

Intermediation and Voluntary Exposure to Counterparty Risk *

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Abstract

I develop a model of the financial sector in which endogenous intermediation among debt financed banks generates excessive systemic risk. Financial institutions have incentives to capture intermediation spreads through strategic borrowing and lending decisions. By doing so, they tilt the division of surplus along an intermediation chain in their favor, while at the same time reducing aggregate surplus. I show that a *core-periphery* network – few highly interconnected and many sparsely connected banks – endogenously emerges in my model. The network is inefficient relative to a constrained efficient benchmark since banks who make risky investments “overconnect”, exposing themselves to excessive counterparty risk, while banks who mainly provide funding end up with too few connections. The predictions of the model are consistent with empirical evidence in the literature.

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1 Introduction

The years following the financial crisis resulted in an intense scrutiny of the architecture of financial markets. Many prominent economists have argued that the existing financial structure was socially suboptimal due to high systemic risk that emerged from excessive interconnectedness between financial intermediaries.¹ A relatively new, but fast growing, body of work tries to understand the optimal regulatory response to such financial structure. This literature mostly takes the financial structure as given, and assesses appropriate policy responses which minimize the systemic risk.² However, any policy which is implemented to mitigate the risk in the current financial architecture could feedback into bank decisions and influence the choice of inter-linkages. Alternative policy should account for endogenous changes to the financial structure. In this paper I develop a new model where the bilateral exposures of financial institutions emerge endogenously from their profit maximizing decisions. In doing so, I generate the underpinnings of interconnectedness in the financial sector, which allows me to evaluate formally the efficiency of the current financial architecture.

I develop a model of the financial sector in which endogenous intermediation among debt financed banks generates excessive systemic risk, which is measured as the distribution of total value lost due to bank failures. Financial institutions have incentives to capture intermediation spreads through strategic borrowing and lending decisions. By so doing, they tilt the division of surplus along an intermediation chain in their favor, while at the same time reducing aggregate surplus. I show that a *core-periphery network* – few highly interconnected and many sparsely connected banks – endogenously emerges in my model. In other words, my model predicts that there is a small number of very interconnected banks that trade with many other banks and a large number of banks that trade with a small number of counterparties.

There is overwhelming recent evidence that interbank markets exhibit a core periphery structure.³ Moreover, banks at the core has high gross exposures and low net exposures

¹A high degree of interconnectedness among financial institutions has been frequently recognized by policy makers. Federal Reserve chairman Ben Bernanke and undersecretary of finance Robert Steel, in their senate testimony on April 3, 2008, alluded to potential risk of system wide failure due to mutual interconnections of financial institutions in defending Bear Stearns bailout.

²Notable examples are stress tests designed by the Fed. See Fed [2012], Fed [2013] for more detail.

³ See Bech and Atalay [2010], Allen and Saunders [1986], Afonso and Lagos [2015] and Afonso et al. [2011] for evidence on federal funds market, Boss et al. [2004], Chang et al. [2008], Craig and Von Peter [2014] and van Lelyveld and in 't Veld [2014] for interbank market in other countries, Peltonen et al. [2014] and Vuillemeij and Breton [2013] for OTC derivatives and Di Maggio et al. [2015] for corporate bond inter-dealer market.

among themselves. My model not only provides a theoretical framework that jointly explains these empirical stylized facts; its main contribution is to do so by explicit modeling of intermediation among banks and its frictions.

In the model, the financial network consists of banks and their lending decisions. Banks need to raise resources for investment either from households or from other banks. My model endogenously generates indirect lending and borrowing in the interbank market, which is a prominent feature of both the federal funds market and over-the-counter market for derivatives.⁴ If the investment fails and the borrowing bank does not have sufficient funds to pay back her lender(s), it fails and potentially triggers a cascade of failures to the lenders, lenders of lenders and so on.

Banks are profit maximizers. There are two groups of banks in the model: those who have access to a risky investment opportunity, and those who do not. Each bank chooses its lending and borrowing relationships to get the highest expected possible rate on the funding it lends out and the investment it undertakes, net of cost of failure. When there are positive intermediation rents in the system, profit maximization creates private incentives to provide intermediation, which in turn leads to a particular structure for the equilibrium network. Since intermediation is profitable per-se, in equilibrium, competition implies that the banks who are able to offer the highest expected returns become intermediaries. These banks are exactly the ones who have access to the risky investment technology. On the other hand, a bank who is not an intermediary still wants to earn the highest possible returns, thus opting for the shortest connecting path to investing banks to avoid paying intermediation spread as often as possible. These two forces give rise to a core-periphery equilibrium network (definition 7) in which a subset of banks with risky investment opportunities constitute the core (theorem 1).

The interbank network generated by the model is socially inefficient. Banks who make risky investments “overconnect”, exposing themselves to excessive counterparty risk, while banks who mainly provide funding end up with too few connections.⁵ In other words, when default is costly, efficiency requires reaching the optimal scale of investment while minimizing the loss of failure, which leads to a different structure from the one which arises in equilibrium (theorem 2). This is in contrast to Gale and Kariv [2007] and Blume et al. [2009] who suggest that the financial architecture does not matter for efficiency. The main driving force behind this difference is the presence of intermediation rents which prevent social and private incentives from being aligned.

⁴Bech and Atalay [2010], Gofman [2012] and Di Maggio et al. [2015].

⁵The socially optimal structure is the one which maximizes the equally weighted sum of all bank expected profits.

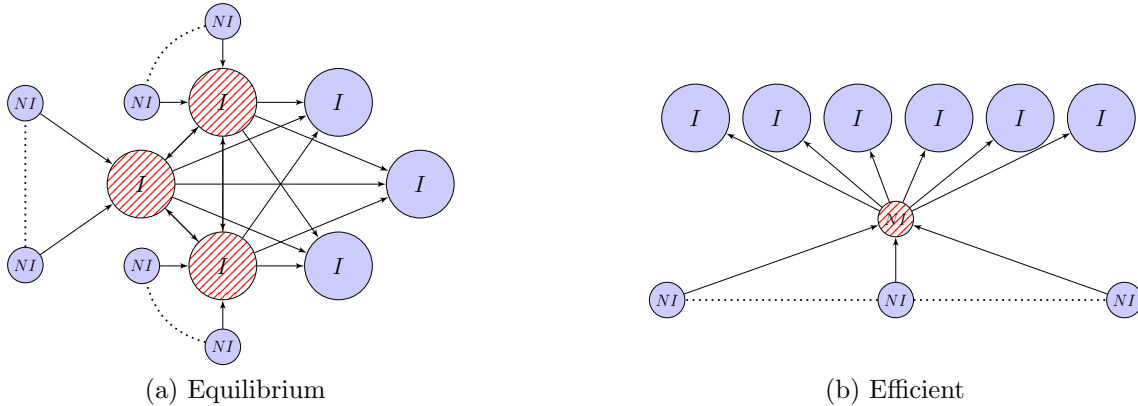


Figure 1: Equilibrium versus efficient structure of the interbank network⁶

1.1 Model Implications

The model predicts that multiple banks can be at the core of the financial system, with high gross and low net exposures among core banks. Consistent with this prediction, there is direct evidence from the financial crisis on substantial exposure among large financial institutions, which entailed runs and subsequent failure of one entity following its counterparty's failure.⁷

Equilibrium intermediaries are exposed to excessive risk since they do not contribute to the investment except through intermediation. The social planner prefers leaving such intermediaries out of the chain, replacing them with intermediaries who take minimal extra risk by intermediating. This minimizes the systemic risk without hurting the scale of investment. Thus social planner balances the net gain from investment with the expected loss of default. In contrast, private incentives compare rents, partially in the form of intermediation spreads, with the cost of default. The cost of default is a real cost while intermediation spreads are a mere redistribution of surplus. Consequently, I illustrate that the social and private incentives diverge in several situations. The intuition can be obtained by focusing on figure 1 that compares the equilibrium interbank network with the efficient one. Banks at the core are hatched in red in each structure.

⁶The labels I and NI refer to banks with and without potential risky investment, the latter solely raising funds from households and intermediating them to investing banks. See the model for the detail. The dots represent more NI banks.

⁷A prominent example, as reported in the FCIC report on the financial crisis, is the immediate run on holders of Lehman unsecured Commercial Paper (CP) and lenders to Lehman in tri-party repo, such as Wachovia's Evergreens Investment and Reserve Management Company's Reserve Primary Fund, after Lehman failed on September 15, 2008. The first wave of runs was followed by a second wave of withdrawal from Lehman OTC counterparties, most notably UBS and Deutsche Bank. For more details please see FCIC [2011].

One can also interpret the implications of the model in terms of *contagion*. In the model, investment and funding opportunities arise at different banks, which requires funding to be channeled from banks with liquidity surplus to the ones with investment opportunities. This decentralized distribution of resources and investment opportunities gives rise to endogenous interbank intermediation. Moreover, the return to risky investment is not contractible, so all the bank liabilities are in the form of debt, which leads to failure if obligations are not met. As a result, lenders and intermediators are exposed to counterparty risk. Because investment is positive NPV, there is an *optimal level* of contagion, due to counterparty risk exposure, in order to provide funding for the projects. In other words, even the financial structure chosen by a social planner involves a certain level of contagion when risky investment fails. The important prediction of the model is that the equilibrium interbank network involves *excessive* contagion, more than what is necessary to support the optimal level of investment.

Furthermore, an improvement in fundamentals of the economy does not necessarily translate to higher equilibrium welfare. An increase in project returns, as well as higher success probability, encourages risk taking behavior, enlarges the core of the financial network, and can decrease welfare through the endogenous change in the interbank structure.

The core-periphery structure implies that many banks are connected to each other only indirectly, a similar notion to weak ties as defined in Granovetter [1973]. In the context of the model, the weak ties are intermediary's borrowing and lending relationships. As these relationships are associated with rents, every bank prefers to have many weak ties. In equilibrium, banks who are able to pledge the highest return to their investors have many weak ties and are in the core.

The model not only provides predictions on the global structure of the interbank network, but also has implications about the bilateral interbank rates. Consistent with findings of Di Maggio et al. [2015] who empirically study the inter-dealer market for corporate bonds, my model predicts that core dealers charge higher average prices to the peripheral dealers than to other core ones. I also explore diversification incentives of banks in equilibrium, which uncovers a different channel for inefficiency, due to under-provision of insurance in the network (similar to Zawadowski [2013]).

Finally, I use the model to shed light on several policies related to the architecture of the financial networks. The model provides a framework to assess bailouts, as well as policy proposals to impose a cap on the number of counterparties and swaps. Moreover, it provides a new rationale for introduction of a Central Clearing Party (CCP) (section 6).

1.2 Literature Review

As a model of interbank networks, my paper is closely related to application of networks in economics (three early seminal papers are Jackson and Wolinsky [1996], Bala and Goyal [2000] and Aumann and Myerson [1988]). Jackson [2005], Jackson [2010] and Allen and Babus [2009] provide excellent reviews of the existing work.

There is also a fast growing literature on contagion and systemic risk in financial networks, started by the seminal work of Allen and Gale [2000] who studies the propagation of negative shocks in simple financial networks. A large part of this literature either focuses on properties of large networks, or take the structure of the network as given.⁸ More recent work in this area focuses on strategic link formation among financial institutions.⁹ Acemoglu et al. [2014], by locating banks on a ring, predicts that the equilibrium network can exhibit both under and over connection. Zawadowski [2013] uses the same ring network to provide a rational for under-insurance due to the high market price of insurance. Related to this literature is Kiyotaki and Moore [1997], who is one of the first papers that look at the formation of credit networks. Although the modeling assumptions of this paper are more closely related to supply chain networks, the implications for contagion and under-insurance can be interpreted in the context of financial networks.

