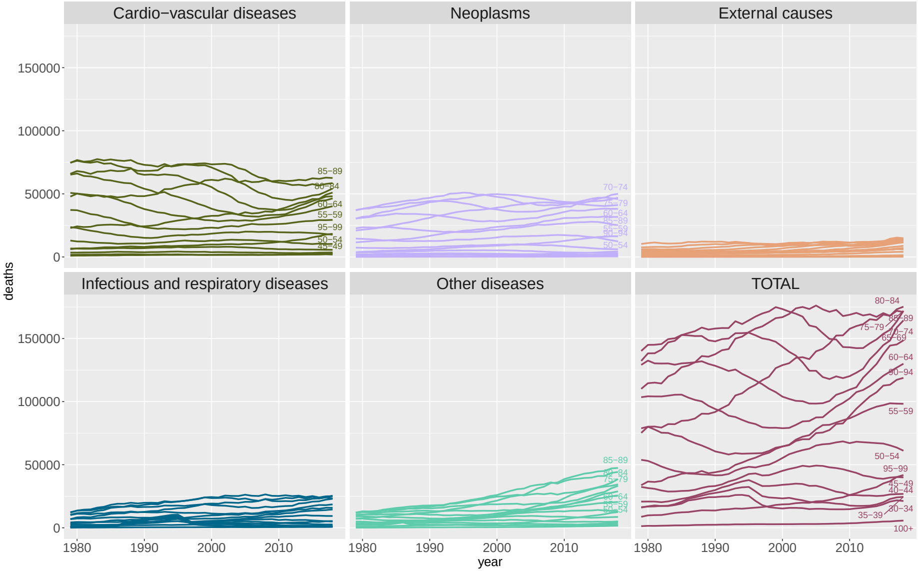

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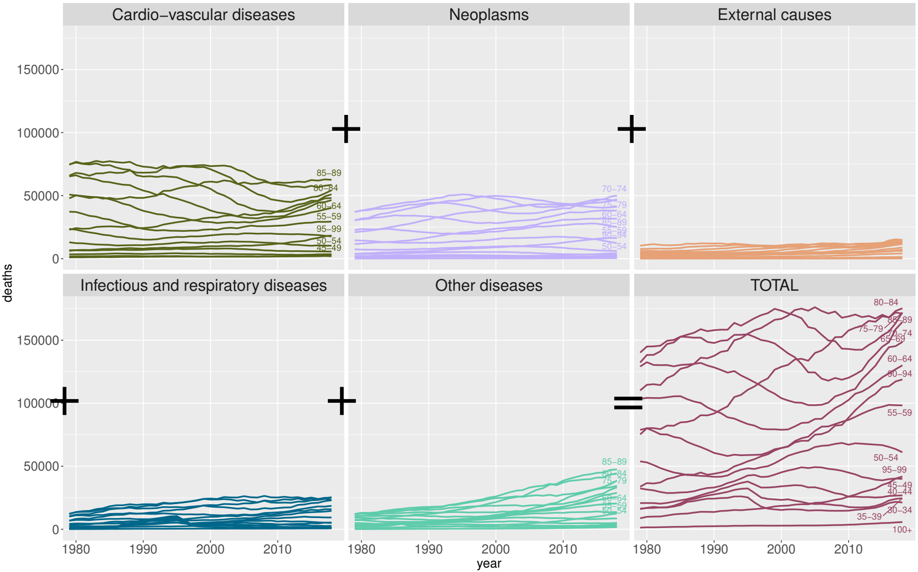

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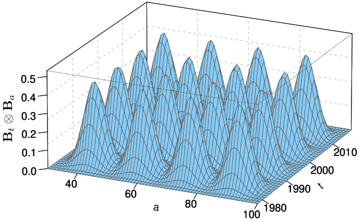

Adding constraints on death counts

- Additional constraints needed as the deaths-level:

$$\boldsymbol{C}\,\mathbb{E}[\boldsymbol{y}] = \mathbf{0} \quad \Rightarrow \quad \boldsymbol{C}\,(\boldsymbol{e} * \boldsymbol{\mu}) = \mathbf{0} \quad \Rightarrow \quad \boldsymbol{C}(\boldsymbol{e} * \exp(\boldsymbol{B}\boldsymbol{\alpha})) = \mathbf{0}$$

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- $\Rightarrow$ All inner operations can be embedded in a Generalized Linear Array Model (GLAM) framework (Currie et al., JRSS-B, 2006)

About constraints and singularity

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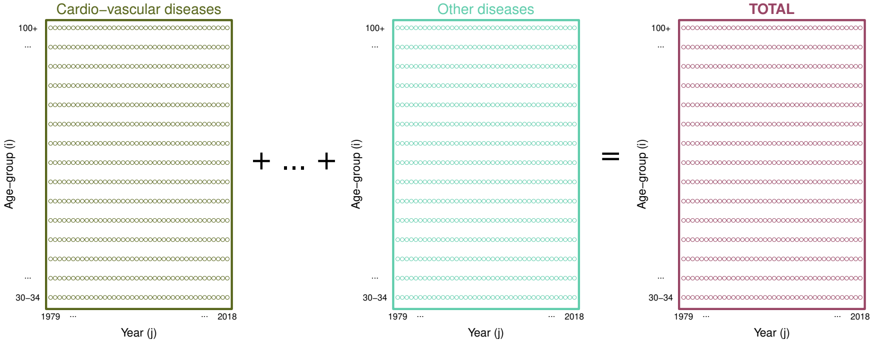