Most relevant to my paper are Hojman and Szeidl [2006], Hojman and Szeidl [2008] and Babus and Hu [2015], which predict minimally connected star equilibrium structures, based on costly link formation.¹⁰ Moreover, unlike mine, these papers focus on undirected networks which is less suitable to model interbank, often asymmetric, relationships. Erol and Vohra [2014], Elliott and Hazell [2016] and Wang [2017] also study network formation, but do not consider intermediation. The two former papers predict clusters of unconnected or weakly connected cliques as equilibrium outcome, while the latter predicts a two-tiered structure where the periphery is strongly connected to multiple banks at the core. Chang and Zhang [2016] allows for intermediation among banks with no default risk, and predicts a star or forest as the equilibrium network. Neither Wang [2017] nor Chang and Zhang [2016] model contagion. My model contributes to this literature by providing rich predictions consistent with stylized facts about global structure of interbank networks missing from the previous work, and does that by underpinning a microfoundation for endogenous cost and benefit of interbank relationships.

⁸See Acemoglu et al. [2015], Eisenberg and Noe [2001], Elliott et al. [2014], Gofman [2011], Gai and Kapadia [2010] and Caballero and Simsek [2013].

⁹See Acemoglu et al. [2014], Blume et al. [2011], Babus [2016], Allen et al. [2012], Moore [2011], Rotemberg [2008], Zawadowski [2011], Zawadowski [2013], Bluhm et al. [2013] and Cabrales et al. [2012].

¹⁰Babus and Hu [2015] can have an equilibrium which is an interlinked star network as well.

There is also an emerging literature on bargaining and intermediation in (financial) networks (Gale and Kariv [2007], Manea, Gofman [2011] and Babus and Hu [2015]). In all of these models except Babus and Hu [2015] intermediaries are determined exogenously. In my model, certain agents endogenously assume the role of intermediaries, which can lead to welfare losses in equilibrium.

Finally, my paper is also related to the literature which studies the role of banks as intermediaries, their balance sheet structure and issues related to insolvency.¹¹ In this literature, banks are intermediaries between investors and entrepreneurs. I add to this literature by specifically modeling the role of banks as intermediaries among each other, and study the corresponding implications for the structure and efficiency of financial sector, as well as systemic risk.

The rest of the paper is organized as follows. Section 2 lays out the basic environment. Section 3 provides a simplified version of the economy with four banks and solves for the equilibrium and constraint efficient structure. Section 4 specifies the detail of the lending contracts for general network structures. Section 5 provides the general results. Section 6 discusses policy implications of the model, and section 7 solves an extension of the model with diversification. Section 8 concludes.

2 Model

There are three periods, $t = 0, 1, 2$, and one good which I refer to as funding. There are two types of agents: banks and households. There are K banks in the economy, which are one of the following two types. Banks $I \in \mathbb{I}$ randomly get risky investment opportunities, while banks $NI \in \mathbb{NI}$ do not. Let $\mathbb{N} = \mathbb{I} \cup \mathbb{NI}$, $k_I = |\mathbb{I}|$ and $k_{NI} = |\mathbb{NI}|$, and assume $k_{NI} \geq k_I$.

The financial system consists of banks and their bilateral exposures. The bilateral exposures represent lending and borrowing relationships among banks through debt contracts. Bank i who lends to bank j through a debt contract is exposed to bank j since if bank j fails, it will not be able to pay bank i back, which affects the balance sheet of bank i and might cause i to fail.

The investment opportunity is a risky asset, and is linearly scalable. Each bank I receives the opportunity to invest in the risky asset, with iid with probability q . Let

¹¹An incomplete list includes Diamond [1984], Rochet and Tirole [1996], Kiyotaki and Moore [1997], Moore [2011], Lagunoff and Schreft [2001], Leitner [2005], Cifuentes et al. [2005], Dang et al. [2010], Dasgupta [2004], Acharya et al. [2012], Acharya and Yorulmazer [2008] Bhattacharya and Gale [1987], Bolton and Scharfstein [1996], Diamond and Rajan [2005], Farhi and Tirole [2013] and Gorton and Metrick [2012].

$\tilde{\mathbb{I}}_R \in \mathbb{I}$ denote the random variable corresponding to the subset of banks that receive the opportunity, and let \mathbb{I}_R be the realization of such subset.¹²

Let $\tilde{R}_i \in [0, \bar{R}]$ denote the (per-unit) random return of bank i 's investment in the risky asset, which is iid across banks. The project succeeds with probability p and returns R , and fails with probability $1 - p$ and returns 0.

Besides the risky investment opportunity, each bank i (of type I or NI) has a value V_i , which is the value of the other businesses, assets, and services the bank provides. If the bank fails for any reason, this value is lost.¹³¹⁴ For simplicity, I assume $V_i = V_I$ for every $i \in \mathbb{I}$ and $V_j = V_{NI}$ for every $j \in \mathbb{NI}$.

Bankers do not have any wealth. They can raise funding from two sources in the form of debt. At $t = 0$, each bank NI_j raises resources from a continuum of households hh_j , of measure one. Each household is endowed with one unit of funding. Because each set of households is a continuum, they are competitive and they lend their endowment to their corresponding bank as long as they break even. Second, a bank can borrow from other banks at $t = 1$. To do so, at $t = 0$, it must have established a *potential borrowing relationship* with them.

I model the financial system as a network. The financial network is a directed graph $G = (\mathbb{N}, E)$, where $\mathbb{N} = \{1, 2, \dots, K\}$ is the set of nodes and $E = \{e_{ij}\}_{i,j \in \mathbb{N}}$ is the set of edges. Each node is a bank, and edge $e_{ij} \in E$ is a potential lending relationship from bank i to bank j . $e_{ij} \in E$ only if at $t = 1$ funding is lent along this potential lending relationship with non zero probability. Otherwise e_{ij} is removed from E .

Each bank chooses its potential borrowing and lending relationships, that is, links over which he can borrow or lend, to maximize its expected profit net of failure cost.

The timing of the model is as follows: At $t = 0$, banks raise funding from households and the potential lending and borrowing relationships are formed. A link e_{ij} means bank j can borrow from i in the period that follows. At $t = 1$, investment opportunities are realized and actual lending happens only along (some of) the links formed at $t = 0$. At $t = 2$ random returns are realized, and banks that are not able to pay back their creditors fail. Holding precautionary liquidity is ruled out, so banks lend or invest as much resources as they are able to raise.

¹²Throughout the paper, I will use the following convention: \tilde{x} denotes a random variable, and x denotes the realization of that random variable.

¹³This value accrues to the banker himself. This model is isomorphic to one with bankruptcy costs that are borne by the bankers in the event of failure.

¹⁴James [1991] finds that losses due to bank failure are substantial, losses on assets and direct expenses averaging 30% and 10% of the failed bank's assets, respectively.

3 Economy with Four Banks

Starting with a simplified version of the model before formal description of the contracts is useful. Here I characterize all the equilibria in an economy with four banks, which illustrates the main forces of the model. In section 4 I provide a description of the contracts under which the same intuition carries over to the general case.

There are two I and two NI banks, $\mathbb{I} = \{I_1, I_2\}$ and $\mathbb{NI} = \{NI_1, NI_2\}$. Each NI bank raises one unit from a continuum of households, while I banks do not raise any outside funding. As a result, each bank I needs to secure funding on the interbank market at $t = 0$ to be able to invest in its project later, at $t = 1$, if it gets an investment opportunity.

To borrow on the interbank market at date $t = 1$, banks need to enter potential agreements at $t = 0$. Potential agreements are similar to credit lines, except that they do not have a limit. I use the two terms interchangeably. Each agreement (established at $t = 0$) is a promise by the lender to deliver at least one unit (at $t = 1$) if the borrower receives an investment opportunity, or if the borrower has a credit line to another bank that has received an investment opportunity.

Definition 1. [Eligibility] *Any potential borrower who has a direct or indirect access to a realized investment opportunity is eligible to draw on his credit line.*

With some abuse of language, I use *eligible* for potential relationships as well. I assume a bank cannot default on its eligible promises, i.e., for any realization of investment opportunities, a potential lender must have sufficient funds to lend each eligible potential borrower at least one unit.

For a concrete example, consider figure 2. In 2a, NI_1 has the unit it has raised from households, but no other source of funding. In particular, credit line $NI_2 \rightarrow NI_1$ does not exist. Moreover, NI_1 has a credit line to both I bank, so both $NI_1 \rightarrow I_1$ and $NI_1 \rightarrow I_2$ exist. In 2b, NI_1 has two units pledged to it, one from households and one through credit line $NI_2 \rightarrow NI_1$. Now consider the $t = 1$ state where both I banks receive investment opportunities. In both structures, NI_1 has promised one unit to each I bank. However, in 2a, it will not be able to keep its promise. 2a is not a feasible structure, and is ruled out. This restriction is formalized in the following assumption.

Assumption 1. [Feasibility] *Each realized lending has a minimum size, normalized to one unit. Each potential lender must satisfy his eligible potential lending promises.*

This assumption implies an opportunity cost for forming potential lending relationships, and puts an endogenous limit on the number of potential relationships a bank can establish.



Figure 2: Two possible sets of potential relationships between two NI and two I banks.

At $t = 0$ a bank can enter into as many potential lending relationships as he chooses, as long as, for each realization of uncertainty at $t = 1$, he is able to raise sufficient funding either from households ($t = 0$) or on the interbank market ($t = 1$), through his potential borrowing contracts, to service them.

There is an exogenous division of expected net surplus that allocates a strictly positive share to every bank involved in an intermediation chain, for each realization of investment opportunities. When bank i raises funding from households and lends directly to bank j who makes the investment ($i \rightarrow j$), j and i receive in expectation a share $1 - \alpha$ and α of expected net surplus of the project, respectively. Alternatively, if i raises the funding, lends to k who in turn lends to j who invests ($i \rightarrow k \rightarrow j$), then j , k , and i receive $1 - \alpha$, $\alpha(1 - \alpha)$, and α^2 shares, respectively.¹⁵¹⁶

All the contracts are bilateral. The final return of the project at $t = 2$ is not contractible, so all the contracts are in the form of debt. However, the contract can be written contingent on all date $t = 0$ and $t = 1$ outcomes, specifically on the network structure, as well as the realization of investment opportunities. So the bilateral contracts are *contingent debt* contract in which the face value of debt is set such that given the network and the realization of investment opportunities, each bank along the intermediation chain receives its appropriate share, as described above.

I assume the participation constraints for the lender and borrower are satisfied in a direct lending relationship $NI \rightarrow I$; $(1 - \alpha)(pR - 1) > (1 - p)V_I$, and $\alpha(pR - 1) > (1 - p)V_{NI}$. Note that such relationship is socially desirable if $pR - 1 > (1 - p)(V_I + V_{NI})$, which is implied by the pair of participation constraints. Moreover, I assume that lending via one

¹⁵Multiple papers provide evidence for existence of intermediation rents. Examples includes Di Maggio et al. [2015] and Li and Schürhoff [2014] who document intermediation spreads charged by dealers in the dealer network for corporate bonds and municipal bonds, respectively.

¹⁶The rule for division of surplus can be micro-founded to endogenize the prices. Section 9.3.1 outlines a micro-foundation using a moral hazard constraint, i.e. limited commitment at the bank level, as in Babus and Hu [2015]. Details are available upon request.

intermediator is viable: $\alpha^2(pR-1) > (1-p)V_{NI}$. Throughout the paper I assume analogous conditions to be satisfied for more general rules for division of surplus.

As noted above, with only two banks individual rationality is sufficient for efficiency, but not necessary. This implies that with only one lender and one borrower, equilibrium can only exhibit under-investment, in the form of under-lending. Remarkably, I show that with more banks and the possibility of multiple investment opportunities, the equilibrium involves over-lending among a certain group of banks.

3.1 Equilibrium

The equilibrium concept is group stability, as defined in Roth and Sotomayor [1990]. It is a generalization of pairwise stability defined in Jackson and Wolinsky [1996], generalized to allow for any number of banks to participate in the deviation.¹⁷

Definition 2. *A network structure G is blocked by a coalition B of banks if there exists another (feasible, individually rational) network structure G' and a coalition B such that*

(a) *G' can be reached from G by a set of bilateral deviations by $b, b' \in B$ and unilateral deviations by $b \in B$.*

(b) *Every bank $b \in B$ is strictly better off in G' than in G .*¹⁸

Definition 3. *A group stable network is one that is not blocked by any coalition of banks.*

The only viable group deviations are those in which the resulting network G' is feasible, and every $e_{ij} \in G'$ is traversed with positive probability at $t = 1$. If $\exists e_{ij} \in G'$ such that lending over e_{ij} always violates either the lender or borrower individual rationality, and never happens in equilibrium, then e_{ij} is removed from G' and the corresponding B is not a blocking deviation.

Before moving to equilibrium characterization, consider the following two lending arrangements: in the first arrangement bank i lends one unit, at face value D , directly to j who

¹⁷A subtlety exists in adopting a concept from matching to my framework. In most matching models, the utility of each agent is only own-match dependent and does not depend on the rest of the matching. However, in my model, utility of the blocking coalition can depend on the rest of the network, so what banks outside the coalition do matters. I assume they don't change their actions. This equilibrium concept is different from the one in which the blocking coalition goes to autarky, which is referred to as the β -core in the network literature.

This more complex equilibrium concept is better suited to think about the core mechanism of the model, compared to pairwise stability. Interbank intermediation generically involves more than two banks, so the latter may not be the best notion for addressing the relevant deviations.

¹⁸Strict preference is equivalent to weak preference with an $\epsilon \rightarrow 0$ cost of deviation.

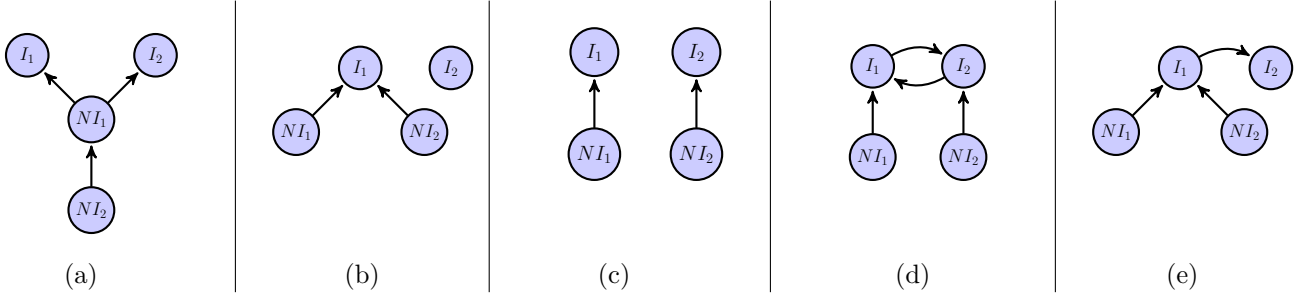


Figure 3: Possible equilibria for an economy with two I and two NI banks

invests the unit. In the second arrangement i lends the one unit to k at face value D_1 , who in turns lends the unit, at face value D_2 , to j who invests the unit. The face value of debt is set to ensure that in expectation, each party (including the intermediary) receives its share of expected net surplus. Thus $D_1 = (\alpha^2(pR - 1) + 1)/p < D_2 = D = (\alpha(pR - 1) + 1)/p$, and $D_2 - D_1$ represents the *intermediation spread*.

Main mechanism. Figure 3 depicts all the possible equilibria of the economy with four banks. To gain some intuition about individual bank incentives and coalitional deviations, consider two specific equilibria, 3a and 3d. In 3d, regardless of which bank(s) have the investment opportunity, all the banks are involved as either investor, intermediary, or final lender in every investment, so they all make profits if projects are successful. On the other hand, in 3a, if only one I invests, the other I bank is not involved and not exposed to the risk of investment failure. Since failure to repay debt obligation entails costly default, rents come at a cost. As such, being part of the intermediation chain is profitable in success, but costly in failure.

Now consider the joint deviation by $\{I_1, I_2, NI_2\}$ in 3a, which leads to 3d, as depicted in figure 4. When only I_2 receives the investment opportunity, I_1 serves as the intermediary for NI_1 and captures the intermediation rents. However, it fails if I_2 fails. The incremental cost of default in 3d (compared to 3a) is born by I banks, that is, precisely the banks that can choose to be out of the chain of intermediation (as in 3a). Yet if the intermediation spread ($D_2 - D_1$) is sufficiently high, I banks voluntarily choose to expose themselves to this incremental cost, and deviate from 3a to 3d in order to earn the spread. In other words, when capturing intermediation spreads requires exposure to counterpart risk, if these rents are high enough, banks prefer to incur the additional risk to capture them.

Finally, NI_2 must benefit from joining the coalition. Each bank chooses to lend to counterparties that offer the highest rate of return. Given the intermediation spreads, being “close” to the banks who invest translates into higher returns, and in 3a, NI_2 is always



Figure 4: Joint deviation by $\{I_1, I_2, NI_2\}$ leading from structure 3a to 3d.

far from the banks who invest. As a result, NI_2 also has an incentive to join a deviation which leads to a structure in which it sometimes avoid paying intermediation spread. Thus if intermediation is sufficiently profitable, I banks attract NI banks as direct lenders by offering (occasional) higher returns, and by doing so expose themselves to counterparty risk in order to capture the intermediation spreads.

Formally, let $X = pR - 1$ be the net expected return of one unit investment in the project. Also, let $\kappa = \frac{\alpha(1-\alpha)X}{(1-p)V_I}$, which is the ratio of the intermediation spread per unit intermediated over the expected cost of default due to intermediation for an I bank. The next proposition provides a characterization of equilibria in the economy with four banks.

Proposition 1. *For every set of parameter values $(q, p, R, \alpha, V_I, V_{NI})$, $q < \bar{q}$, there exist constants $(\tilde{\kappa}, \bar{\kappa})$ such that the following conditions characterize all the equilibria:*

- (a) *If $\kappa \geq \bar{\kappa}$, every equilibrium has a core-periphery structure and is inefficient.*
- (b) *If $\tilde{\kappa} \leq \kappa \leq \bar{\kappa}$, constraint efficient and inefficient equilibria coexist. The inefficient equilibria are either core-periphery or they feature under-investment. The constraint efficient equilibrium is a star network.*
- (c) *If $\kappa < \tilde{\kappa}$, the only equilibrium is empty network.*

The proof characterizes all possible equilibria as a function of κ , depicted in figure 5.

It is useful to start by the constraint efficient network, i.e. the network that maximizes the total surplus subject to feasibility and individual rationality. This is a star network with one NI bank at the center.¹⁹ Fixing the expected default loss of the economy, maximizing scale of investment is efficient because the return on the asset exhibits constant return

¹⁹Star is a special case of core-periphery, with one bank at the core.

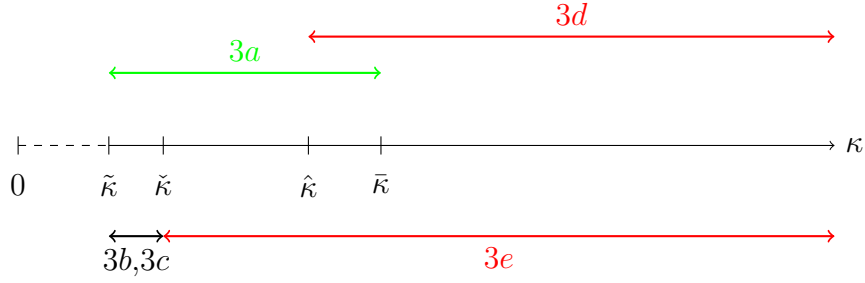


Figure 5: Equilibria of the economy with four banks as a function of κ , the ratio of per-unit intermediation spread over the expected cost of default for I . The green equilibrium is efficient. The two red ones are inefficient due to overconnection, and the black one is inefficient due to underconnection.

to scale. Thus given the maximum investment size, the social planner's problem reduces to minimizing expected loss of default due to failure of project(s). When a project fails, *contagion* also occurs: I is not able to pay its lenders back, and lenders might fail as well depending on their asset composition. Consequently, the constraint efficient solution has one NI bank be the intermediary, borrow from the other NI , and lend to both I s (3a).

This result is quite intuitive: the social planner's objective is to maximize total net return from the projects minus the expected loss, and he does not care about how the return is distributed among agents. Let NI_1 be the "intermediator" NI . Since maximizing the scale of invest requires that NI_1 's funding is channeled to I_1 and I_2 when either of them has an investment opportunity, NI_1 can as well intermediate the funding raised by NI_2 . In other words, intermediation does *not* expose NI_1 to any extra counterparty risk. As such, the scale of investment is maximized while minimizing the cost conditional on failure. A similar intuition goes through in the general case.

Next, consider the region $\kappa > \bar{\kappa}$, where both equilibria, 3d and 3e, are core-periphery and inefficient. To gain some intuition about how $\bar{\kappa}$ is determined, consider the incentives of core and peripheral banks. A key feature of the inefficient core-periphery equilibria is that the core consists of I banks. The first order inefficiency comes from core I banks' exposure to excessive counterparty risk. They are willing to take this risk and expose themselves to cost of contagious default only if intermediation spreads are sufficiently high, above a certain threshold. Moreover, in order for these equilibria to be sustainable, peripheral NI banks should be willing to directly lend to the I core banks. They compare the benefit of circumventing frequent intermediation spread with potentially lower diversification benefit, and the former is larger exactly when intermediation spreads are high, above a second threshold. $\bar{\kappa}$ is determined by the maximum of the two thresholds.

Region $\hat{k} < \kappa < \bar{\kappa}$ is interesting: in this region, there is an inefficient equilibrium in which the banks with risky investment opportunities are in fact willing to decrease their (inefficient) exposure to counterparty risk and deviate to a more efficient equilibrium. However, they are not able to convince their peripheral lender banks to agree to a lower rate and keep funding them, since that implies some peripheral banks have to pay expected intermediation spreads that they deem to high. As a result the economy can be *stuck* in the bad, high-risk equilibrium. Finally, consider the lowest region for κ where the equilibrium network is not yet empty (3b and 3c). These equilibria have the same feature of under-investment due to under-lending.