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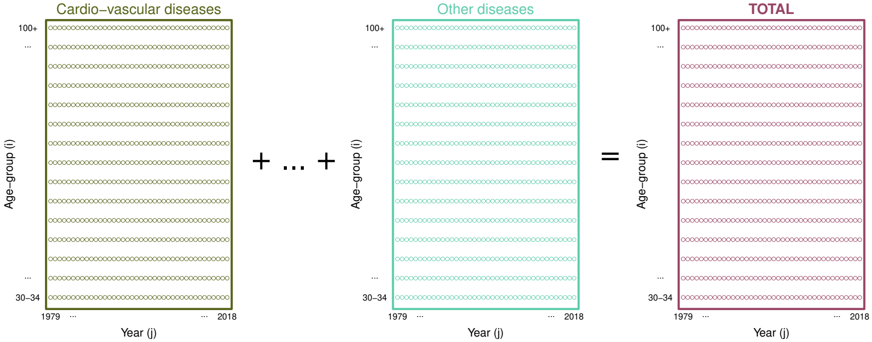

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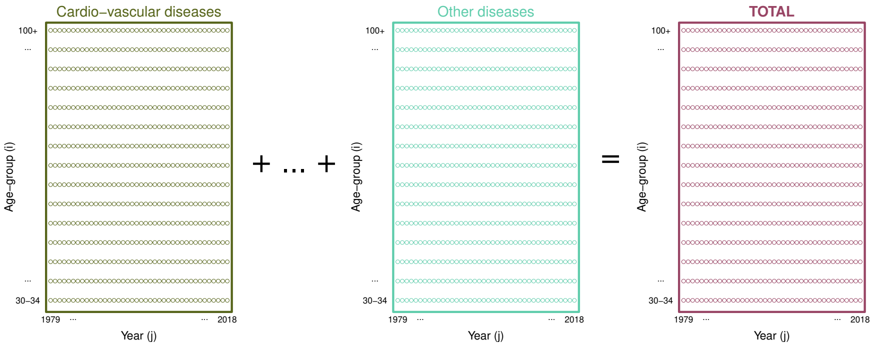

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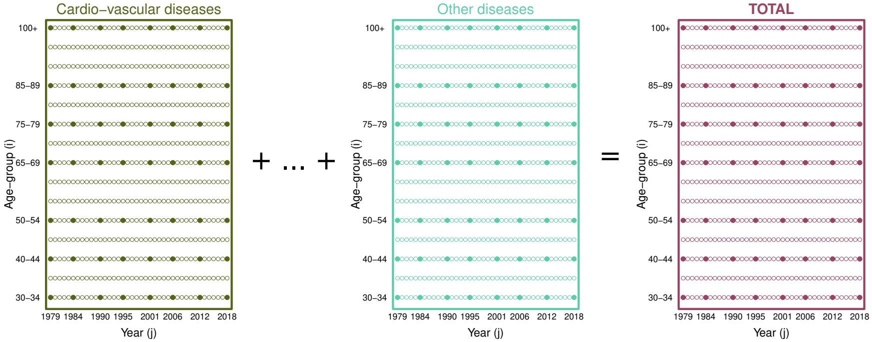

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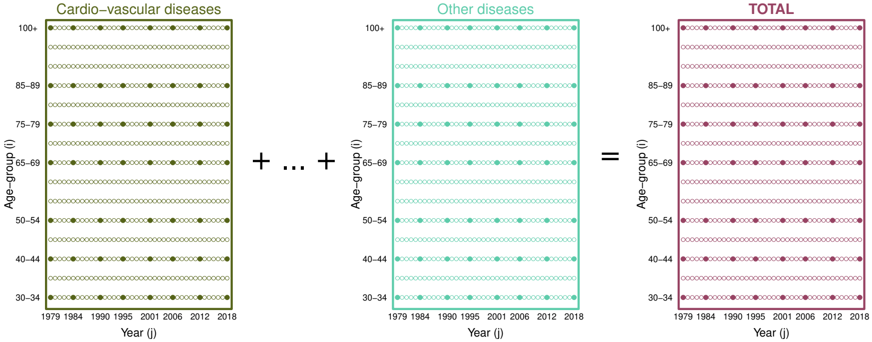

Likelihood function

- We maximize a Poisson likelihood:

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- Constraints are enforced by Lagrange multipliers $\boldsymbol{\omega}$

Estimation procedure (for equation enthusiasts)

- To maximize

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- We derive the following scoring algorithm:

$$\begin{bmatrix} \boldsymbol{B}'\tilde{\boldsymbol{W}}\boldsymbol{B} + \boldsymbol{P} + \boldsymbol{B}'\mathtt{diag}(\boldsymbol{C}'\tilde{\omega})\tilde{\boldsymbol{V}}\boldsymbol{B} & \boldsymbol{B}'\tilde{\boldsymbol{V}}\boldsymbol{C}' \\ \boldsymbol{C}\tilde{\boldsymbol{V}}\boldsymbol{B} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{\alpha}} \\ \tilde{\omega} \end{bmatrix} =$$