To recapitulate, there are always multiple equilibria: if intermediation spreads are low there are both efficient and inefficient equilibria; once the intermediation rents become sufficiently high, $\kappa > \bar{\kappa}$, all the equilibria become inefficient.

4 General Specification

Because few constraints are imposed on the structure of the interbank network, complex networks can form. In particular, multiple intermediation chains can exist between two banks. As a result, I need a rich set of contracts to specify how the funds flow in the network given a network structure and a realization of investment opportunities.

4.1 Lending Contracts

Lending contracts are formed before banks receive their investment opportunities. e_{ij} represents bank i 's potential lending relationship to bank j , subject to a generalized notion of feasibility explained shortly. In this sense, lending contracts are *conditional credit lines*.

There is perfect information. Every bank knows the set \mathbb{I} and \mathbb{NI} , the structure of the formed lending contracts, the realization of the investment opportunities and the realization of final returns. However, markets are incomplete. First, the realization of returns are not contractible, so all the contracts are of the form of debt. Second, the potential lending contracts are formed before investment opportunities are realized.

The only restriction on lending relationships is that assumption (1) has to be satisfied, financial network G can be quite complex. The following definitions are useful to explain the contracts.

Definition 4. *Given financial network G , a “path” from bank i to bank j is a sequence of banks $\{i_1, \dots, i_m\}$ such that $e_{i_d i_{d+1}} \in E$ for $\forall d = 1, \dots, m - 1$.*

A “cycle” is a closed path; that is, $i_m = i_1$.

A “leaf” bank is a bank that only lends to other banks and does not borrow.

Bank i is “connected” to bank j if a path exists from bank i to bank j .

For every unit of funding raised from households at bank i , invested by bank j , and intermediated through a number of intermediaries $\{i_1, \dots, i_m\}$, the sequence of banks involved $\{i, i_1, \dots, i_m, j\}$ (or any subsequence of it) is called an “intermediation chain” (or simply a “chain”). Banks $\{i_1, \dots, i_m\}$ are “intermediators” along the chain.

The “shortest path” from bank i to j , $SP(i, j)$, is the (set of) path(s) that involves the minimum number of intermediaries. With some abuse of notation, I use $SP(i, \mathbb{J})$ to denote the union of shortest paths of i to all banks $j \in \mathbb{J}$, $SP(i, \mathbb{J}) = \{SP(i, j)\}_{j \in \mathbb{J}}$.

The “distance” from bank i to j is the number of edges along the shortest path between i and j , $dist(i, j) \equiv |SP(i, j)|$.

Banks are not competitive. For each set of investment opportunities, \mathbb{I}_R , and set of lending contracts, E , a subset of potential lending contracts will be realized. There is a fixed distribution of expected total surplus over all the banks involved in raising, intermediating, and investing the funds, denoted by $\mathcal{L}(G, \mathbb{I}_R)$, which is a primitive of the model. With a slight abuse of notation, let $\mathcal{L}(i; G, \mathbb{I}_R)$ denote the share of bank i .

$\mathcal{L}(\cdot)$ satisfies the following (sufficient but not necessary) properties. First, the rule is anonymous, and the net expected surplus from each unit of investment is divided only among the banks in the corresponding intermediation chain, as a function of length of the chain and bank position. Second, for every unit of funds, every member of the corresponding *intermediation chain* receives strictly positive shares of net surplus generated by that unit. Third, eliminating an intermediary from an intermediation chain weakly increases the share of every other bank along the chain, and strictly increases the share of the initial lender. Moreover, renegotiation and side payments are ruled out.

Let $B(i; G) = \{j | e_{ij} \text{ exists}\}$ and $C(i; G) = \{j | e_{ji} \text{ exists}\}$ denote the set of *relationship borrowers* and *relationship creditors (lenders)* of bank i in interbank network G , respectively. For each realization of \mathbb{I}_R , bank i can be connected to each $I \in \mathbb{I}_R$ through multiple intermediation chains of different lengths, which makes a generalization of definition (1) necessary.

Definition 5. [Eligibility Revisited] *Given the interbank network and each realization of investment opportunities, each borrower of i that is on at least one of i ’s shortest paths to the set of banks with realized investment opportunities receives at least one unit from i .*

$$(1) \quad \forall \mathbb{I}_R, \forall j \in B(i; G) \text{ if } \exists I \in \mathbb{I}_R \text{ s.t. } j \in SP(i, I) \Rightarrow i \text{ lends } j \text{ at least one unit.}$$

The financial network has to satisfy feasibility as defined in assumption (1), incorporating the above more general notion of eligibility.²⁰

This notion of eligibility ensures that in the interim period, if i 's fund is (directly or indirectly) lent to $I \in \mathbb{I}_R$, it is intermediated through i 's shortest path to I ; that is, minimum intermediation rents are paid. The intuition is that when bank i can lend to a bank with an investment opportunity through multiple intermediation paths, at $t = 1$, it chooses the option that provides it with the highest possible rate. What the lender is not able to do in the interim period is to add a new lending. After the investment opportunities are realized, if j wants to be able to borrow from i , link e_{ij} needs to exist in G . Moreover, only lending contracts along the shortest paths are realized at $t = 1$.

Intermediator $j \in SP(i, \mathbb{I}_R)$, who receives the unit raised at i , must lend the unit along (one of the) $SP(i, \mathbb{I}_R)$ paths on which it lies. Within $SP(i, \mathbb{I}_R)$, j has discretion to allocate i 's unit so that j satisfies the minimum size constraint over all its realized lending contracts. The unit j has raised from outsiders receives equal treatment. Starting from leaf banks, at every bank, units are lent accordingly to satisfy the minimum size constraint. Any excess unit is divided equally among all the corresponding shortest paths. The process is done recursively starting from the leaf nodes until either all the units are allocated to investment opportunities, or no credit line exists along which a unit can be lent.²¹

The face value of debt is contingent on the network G and the realization of \mathbb{I}_R , thus on set of realized lending contracts. It is set such that in expectation (over realizations of random returns $\{\tilde{R}_k\}_{k \in \mathbb{I}_R}$), each bank i receives $\mathcal{L}(i; G, \mathbb{I}_R)$.

At $t = 1$, given the equilibrium network G and each realization of investment opportunities, \mathbb{I}_R , the contracts determine the number of units lent along each potential lending agreement, as well as the face value of debt corresponding to this realized lending. Let $m_{ij} = m(i, j; G, \mathbb{I}_R)$ denote the size of the loan from bank i to j , and let $D_{ji} = D(j, i; G, \mathbb{I}_R)$ denote the per-unit face value corresponding to this loan, payable by bank j to bank i . Moreover, let $D_j^h = D(i; G, \mathbb{I}_R)$ be the face value of debt payable by bank j to households.

At $t = 2$, given any realization of project returns $\{R_k\}_{k \in \mathbb{I}_R}$, a borrower may or may not be able to pay lenders back in full. Let $d_{ji} = d(j, i; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R})$ and $d_j^h = d(j; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R})$ denote the per-unit (partial) repayment of bank j to bank i and house-

²⁰ $e_{ij} \in G$ does not imply i lends at least one unit to j for every realization of investment opportunities at $\hat{I} \in \hat{\mathbb{I}}$ banks to whom j has a path, as j might not be part of $SP(i, \hat{I})$ for any $\hat{I} \in \hat{\mathbb{I}}$.

²¹This detail can be specified differently without altering the results as long as the network is feasible, where the general notion of eligibility, equation (1) is satisfied. The reason is that contracts can be written on what happens at date $t = 1$. At $t = 0$, banks correctly forecast the expected rates they will be pledged, as well as their expected probability of default given any set of rules and adjust their connections accordingly. This particular choice helps explain the deviations.

holds, respectively. As a convention, $D_j^h = d_j^h = 0$ if j has not borrowed from households. By construction, $d_{ji} \in [0, D_{ji}]$. Finally, let $L(i; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R})$ and $A(i; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R})$ denote the total liabilities and assets of bank i at date 2 when all the uncertainty is resolved:

$$L_i = L(i; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}) = \sum_{j \in \mathbb{N}} m_{ji} d_{ij} + d_i^h$$

$$A_i = A(i; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}) = \mathbb{1}[i \in \mathbb{I}_R] \left(R_i \left(\sum_{j \in \mathbb{N}} (m_{ji} - m_{ij}) \right) \right) + \sum_{j \in \mathbb{N}} m_{ij} d_{ji},$$

where $\mathbb{1}[i \in \mathbb{I}_R]$ is the indicator function that takes value one if i has access to an investment opportunity. Consequently, the per-unit (partial) repayment from j to i in each state of the world can be written as

$$(2) \quad d_{ji}(j, i; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}) = \max \left\{ 0, \min \left\{ D_{ji}, D_{ji} \frac{A_j}{L_j} \right\} \right\},$$

and a similar expression holds for d_j^h . The above expression simply means that if a borrower does not have sufficient funds to repay its lenders, each lender will be paid back pro-rata, and there is limited liability.²²

Given the solution to the system of (partial) debt repayments at $t = 2$, specified by (2), using backward induction, the face value of each debt contract at date $t = 1$ is set such that in expectation, each bank i receives its share of surplus according to $\mathcal{L}(i; G, \mathbb{I}_R)$. This completes the specification of contracts.

4.2 Bank Optimization Problem

Let $S(i; G, \mathbb{I}_R, \{R_i\}_{i \in \mathbb{I}_R})$ denote the ex-post profit of bank i ,

$$S(i; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}) = A(i; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}) - L(i; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}).$$

Let $\mathbb{1}[i \text{ survives}; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}]$ denote the indicator function that takes value one if bank i survives at $t = 2$, given the financial network G formed at $t = 0$; and let $P(i; G) = \mathbb{E} \left[\mathbb{1}[i \text{ survives}; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}] \right]$ denote the corresponding probability. Banker i 's optimiza-

²²This definition implies that all debt is pari passu. Junior household debt can be interpreted as capital and be used to study the effect of capital requirements.

tion problem at $t = 0$ can be written as

$$(3) \quad \begin{aligned} \max_{\{e_{im}, e_{mi}\}_{m \in \mathbb{N}, m \neq i}} \quad & \mathbb{E} \left[S(i; G, \mathbb{I}_R, \{R_k\}_{k \in \mathbb{I}_R}) \right] + P(i; G) V_i \\ \text{s.t.} \quad & \text{Feasibility (1) ;} \\ & \text{Participation Constraint.} \end{aligned}$$

where the expectations are taken over realization of \mathbb{I}_R , determined by realization of investment opportunity at $t = 1$, and realization of project returns $\{R_k\}_{k \in \mathbb{I}_R}$ at $t = 2$. The choices of other banks are reflected in G .

Since the banking sector is non-competitive and each bank gets strictly positive share of the surplus, the banker uses the structure of its inter-bank connections to extract more rents. Each bank balances the costs and benefits of exposure to more risk via intermediation, and chooses the set of lending and borrowing relationships that maximizes its total expected profit.

4.3 Lending Structure and Division of Surplus

In this section, I specify a highly tractable rule for surplus division, α -rule, the general version of the rule in section 3. I use α -rule throughout the paper, and show in theorem 3 that the main results hold for any fixed surplus division \mathcal{L} that satisfies the properties of section 4.