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where

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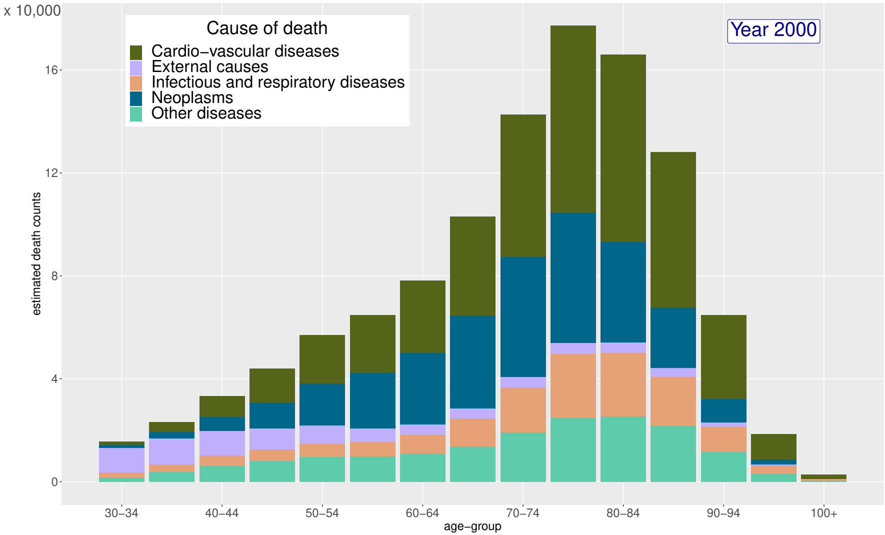
- With no summation constraint the model simply reduces to a series of two-dimensional GLAMs

Checking constraints



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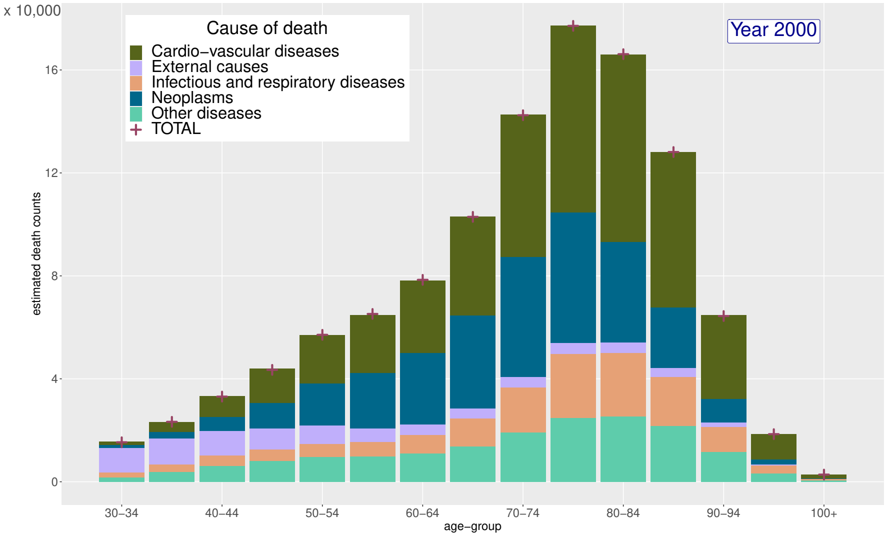

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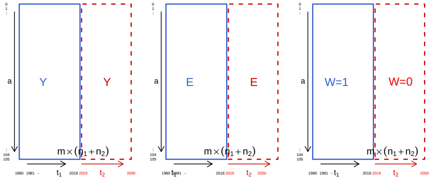

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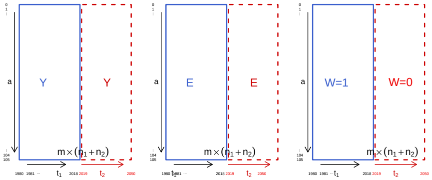

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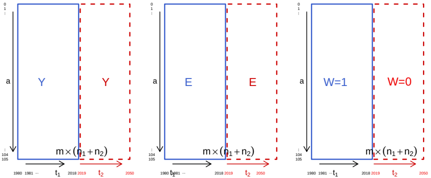

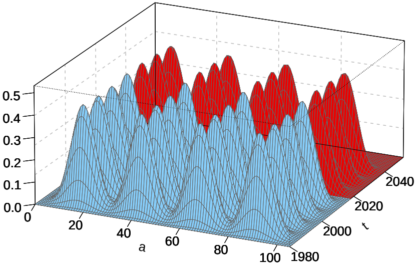

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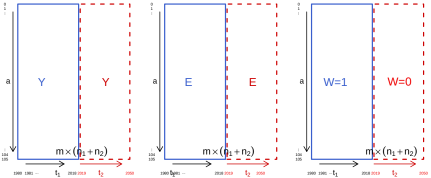
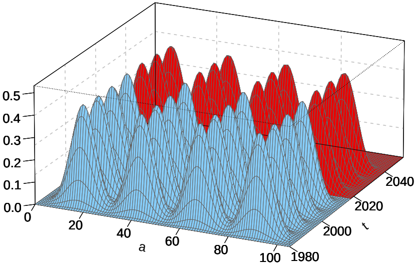
- We add 0/1 weights in the scoring algorithm





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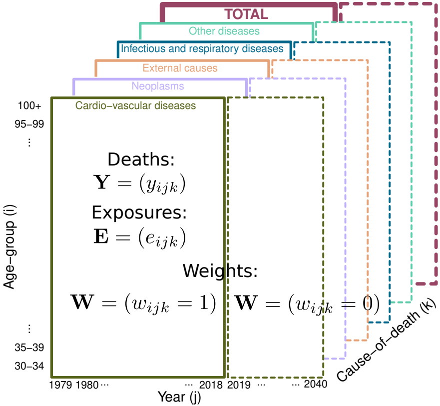

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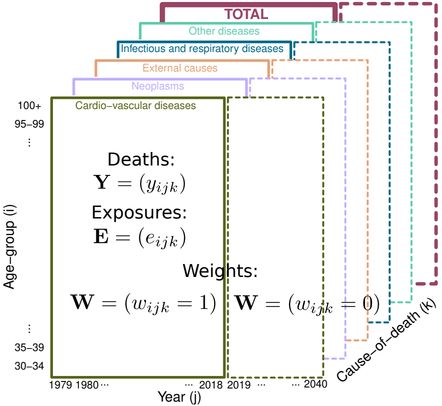

- We generalize in 3D the expanded B-spline bases:

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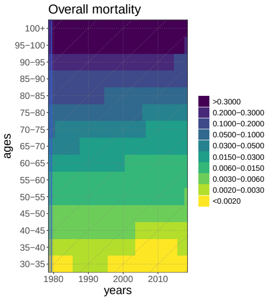
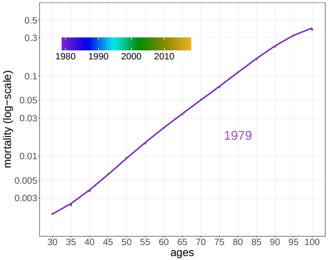
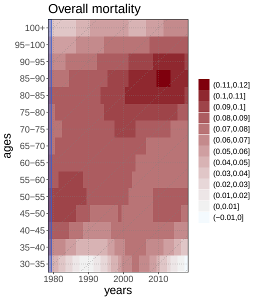
Observing mortality patterns (over ages)

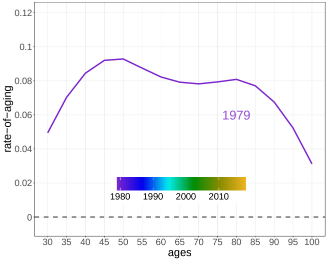
Actual and Smooth log-mortality





Rate-of-aging from smooth mortality





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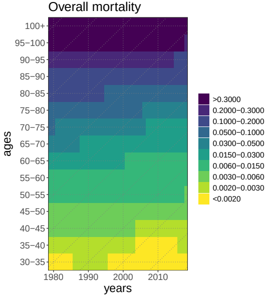

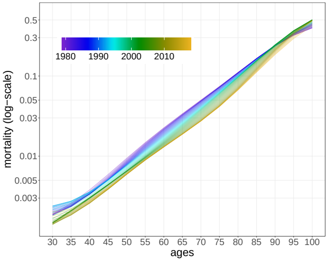

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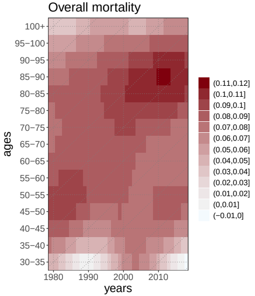

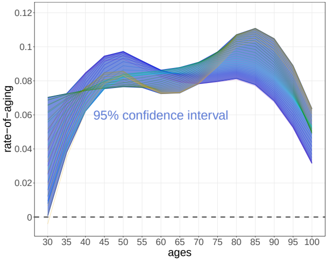

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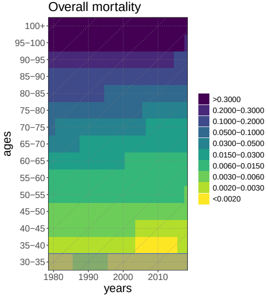

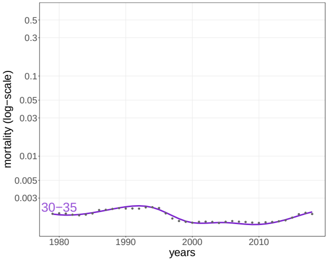

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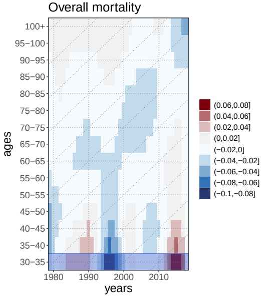

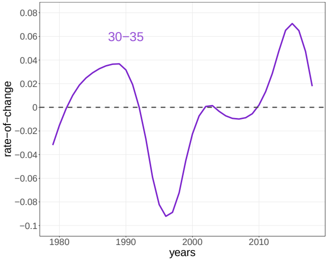

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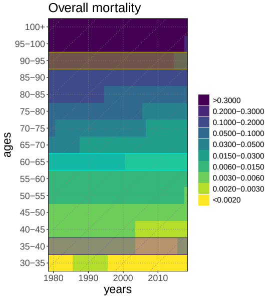