Consider an intermediation chain of infinite length, and one unit of funding intermediated along the chain. The share of net surplus received by each bank along the chain, starting from the final borrower, falls at rate α , so that the initial lender (who is infinitely far away) receives a negligible share of net surplus and breaks even. Since the sum of the shares adds up to one, the final borrower receives share $(1 - \alpha)$, the immediate intermediary receives $(1 - \alpha)\alpha$, and the intermediary at distance d receives $(1 - \alpha)\alpha^d$. Now suppose the initial lender is at distance k (instead of being infinitely far away). It receives the cumulative share of all *hypothetical* intermediators at distance k and further, i.e. it gets α^k share of net surplus plus the cost of initial investment. This particular division implies the lender bears all the cost of intermediation, and makes the face value of a unit of debt payable to each lender independent of the source of funding in the chain.²³

²³In section 9.3.3 I solve the model with α -rule augmented to incorporate expected default costs and show that the same results hold.

5 Results

The first proposition provides bounds on the flow of funds at date $t = 1$, given the realization of investment opportunities. To do so, construct the following auxiliary graph \hat{G} from G , given the realization of \mathbb{I}_R . Remove all edges among I banks. Moreover, remove $\mathbb{I} \setminus \mathbb{I}_R$ and all the remaining edges incident on them from G . Define the weight of edge e_{ij} to be m_{ij} . Finally, reverse the direction of all edges. Let i_1 be i_2 's parent if $e_{i_1 i_2}$ exists in \hat{G} .

The following definition is useful to state the proposition.

Definition 6. A “cut” is a partition of the nodes of a graph into two disjoint subsets that are joined by at least one edge.

The “cut-set” of the cut is the set of edges whose end points are in different subsets of the partition. Edges are said to be “crossing” the cut if they are in its cut-set.

In a flow network, an “s-t cut” is a cut that requires the source and the sink to be in different subsets, and its cut-set only consists of edges going from the source’s side to the sink’s side.

In a weighted graph, the “size” of a cut is the sum of the weights of the edges crossing the cut.

Proposition 2. Consider the auxiliary graph \hat{G} . For every subset $\hat{\mathbb{I}}_R \subset \mathbb{I}_R$, let $\hat{\mathbb{I}}_R$ be the source(s) and let different subsets of leaf bank(s) be the sink(s). Consider each s-t cut $C(\hat{\mathbb{I}}_R)$ with the following property: if b is on the source side of the cut, all parents of b are also on the source side. Let $C_o(\hat{\mathbb{I}}_R) \in C(\hat{\mathbb{I}}_R)$ be the one that only has $\hat{\mathbb{I}}_R$ on the source side of the cut. Let $\text{Size}(C)$ denote the size of the cut, $\text{Size}(C) = \sum_{e_{ij} \in C} m_{ij}$. Moreover, let $X_S(C)$ denote the number of banks on the sink side of the cut, and $\text{Count}(C)$ the number of edges in the cut set, $\text{Count}(C) = \sum_{e_{ij} \in C} 1$. Then

$$\begin{cases} \text{Size}(C) \leq X_S(C) & \forall \hat{\mathbb{I}}_R \forall C(\hat{\mathbb{I}}_R) \\ \text{Count}(C_o) \leq \text{Size}(C_o) & \forall \hat{\mathbb{I}}_R \end{cases}$$

where the first inequality hold with equality when C is such that only leaf nodes are on the sink side.

The main intuition is that each bank in \mathbb{I}_R is entitled to at least one unit from each of its lenders, which gives the lower bound. These lenders then draw their credit lines from their own lenders, and so on. As a result, the amount of money that flows into each set of banks cannot be more than the amount of money that their lenders (direct and indirect)

have. Note that these bounds do not uniquely determine each m_{ij} .²⁴

The remainder of the results focus on the network formation stage. I analyze the general model under the following assumption, and later relax it in section 7.

Assumption 2. *If a bank i owes funds to multiple banks, all of its funding is randomly assigned to exactly one of them such that in expectation, each borrower receives the amount determined by \mathcal{L} . An I bank that receives an investment opportunity invests all of its funds in its own project.*

This assumption substantially simplifies the exposition. Furthermore, it allows me to analyze pure intermediation and diversification separately. Consider bank i with sufficient funding. i can establish potential relationships to different banks with potential investment opportunities, in order to channel funds to different points of the financial system where investment opportunities arise, increase the scale of investment, and capture profits. In other words, i has an incentive to *intermediate* as often as possible through having many potential relationship, and this incentive is not affected by the degree of correlation across success of different investment opportunities.

Alternatively, the same set of relationships allows i to *diversify* his portfolio by being a lender to multiple projects. As such, depending on the parameters, even if only some of the projects are successful, i could be able to service all of his liabilities and avoid default. In other words, i has incentives to establish potential lending relationships to multiple banks in order to benefit from diversification when many of them have (direct or indirect) access to different realized investment opportunities simultaneously. The more correlated the projects' success across different banks are, diversification incentives are weaker as the gains to diversification deteriorates.

Assumption 2 disables diversification, as defined above, and allows me to focus on intermediation. Since diversification is relatively well-studied in a number of different contexts, I choose to abstract away from it in order to focus on the novel insight of the model. I will re-introduce diversification in section 7 and show how it interacts with the main intermediation mechanism introduced in this paper.²⁵

The first lemma addresses the length of intermediation chains, and characterizes an endogenous maximum length for any intermediation chain.

²⁴The extra funds at each lender are randomly allocated over corresponding shortest paths precisely in order to resolve the above indeterminacy in a way that is not consequential to results.

²⁵Random allocation of all funds is not sufficient to switch off diversification at an I bank because I banks can have cross-lending contracts. I need to make an assumption that implies each I bank is involved in at most one project, which is achieved through the second clause of the above assumption.

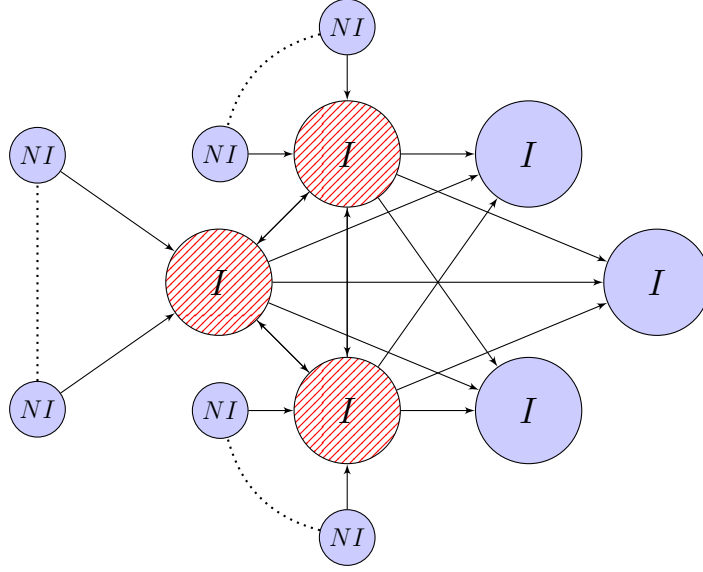


Figure 6: Core-periphery equilibrium interbank lending structure with sufficiently many NI banks. The hatched red banks are banks $I \in \mathbb{C}_G$ in definition 7.

Lemma 1. *There is no intermediation chain of length more than l_{max} , such that*

$$\alpha^{l_{max}} X \geq (1 - p)V_{NI} \text{ and } \alpha^{l_{max}+1} X < (1 - p)V_{NI}.$$

The surplus share of each bank along the chain falls as the length of the chain grows, whereas the expected cost of default is constant. Under assumption 2, each bank j fails if the project at the single I bank to which j lends (directly or indirectly) fails, so the expected cost of default is $(1 - p)V_{NI}$. The trade-off between a benefit that geometrically decreases in distance, and a constant cost, determines the endogenous maximum length of the intermediation chain.

Before proceeding to equilibrium characterization, it is useful to introduce a class of core-periphery networks.

Definition 7. [Core-Periphery Network] *A financial network G is “core-periphery” if it has the following structure: choose a subset $\mathbb{C}_G \in \mathbb{I}$, referred to as “core”. Banks $I \in \mathbb{C}_G$ form a complete digraph. Each NI bank lends to exactly one $I \in \mathbb{C}_G$, such that at least k_I NI banks lend to each $I \in \mathbb{C}_G$. Every $I \in \mathbb{C}_G$ lends to every other I bank, and every $I \notin \mathbb{C}_G$ does not lend to any bank.*

\mathbb{G}_{cp} is the set of all such core-periphery networks, and \mathbb{G}_{cp}^s is the set of core-periphery networks with core size s ; $\forall G \in \mathbb{G}_{cp}^s, |\mathbb{C}_G| = s$. $\mathbb{G}_{cp} = \bigcup_{s=1}^{k_I} \mathbb{G}_{cp}^s$.

A representative core-periphery network of the above family is depicted in figure 6.²⁶
The next theorem states the first main result of the paper.

Theorem 1. *Assume $k_{NI} > k_I^2$, and surplus is divided via α -rule.*

1. *There exists a constant M such that for $\kappa > M$, every $G \in \mathbb{G}_{cp}$ is an equilibrium.*
2. *There exist a sequence of strictly increasing constants $\{M_s\}_{s=1, \dots, k_I}$, $M = M_{k_I}$, such that all financial networks $G \in \mathbb{G}_{cp}^s$ are equilibria if and only if $\kappa > M_s$.*

Core-periphery equilibria emerge when highly profitable projects trigger rent-seeking behavior by I banks and induce voluntary exposure to counterparty risk in order to capture intermediation spreads. Moreover, each core I bank is able to channel funding to every other I bank if it has enough peripheral lenders. On the other hand, in this equilibrium an NI banks enhances its return by first circumventing high spreads as often as possible by lending directly to an I bank, and second receiving more frequent positive returns by lending directly or indirectly to as many I banks as possible. With sufficiently many NI banks, there are configurations in which each I bank is able to be connected to every other I bank (a *well-connected* I bank). As such, any subset of well-connected I banks can act as intermediators in a stable structure.

The second part of the theorem argues that core-periphery equilibria with smaller core sizes exist for a wider range of parameters. When the core is smaller, each I bank in the core absorbs more intermediation spreads, which in turn covers a higher expected cost of default.

Recall that the rule for division of surplus is exogenous conditional on the endogenous network. Nevertheless, the pricing implications of the model is reassuringly consistent with recent empirical evidence. Di Maggio et al. [2015] empirically investigates the inter-dealer market for corporate bonds and documents that it exhibits a clear core-periphery structure. Moreover, consistent with the current model, they show that core dealers on average charge higher prices to the peripherals than to other core dealers.

The next proposition provides an existence result.

Proposition 3. *An equilibrium exists.*

The proof is in two steps. First I show that taking network structure as given, for any resolution of uncertainty, the system of equations for interbank repayments has a unique

²⁶Note that not all the possible core-periphery networks have the exact above structure, for instance those with NI banks in the core. As such, definition 7 is not an if and only if statement.

solution (a la Acemoglu et al. [2015]). Then I provide a constructive proof of the equilibrium structure in the network formation stage.

In order to gain a better understanding of the core-periphery equilibrium family, it is useful to examine how the equilibria change as a function of parameters.