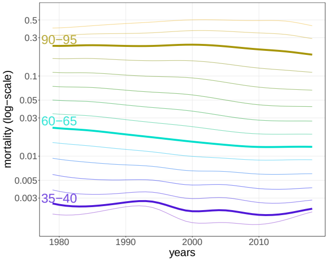

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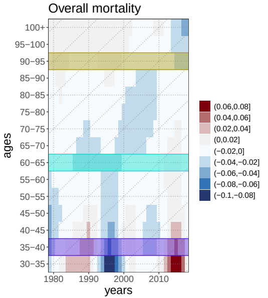
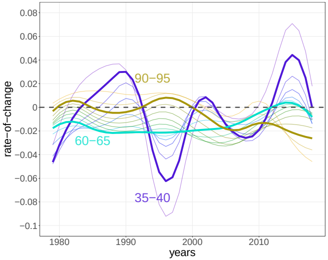
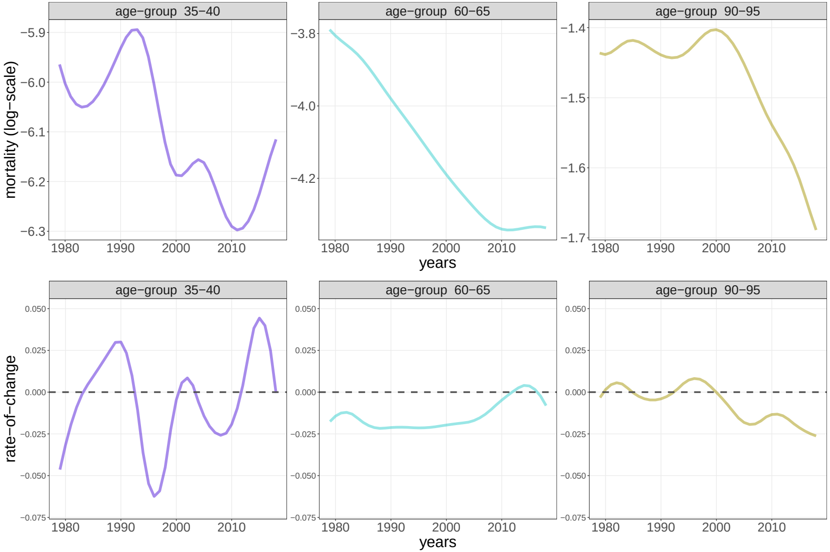




Rate-of-change, selected ages



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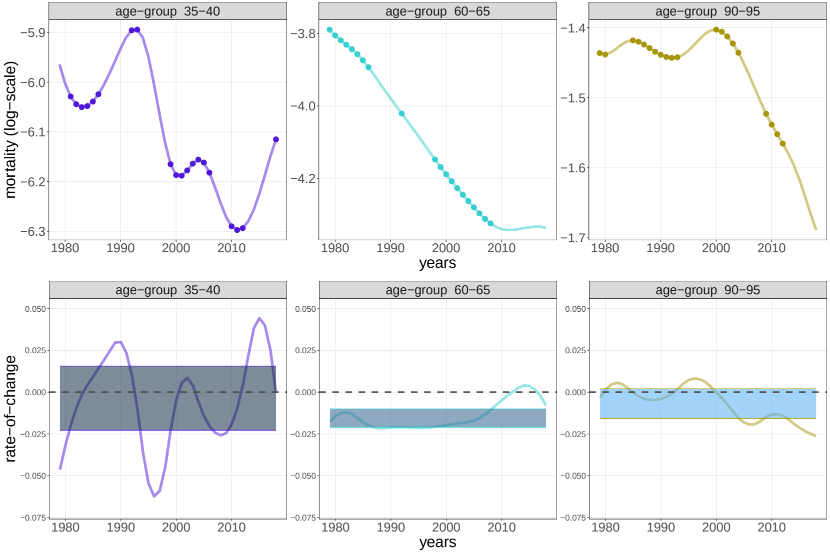

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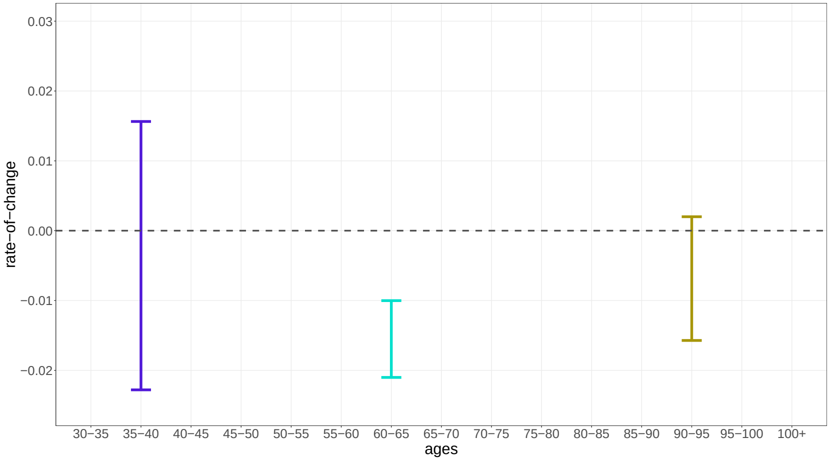
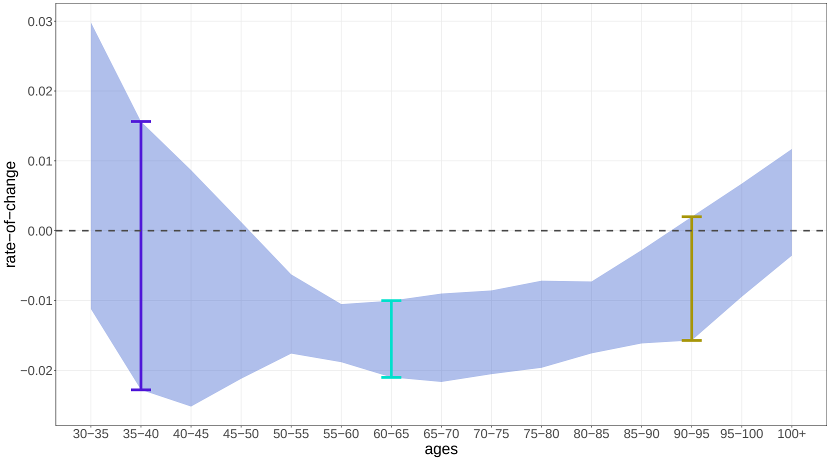
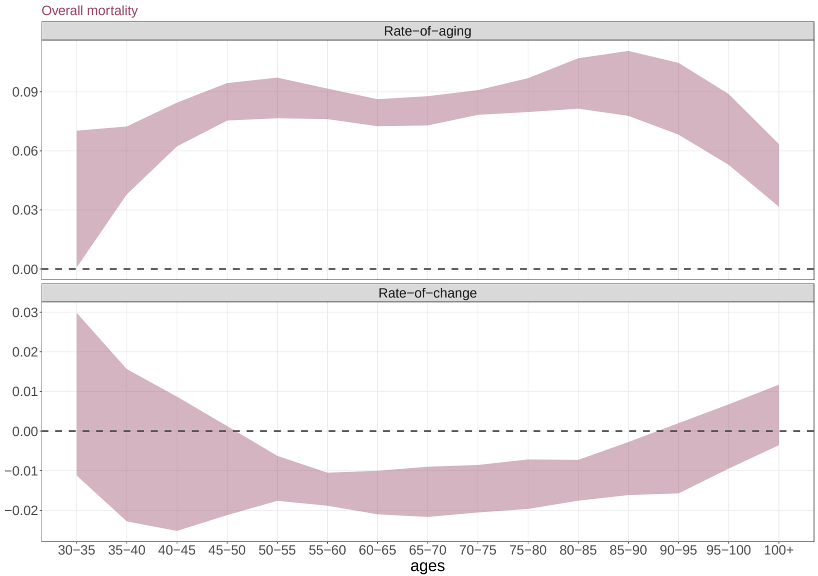
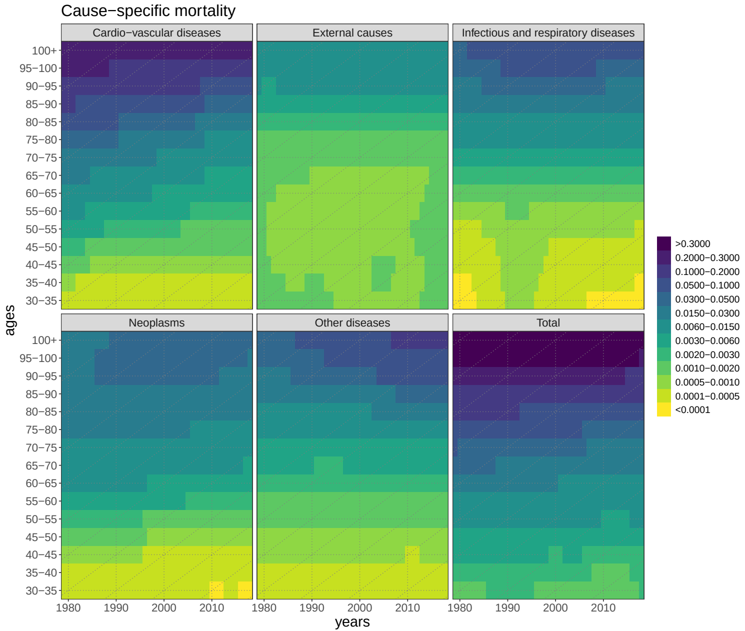


Rate-of-change, all ages




Overall mortality
Rate−of−aging
0.09
0.06
0.03
0.00
Rate−of−change
0.03
0.02
0.01
0.00
−0.01
−0.02
30−35
35−40
40−45
45−50
50−55
55−60
60−65
65−70
70−75
75−80
80−85
85−90
90−95
95−100
100+
ages

Looking at cause-specific mortality



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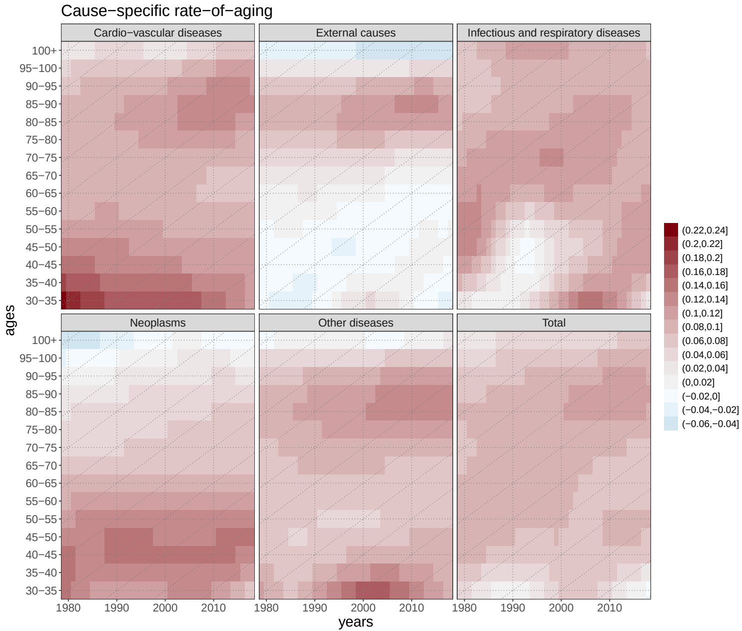