Lemma 2. *For a set of parameters $\Theta = (p, R, \alpha, V_I, V_{NI}, k_I, k_{NI}, q)$, $k_{NI} > k_I^2$, let $\bar{s} = \max_{G \in \mathbb{G}_{cp}} |\mathbb{C}_G|$ denote the maximum sustainable core size among all core-periphery equilibria, given Θ . Then*

$$(4) \quad \bar{s} = \max \left\{ \left\lfloor \frac{k_{NI}}{\max \left\{ \left\lceil \frac{(1-p)V_I}{\alpha(1-\alpha)(pR-1)} \right\rceil, k_I \right\}} \right\rfloor, k_I \right\}.$$

\bar{s} is the maximum number of I banks who can be in the core without violating feasibility and/or individual rationality. In a core-periphery network, each I bank in the core needs at least k_I NI lenders to sustain links to every other I bank, in and outside the core. It also needs at least $\frac{(1-p)V_I}{\alpha(1-\alpha)X}$ lenders to cover his expected cost of default due to contagion.

Proposition 4. *For a set of parameters Θ and the corresponding core-periphery equilibrium family \mathbb{G}_{cp} , the following statements hold:*

- $\frac{d\bar{s}}{dp} \geq 0$, $\frac{d\bar{s}}{dR} \geq 0$, $\frac{d\bar{s}}{dk_{NI}} \geq 0$.
- $\frac{d\bar{s}}{dV_I} \leq 0$
- $\frac{d\bar{s}}{d\alpha} = \begin{cases} \geq 0 & \text{if } 0 < \alpha < \frac{1}{2} \\ < 0 & \text{if } \frac{1}{2} \leq \alpha < 1 \end{cases}$

The above proposition describes how the largest sustainable core-size within equilibrium family \mathbb{G}_{cp} changes with the parameters. Intuitively, any force that leads to higher intermediation spreads, lower cost of default, or more spreads, incentivizes more I banks to expose themselves to counterparty risk, and allows for a larger core-size to be sustainable in equilibrium. Higher probability of successful project outcome, p , both increases the expected intermediation spread and decreases expected cost of default. Higher per-unit return, R , increases the expected spread. An increase in the number of NI banks allows each I bank in the core to earn weakly more intermediation spreads. A decline in charter value, V_I decreases expected cost of default. Lastly, given the rule for division of surplus, intermediation spreads are maximized at $\alpha = \frac{1}{2}$ and are strictly convex in interval $0 < \alpha < 1$.

The next theorem provides the constraint efficient benchmark, which maximizes total net surplus subject to feasibility and individual rationality.

Theorem 2. Assume $k_{NI} > k_I$.

1. Every constraint efficient network is a tiered network with the following structure: there is one NI bank, NI_c , who borrows from every other NI bank, directly or indirectly, and lends to every I bank, directly. All the paths from each bank NI_j to NI_c have the same length, $dist(NI_j, NI_c)$.
2. $\forall G \in \mathbb{G}_{cp}$ is inefficient. Moreover, there are constant \bar{M} and \bar{K} such that for $\kappa > \bar{M}$ and $\frac{k_{NI}}{k_I} > \bar{K}$, no constraint efficient equilibrium exists.

A constraint efficient financial network is depicted in figure 1b. The defining feature of this structure is that no I bank intermediates, so the unnecessary defaults of theorem 1 are avoided. Moreover, all the funding is allocated via the same NI so that maximum concentration is achieved. NI_c , the red NI , acts like a central clearing house as all of the lending goes through this particular bank. Since diversification is assumed away in this section, what makes the existence of the central clearing party (CCP) optimal is not the gains to diversification. Instead, the CCP is an entity that channels all the available resources to all the investment opportunities optimally without being exposed to *excessive* counterparty risk.

Comparing the constraint efficient and core-periphery financial network structures reveals an important insight about the nature of the inefficiency. Note that a star networks is a special case of a core-periphery network with a single core. The constraint efficient structure, as well as the single-core member of the inefficient equilibrium family are both star networks. As such, they do not differ in their network structure, but they differ in the identity of the the core bank.

The exact same force that pins down the structure of equilibria in theorem 1 is the key source of inefficiency in the model. Consider $G \in \mathbb{G}_{cp}$ with a single I bank at the core, and the constraint efficient network. In both network structures the core bank is exposed to counterparty risk of I banks he intermediates to. The difference is that when an NI bank is at the core, this exposure to counterparty risk is efficient as the NI bank is also channeling outside funding into the system, and increasing the scale of investment which is only possible through taking some risk. On the contrary, when the I bank is at the core the exposure to counterparty risk is excessive, and driven by pure rent seeking motives.

Corollary 1. Fix parameters $\hat{\Theta} = (R, \alpha, k_I, k_{NI}, q)$, $k_{NI} > k_I^2$, and let $V = \min\{V_I, V_{NI}\}$. Then $\forall V, \exists \bar{p}$ such that for $p > \bar{p}$, ex-post cost of failure strictly exceeds $(k_I + k_{NI})V$. Moreover, $\frac{d\bar{p}}{dV_I} \geq 0$, $\frac{d\bar{p}}{dR} \leq 0$, and $\frac{d\bar{p}}{d\alpha} \leq 0$ iff $\alpha < \frac{1}{2}$.

This result follows directly from lemma 2 and proposition 4, using $k_{NI} \geq k_I^2$ and $\bar{p} = \frac{V_I + \alpha(1-\alpha)(k_I-1)}{V_I + R\alpha(1-\alpha)(k_I-1)}$, so the core-periphery equilibrium with the largest possible core size, k_I , is sustainable. This is a striking result as it implies that fully rational agents form a financial network in which the ex-post realized losses are arbitrarily large. Moreover, if V_I is considerably larger than V_{NI} , a considerable portion of the ex-post losses could be avoided if the equilibrium network was constraint efficient. This interpretation rationalizes the high degree of interconnectedness among large financial institutions during the run-up to the financial crisis of 2008, as well as the enormous losses once the financial sector collapsed.

The next two propositions explain how the parameters of the model affect the size of inefficiency through the endogeneity of the network structure.

Proposition 5. *For a set of parameters Θ , let $T\bar{N}S_G$, \bar{n}_G and s_G denote the total expected net surplus, the expected number of defaults, and the size of the core respectively, for $G \in \mathbb{G}_{cp}$. Then, $\frac{dT\bar{N}S_G}{ds_G} < 0$ and $\frac{d\bar{n}_G}{ds_G} > 0$.*

In every core-periphery equilibrium $G \in \mathbb{G}_{cp}$, there is a path from every NI bank to every I bank, so the maximum scale of investment is always achieved regardless of the realization of investment opportunities. As a result, the size of inefficiency is purely determined by the expected number of defaults, which increases with the size of the core. As such, the proposition argues that the degree of inefficiency varies among the equilibria. Equilibrium networks G with smaller s_G (i.e., fewer I banks as intermediators) have in expectation fewer number of banks who default, and thus are less inefficient. This proposition also implies that all networks $G \in \mathbb{G}_{cp}^s$ have the same degree of inefficiency and same expected number of defaults.

Proposition 5 explains how expected number of defaults and degree of equilibrium inefficiency changes as the core size changes, keeping all the parameters constant. However, the size of the core itself is an endogenous object, and changes when parameters change. So it is important to trace back the source of inefficiency to changes in the deep parameters of the model. Note that for any set of parameters, if a core-periphery equilibrium with size s exists, core-periphery equilibria of all smaller core sizes, $\hat{s} < s$, also exist. As a result, in order to understand how changes in the parameters of the model influence equilibrium efficiency, it is most useful to focus on the equilibria with largest core-size, $\mathbb{G}_{cp}^{\bar{s}}$.

Proposition 6. *An increase in the probability of project success, p , as well as per-unit project return, R , weakly increases the expected number of defaults in $\mathbb{G}_{cp}^{\bar{s}}$. Moreover, it can also increase the degree of inefficiency. On the contrary, an increase in the level of loss in*

default, V_I , weakly decreases the expected number of defaults, and can make the equilibrium more efficient.

This result is rather counter-intuitive: an increase in project success probability, and per-unit project return, increases the net surplus of each unit of investment, which improves efficiency. However, it makes the equilibrium financial network less efficient if the maximum sustainable core size increases. On the other hand, although an increase in the cost of default makes each unit of lending less efficient, it can increase the overall efficiency of the financial network. The key to this result is that these changes in parameters influence the incentives of the banks to form their interbank lending relationships. More profitable intermediation, in the form of higher spreads, allows more I banks to take on counterparty risk to capture the spreads, which increases the maximum sustainable size of the core and can lead to more inefficient equilibria. Which force dominates depends on the magnitude of parameters, and the proof characterizes the region where the second, network formation force, dominates and the equilibrium is less efficient over all. Note that the exact same argument holds if we assume a uniform distribution over all $G \in \mathbb{G}_{cp}$ instead of focusing on $G \in \mathbb{G}_{cp}^s$.

The last result shows that theorems 1 and 2 generalize to any rule for division of surplus, $\mathcal{L}(\cdot)$, that satisfies the properties of section 4.1. Let $\bar{\mathcal{L}}(k, K, X)$ denote the share of expected net surplus, implied by rule $\mathcal{L}(\cdot)$, from a unit of investment intermediated through a chain of length K , accrued to the bank in position k . Also, let $k = 1$ denote the position of the I bank who has made the investment.

Theorem 3. *Theorem 1 and 2 hold for any $\mathcal{L}(\cdot)$ satisfying conditions of section 4.1, with $\kappa = \frac{\bar{\mathcal{L}}(2;3,X)}{(1-p)V_I}$.*

This generalization is intuitive. α -rule is a special case of $\mathcal{L}(\cdot)$ along two dimensions: the shares are decreasing along an intermediation chain, and final borrower share is invariant to the length of the chain. None of the two are crucial for theorems 1 and 2.

6 Discussion and Policy Implications

This paper highlights the role of intermediation in interbank networks. Intermediation enhances welfare by allocating funds from parts of financial sector with excess liquidity to parts with profitable investment opportunities, but it can also diminish welfare by triggering excessive voluntary exposure to counterparty risk in order to capture intermediation spreads. The model demonstrates that each bank's motive to absorb intermediation spreads, along with differential abilities of different banks in offering rates of return on their borrowing,

leads to a specific equilibrium structure, a core-periphery network. As such, the model not only delivers the first micro-foundation for core-periphery financial networks, but also provides a novel explanation of why banks choose to expose themselves to counterparty risk, which differs from the existing explanations such as bailouts and ignoring tail risk. This alternative explanations amplify banks' incentives to take risk, but I argue that neither is necessary to explain excessive exposure to counterparty risk.

The key to understanding the equilibrium outcome is to recognize the dual role of spreads. On one hand, spreads make intermediation profitable and provide a motive for intermediation. On the other hand, spreads are a cost born by lender banks, so paying spreads less frequently increases lender profits. As a result, banks who emerge as the core of the financial network and act as intermediators are those who attract lender banks by letting them paying spreads less often.

It is important to realize that welfare is not invariant to financial network structure. For a given aggregate scale of investment, total expected net surplus is independent of the distribution of investment among banks. As a result, as long as the identity of intermediators does not change the scale, the rent-seeking activity translates into a change in the division of surplus in favor of intermediators, without any implications for gross surplus. However, this surplus redistribution is not the only effect of a change in the identity of the intermediary. Intuitively, all banks along the path of intermediation are exposed to the risk of failure if the investment fails, so a change in the set of banks that do the intermediation also changes the cost of default. That is, the identity and characteristics of the intermediaries does not merely have a redistribution effect.