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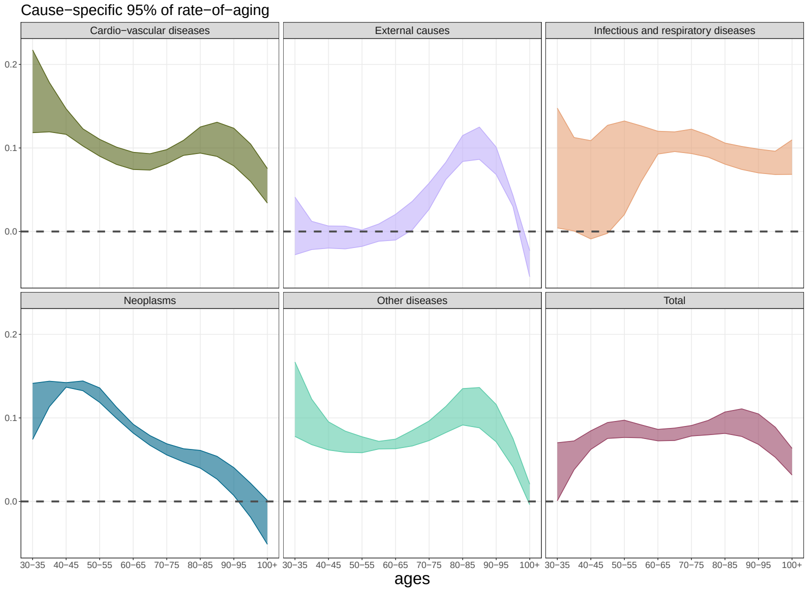

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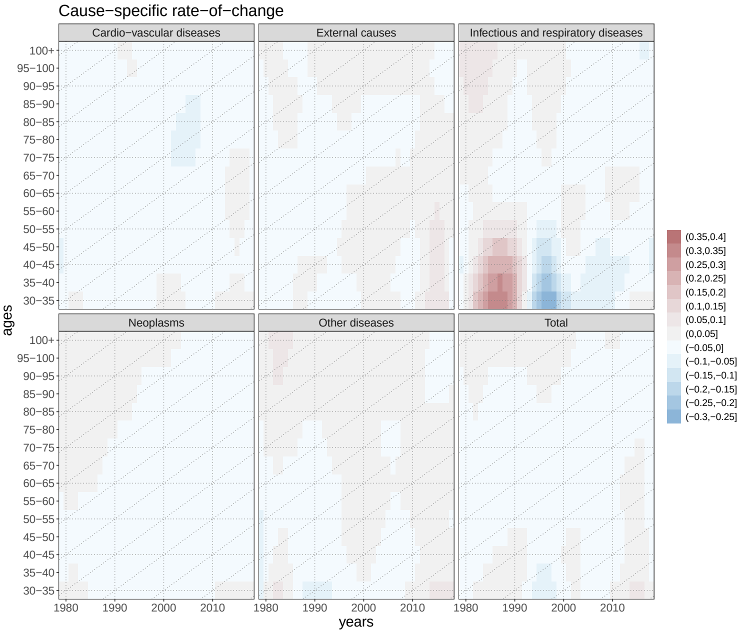

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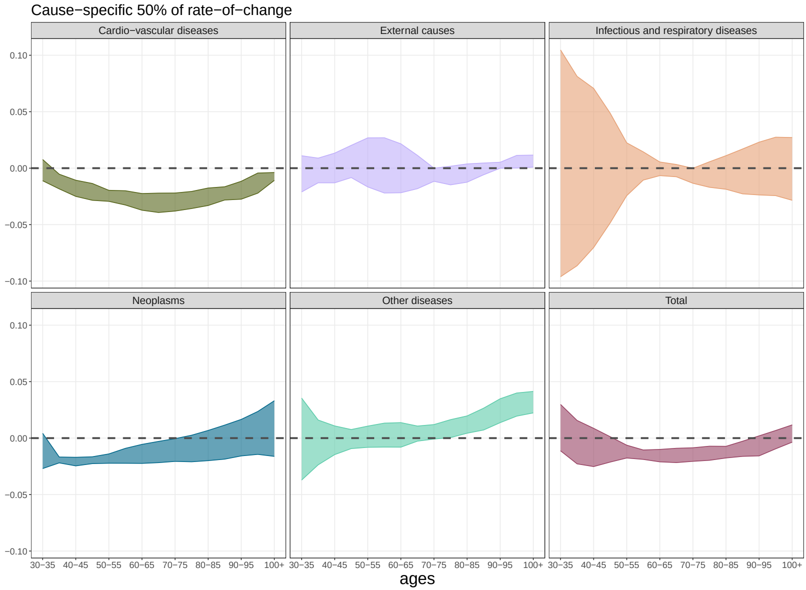

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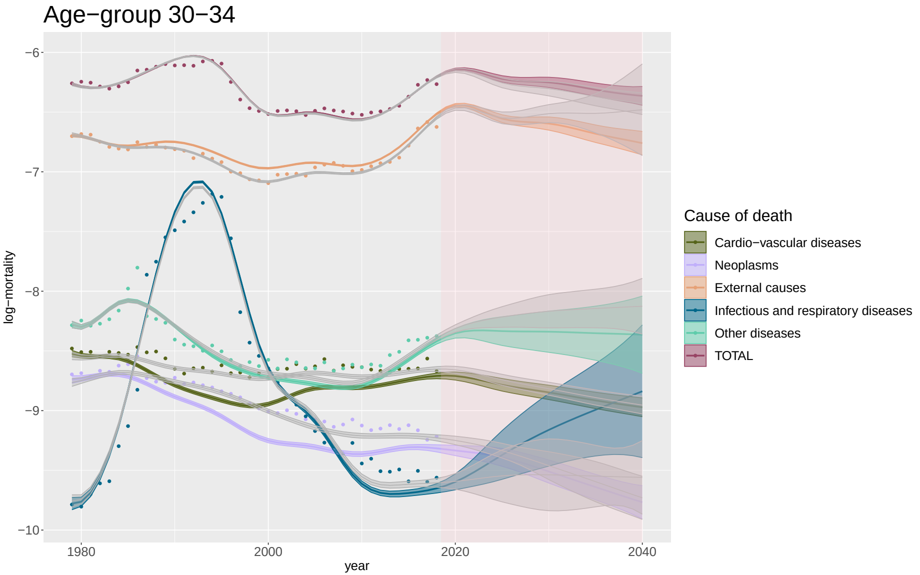
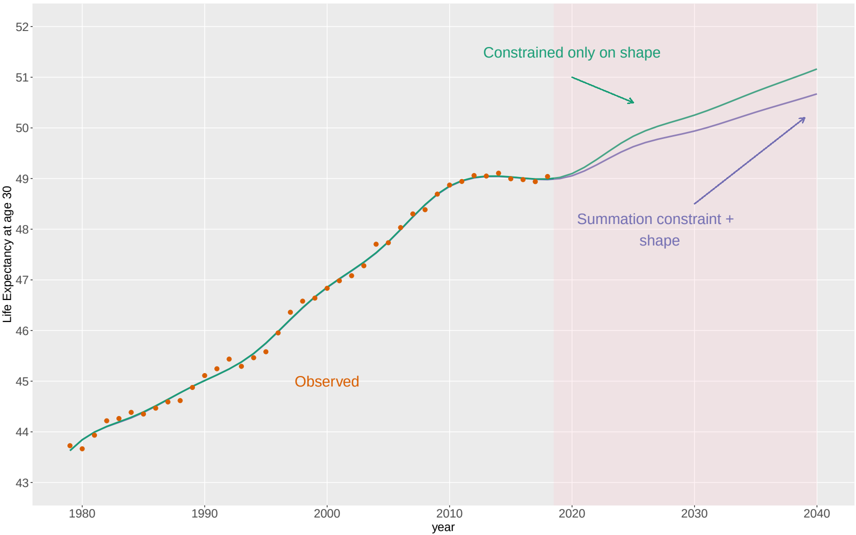
- We generalize *CP*-splines in a 3D setting (age+time+cause)

Outcomes: log-Rates



Grey lines and areas depict unconstrained model

Life expectancy at age 30 for overall mortality



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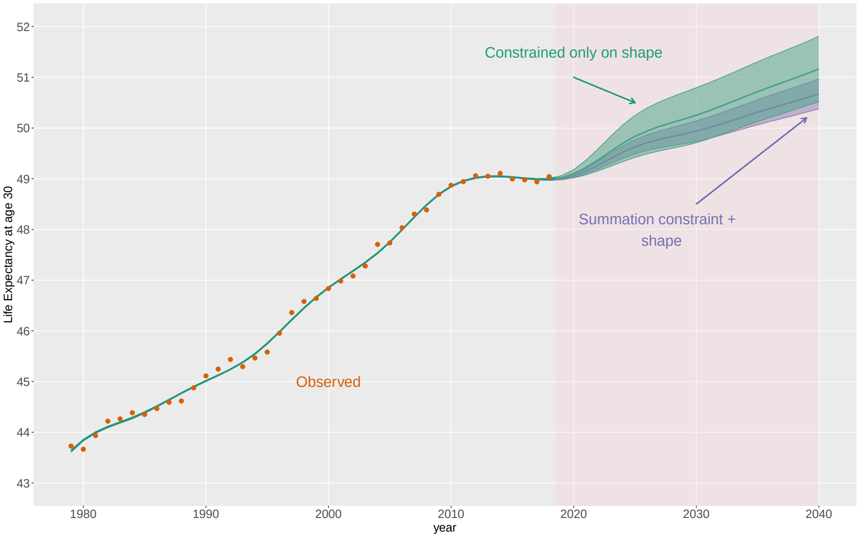

Concluding remarks

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- Non-linear summation constraints at the level death counts are incorporated by Lagrange multipliers
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- Future research:
  - Selection of the $2 \times (k+1)$ smoothing parameters
  - Back-testing for model performance
  - Extension to female/male and regional mortality settings

Giancarlo Camarda
in collaboration with María Durbán

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Thanks for your attention.
Comments and questions?

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