As such, the same rent-seeking behavior which underlies the equilibrium structure, misaligns private and social incentives. The main source of inefficiency is that the gains from intermediation are purely redistributive, whereas the loss is incremental. This is an important insight from the model, missing from the existing literature on interbank interconnectedness, which emphasizes a *downside externality*, contagion, as the main externality in the interbank network: a bank does not take into account that his creditors fail if he fails and does not pay them back. Here, although contagion happens ex-post, it is due to an *upside externality*, i.e. rent-seeking: banks choose to expose *themselves* to an excessive probability of failure in order to absorb intermediation rent. So the direct loss borne by the system is failure of this particular bank, above and beyond any contagion from this bank to other banks. In the next section I show that the two externalities can coexist.

Multiple policies targeting the structure of financial networks can be studied in the context of the model. First, the model provides a new rationale for introduction of a Central

Clearing Party (CCP). Designating a non-investing bank as the CCP and enforcing all the lendings to go through the CCP prevents excessive bilateral exposure among banks with investment opportunities, and enhances welfare particularly as investment returns become more correlated. This effect is different from the channels identified by Duffie and Zhu [2011] and Bond [2004]. Moreover, the model predicts that this network structure is not an equilibrium when intermediation rents are sufficiently high, so intervention is necessary to implement it.

Moreover, the model can be used to study bailouts and how they affect the equilibrium structure. A natural way to incorporate bailouts in the model is as a wedge between true V_i and the loss to the bank if it is in default. Assessing the ex-post cost of bailout at the existing network structure misses the critical point of this paper. Implementing a bailout policy not only has an ex-post cost evaluated at the current financial architecture, but also feeds back into bank decisions and affect the equilibrium interbank network. As the bailout decreases the cost of default borne by banks, it expands the core in two ways, and increases the ex-post cost of failure. First, at the same level of spreads, more banks with default cost at prior-to-bailout level are willing to intermediate, and a larger core is sustainable. Second, banks with larger default costs will also be willing to expose themselves to excessive counterparty risk to capture intermediation spreads. The crucial observation is that the latter group of banks, absent bailout, would optimally choose not take the risk because their opportunity cost was prohibitive. A bailout decreases this individual opportunity cost by shifting it toward the government, and can lead to lower welfare. The next corollary directly follows from proposition (5) and (6) and formalizes the above intuition.

Corollary 2. *Consider the following bailout policy. For any bank i with cost of default V_i who defaults, i bears $\hat{V}_i = \beta V_i$, for some $\beta < 1$, and the difference $(1 - \beta)V_i$ is borne by the government. Implementing such bailout policy weakly increases the maximum size of the core and weakly decreases total welfare.*

Therefore, incorporating the effect of network formation into evaluation of bailout policies uncovers an important amplification effect. Expectation of a bailout not only makes the highly interconnected core of the financial sector larger, but also banks with larger default consequence join the core. More large financial institutions default due to exposure to counterparty risk, and need to be saved, which deepens the financial crisis. Of course the ex-post bailout cost needs to be probability weighted, but correctly estimating the ex-post cost is essential for policy evaluation. To the extent that projects are relatively correlated, and most of the interbank exposure is due to incentives to capture spreads, a larger core

do not substantially decrease the contagion probability, and bailouts increase the expected cost of systemic failure.

Finally, part of Title VII of the Dodd-Frank Wall Street Reform and Consumer Protection Act was a proposed cap on the number of counterparties and swaps, which was later eliminated from the finalized rules.²⁷ The current paper provides sharp theoretical predictions about such a policy: it either leads to under-investment, or more inefficient equilibria with larger cores. First, the constraint efficient financial structures in the model require intermediaries with many connections, which is prohibited under this proposal. Moreover, the proposal has an adverse effect on the core-periphery equilibrium family. With a cap \bar{Z} on number of bank connections ($\bar{Z} < k_I$), no bank would be able to lend to every I bank. When multiple core-periphery equilibria are possible, this policy shifts the family of equilibria toward those with larger cores, so that every core bank is directly or indirectly connected to as many I banks as possible, and in expectation all core banks offer the same rate of return, as high as possible. A larger core increases the cost in the event of failure, and is particularly costly when investment outcomes are highly correlated. This is especially relevant for crisis of 2007-08.

7 Diversification

In this section I study the equilibrium network structure when banks are allowed to hold diversified portfolios, by relaxing assumption 2. The same structure of equilibria emerges, albeit with a twist. I focus on an economy with two I banks, $k_I = 2$, and k_{NI} NI banks, $k_{NI} > 4$. Restricting the number of I banks keeps the problem tractable while incorporating the main intuition associated with diversification.

Assumption 3. *Consider a realization of \mathbb{I}_R . If bank b has access to multiple $I \in \mathbb{I}_R$ through intermediation chains of different lengths, it can use the shortest chain to bargain its share in other chains up to what he gets in the shortest path. b 's (direct and indirect) borrowers in each longer chain divide the remaining share pro-rata.*

Consider the following simple structure. $NI_0 \rightarrow NI_1 \rightarrow I_1$, and $NI_1 \rightarrow NI_2 \rightarrow I_2$. When both I banks have investment opportunities, NI_1 has direct access to one and indirect access to the other. The above assumption implies NI_1 bargains up his share in the chain

²⁷See CFTC/SEC [2012] and Stroock Special Bulletin (<http://www.stroock.com/SiteFiles/Pub1201.pdf>) for more detail.

$NI_1 \rightarrow NI_2 \rightarrow I_2$ to α . I_2 and NI_2 divide the remaining $(1 - \alpha)$ share with proportions $\frac{1}{1+\alpha}$ and $\frac{\alpha}{1+\alpha}$, respectively.²⁸

This assumption is important to keep the incentives of banks to provide funding monotonic. The following lemma formalizes the idea.

Lemma 3. [Dominance] *Consider two banks j_1 and j_2 . Let $SPL_i = \{l_1^i, l_2^i, \dots, l_{z_i}^i\}$ be the set whose elements are lengths of paths in $SP(j_i, \mathbb{I})$, $i = 1, 2$. Assume elements of each set are sorted in increasing order. Also, without loss of generality, assume j_1 has more shortest paths to \mathbb{I} , $z_1 > z_2$. A leaf bank i prefers to lend to j_1 if*

$$\forall k \leq z_2 : l_k^1 \leq l_k^2$$

independent of l_k^1 for $k > z_2$.

Assume parameters are such that, absent diversification, an I bank chooses to intermediate (even) with a single peripheral lender. Moreover, assume banks net out their payments at date $t = 2$.

Consider the 2- I core-periphery structure that is an equilibrium without diversification. When both I_1 and I_2 have realized investment opportunities, probability q^2 , diversification becomes relevant. Assume each I_i has credit lines from Y_i of NI banks, where $Y_1 + Y_2 = k_{NI}$. As described in section 4, I_i lends $\frac{Y_i}{2}$ to I_j . Let D_{ii} denote the face value of debt promised by I_i to each of its NI lenders. Moreover, let D_{ij} denote the face value of the debt payable to I_j by I_i . Due to netting, when $\frac{Y_i}{2}D_{ji} > \frac{Y_j}{2}D_{ij}$, j owes i the difference, namely, $\frac{Y_i}{2}D_{ji} - \frac{Y_j}{2}D_{ij}$.²⁹ So I_j is the *net borrower* and I_i is the *net lender*.

Without loss of generality, let $i = 1$ and $j = 2$ in the above discussion, so that I_1 is the net lender. Assumption 3 is extremely useful in determining D_{12} and D_{21} . Each I_i has access to two investment opportunities: its own investment, which provides it with all the return (out of which he has to pay his lenders); as well as I_j 's investment opportunity. By assumption 3, each I_i receives all the return from investment for each unit it lends to I_j .³⁰ This argument pins down both inter- I face values to be exactly R , $D_{12} = D_{21} = R$. Thus at $t = 2$, bank I_2 owes I_1 a net payment of $\frac{Y_1 - Y_2}{2}R$.

The balance sheets of I_1 and I_2 are depicted in figure 7. The critical observation is that survival of the net borrower solely depends on its own investment, while for the net

²⁸I restrict the analysis to parameters where individual rationality is maintained.

²⁹Both banks lend to each other, and face values of debt are determined in equilibrium.

³⁰This contract is individually rational for I_j . I_j accepts as long as it has funding pledged to it directly by NI banks and the share of that investment covers its expected cost of default.

<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="border-bottom: 1px solid black; padding: 5px;">Assets</th> <th style="border-bottom: 1px solid black; padding: 5px;">Liabilities</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">$\frac{Y_1+Y_2}{2} \tilde{R}$</td> <td style="padding: 5px;">$Y_1 D_{11}$</td> </tr> <tr> <td style="padding: 5px;">$\frac{Y_1-Y_2}{2} D_{21}$</td> <td style="padding: 5px;"></td> </tr> </tbody> </table>	Assets	Liabilities	$\frac{Y_1+Y_2}{2} \tilde{R}$	$Y_1 D_{11}$	$\frac{Y_1-Y_2}{2} D_{21}$		<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="border-bottom: 1px solid black; padding: 5px;">Assets</th> <th style="border-bottom: 1px solid black; padding: 5px;">Liabilities</th> </tr> </thead> <tbody> <tr> <td style="padding: 5px;">$\frac{Y_1+Y_2}{2} \tilde{R}$</td> <td style="padding: 5px;">$Y_2 D_{22}$</td> </tr> <tr> <td style="padding: 5px;"></td> <td style="padding: 5px;">$\frac{Y_1-Y_2}{2} D_{21}$</td> </tr> </tbody> </table>	Assets	Liabilities	$\frac{Y_1+Y_2}{2} \tilde{R}$	$Y_2 D_{22}$		$\frac{Y_1-Y_2}{2} D_{21}$
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	$\frac{Y_1-Y_2}{2} D_{21}$												
(a) Net Lender (I_1)	(b) Net Borrower (I_2)												

Figure 7: Balance sheet of banks I_1 and I_2 when banks net out their payments. There are two I banks and k_{NI} NI banks. Y_i NI banks lend to I_i such that $Y_1 > Y_2$, so I_1 is the net lender and I_2 is the net borrower.

lender, it also depends on whether the net borrower pays back. As a result, when both I banks invest, the net borrower survives exactly with probability p , whereas net lender's survival probability depends on other parameters of the model as well as the structure of the network, and is determined in equilibrium. The following argument outlines how I_1 incentives depends on level of α , which governs the amount of his liabilities.

Conditional on the level of R , there can be two cases, as depicted in figure 8. Panel 8a and 8b correspond to high and low levels of return, respectively. In each plot, the horizontal axis is $0 \leq \alpha \leq 1$, lender share in a chain of length two, and the vertical axis is $0 \leq \frac{Y_2}{Y_1} \leq 1$, the ratio of the number of peripheries of the net borrower to the net lender. As such, the unit square in the first quadrant is the relevant region.

Below the solid red line, liabilities of I_1 are low, so having more peripheries increases the gain to diversification, and I_1 survives with probability $1 - (1 - p)^2$ ($\alpha < \bar{\alpha}$, yellow region in panel 8a). The reverse situation happens below the dashed blue line (green region in both panels). Here the liabilities are so high that I_1 fails unless all of his assets pay, so having many direct lenders increases his liabilities and widens the range with low survival probability p^2 . In the intermediate region, above both lines, I_1 survives if and only if its own investment survives; i.e. with probability p . Finally, on the horizontal axis $y = 0$, I_1 is the only core bank and fails with probability p as well.

Incentives of NI banks are more complicated. First note that they are purely driven by minimizing the probability of default, and default probability of NI banks who are peripheral to the net borrower I_2 is p . The complexity stems from the fact that NI bank's liabilities are independent of α , and consequently its default probability is determined at $\alpha = 0$. Here is the relevant intuition for 8a: the reason I_1 fails more often in certain regions compared to others, with the same successful assets, is that its liabilities are higher, i.e. α is high. However, an NI bank pays the households only one unit in expectation, independent

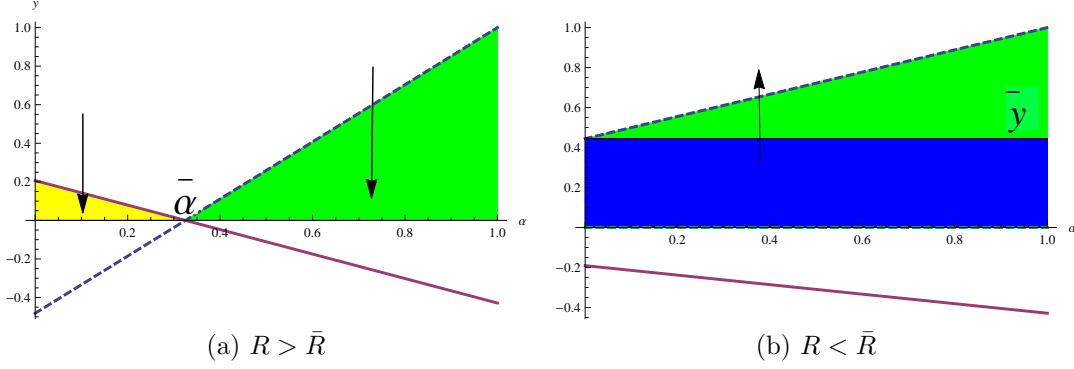


Figure 8: Possible Equilibria with two I banks and k_{NI} NI banks and diversification. The x -axis is α , lender share of expected net surplus in a direct lending, and the y -axis is the ratio of the number of NI peripheries of I_2 to I_1 , y . The arrows show the direction of the deviation for the NI banks.

of α , which in turn α is not relevant in determining failure probability of the NI banks. As a result in 8a all NI banks *migrate* and lend to I_1 , even at $\alpha > \bar{\alpha}$, which increases I_1 's probability of default.

Given the above discussion, the next proposition characterize the equilibrium.

Proposition 7. *Let y denote the ratio of the number of NI peripheries of net borrower to net lender I bank. There is a constant \bar{R} such that*

- *When $R > \bar{R}$, there are two core-periphery equilibria with I banks at the core: $y = 0$ with only I_1 at the core, and $y = \frac{1}{k_{NI}-1}$ with both I_1 and I_2 at the core.*
- *When $R < \bar{R}$, the single-core equilibrium is still an equilibrium. There are multiple two-core equilibria, one for each $y > \bar{y}$, where $\bar{y} = \frac{2}{p^2 R} - \frac{2-p}{p}$.*

Moreover, there are constants $\bar{\alpha}_l, \hat{\alpha}_l < \hat{\alpha}_h$ and $\hat{q} < \bar{q}$, all in $(0, 1)$, such that

- *$R > \bar{R}$ and $\hat{\alpha}_l < \alpha < \hat{\alpha}_h$: 2- I core-periphery equilibrium is inefficient if either $\alpha > \bar{\alpha}_l$, or $\alpha < \bar{\alpha}_l$ and $q < \bar{q}$.*
- *$R < \bar{R}$: 2- I core-periphery equilibrium is inefficient if $q < \hat{q}$.*

A detailed argument is provided in the appendix. A few final points are worth mentioning. First, diversification creates a coordination problem between lenders and borrowers, which can in turn lead to inefficiencies in the financial network. In 8b, for equilibria with y between the $y = \bar{y}$ and the dashed blue line, there are two sources of inefficiency: first, I_1 is

exposed to the risk of default of I_2 when he only intermediates. Second, I_1 is not diversified in the best possible way when he invests as well.

Second, this proposition shows that adding diversification does not alter the incentives to intermediates. Even when the gains from diversification are larger in the 2- I core-periphery network compared to the NI -star network, they can be dwarfed by the extra cost of I banks' failure due to excessive exposure to counterparty risk, and the core-periphery structure remains inefficient. Since the NI -star network is used to find sufficient conditions under which the 2- I core-periphery structure is not efficient, these conditions are not necessary.

Finally, adding diversification enables me to study the interesting question of under-insurance in the context of the model. Consider the $y = 0$ equilibrium, and assume $R > \bar{R}$ and $\alpha < \bar{\alpha}$. Imagine I_1 was able to offer the following deal to I_2 when both have investment opportunities: I_1 lends half of its funds to I_2 in order to fully diversify, and it pays I_2 exactly enough to cover I_2 's expected cost of default, $(1 - p)V_I$. Such an offer increases I_1 and all of NI 's probability of survival from p to $1 - (1 - p)^2$, whereas it imposes some extra cost of default (that of I_2) on the economy. One can show that if $k_{NI} > \frac{V_I}{V_{NI}} \frac{(1-p)}{p}$, the above strategy improves welfare. However, I_1 would not make such an offer even if it could, because its individual gain to diversification, $p(1 - p)V_I$, is lower than the price that it has to pay, $(1 - p)V_I$. This means that I_1 does not internalize the positive externality of it buying insurance on its lenders. In other words, the price of insurance is too high for I_1 , which leads to voluntary under-insurance and contagion.

8 Conclusion

I develop a model of the financial sector in which endogenous intermediation among debt financed banks generates excessive systemic risk. The central feature of the model is that financial institutions have incentives to capture intermediation spreads through strategic borrowing and lending decisions. By doing so, they tilt the division of surplus along an intermediation chain in their favor, while at the same time reducing aggregate surplus. I show that a core-periphery network – few highly interconnected and many sparsely connected banks – endogenously emerges in my model. The network is inefficient relative to a constrained efficient benchmark since banks who make risky investments “overconnect”, exposing themselves to excessive counterparty risk, while banks who mainly provide funding end up with too few connections.

This paper suggests that explicitly modeling the interaction between banks' incentives to capture higher returns, with intermediation, a necessary mechanism to allocate liquidity

within the financial system, jointly explains the stylized facts about global structure of interbank networks, interbank interconnectedness, and gross and net exposures among financial institutions. Moreover, by providing sharp predictions about sources of inefficiency in interbank relationships, the model contributes to the heated policy debate on how to regulate the financial market.

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9 Appendix

9.1 Proofs

I will present all the proofs using α -rule as it greatly simplifies the exposition of the proof. It is easy to verify that all the proofs go through with any rule for surplus division which satisfies the properties of section 4.1. The general proofs are available upon request.

The first lemma shows that the structures in 9 are the only possible equilibria of the economy with four banks.

Lemma 4. *Network structures depicted in Figure 3 are the only possible equilibria with four banks.*

Proof. Any structure in which an NI does not lend to any other bank is trivially not an equilibrium. Aside from those, all the feasible structures with four banks are depicted in 9. Each structure consists of the four banks and credit lines among them depicted in black.

Finally, the deviations which rule out the other structures (9d, 9g and 9h) are depicted as red or crossed out edges. For instance in 9h, NI_1 has two units pledged to him but is only lending to a single I bank. NI_1 and I_2 strictly prefer to jointly deviate together. NI_1 saves on the intermediation rent payed to I_1 when only I_2 has an investment opportunity, while post deviation I_2 gets to invest 50% of time when both I_1 and I_2 get the investment opportunity and prior to deviation I_2 would not invest.³¹ $e_{I_1I_2}$ is removed since nothing is ever lent over that credit line and we move from 9h to 9a.

In 9c, adding the $e_{I_1I_2}$ and $e_{I_2I_1}$ is not always a viable deviation because if $\alpha(1 - \alpha)X < (1 - p)V_I$, in the resulting network, lending over $e_{I_iI_j}$ always violates the participation constraint of I_i , so it would happen with probability zero. So this is not a valid coalitional deviation and 9c is a possible equilibrium. \square

Proof of Proposition 1.

³¹If the bargaining rule is such that both final lender and initial borrower save on intermediation rents when an intermediary is removed the second part of argument is redundant as I_2 also saves on intermediation rents when only he gets the investment opportunity and lending goes through I_1 . However, in α -rule borrower does not care for the source of funds so the second part of argument is necessary.

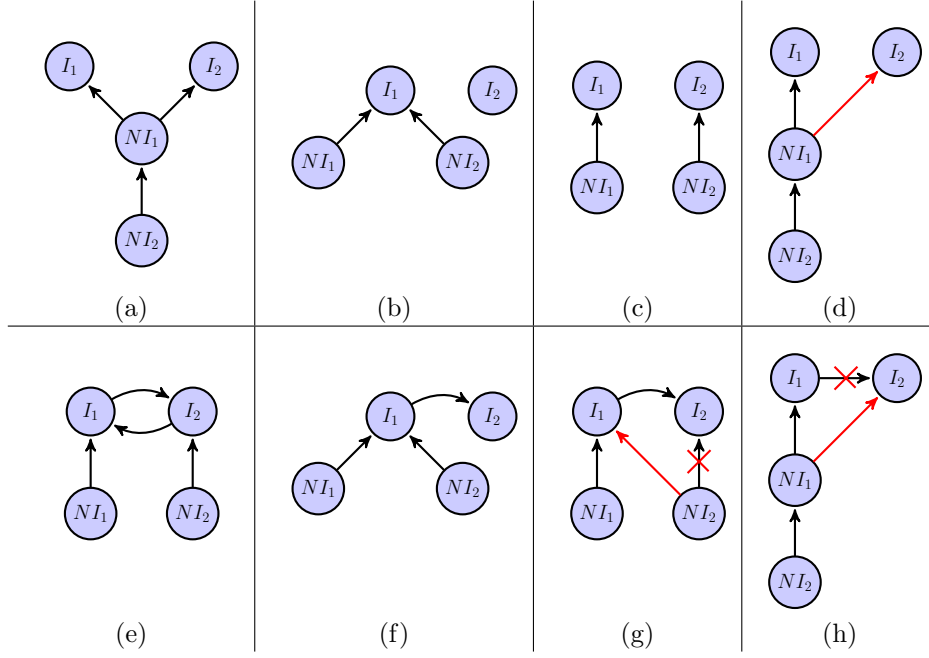


Figure 9: Feasible lending structures for an economy with two I and two NI banks. The black edges are in the feasible structure. The red and crossed-out edges are the deviations which rule out each particular structure as an equilibrium.

Following the notation in the text, let $\kappa = \frac{\alpha(1-\alpha)}{(1-p)V_I}$ and $\tilde{\kappa} = \max\{\alpha, (1-\alpha)\frac{V_{NI}}{V_I}\}$. Individual rationality constraints imply $\kappa \geq \tilde{\kappa}$ is the relevant range of parameters.

Panels 9d and 9g cannot be equilibria because an (NI, I) pair of banks have a bilateral deviation which keeps feasibility, weakly decreases their default probability and strictly increases their expected share of surplus.

Next, in any structure in figure 3, if only one I bank receives an investment opportunity, face values of debt are given by D, D_1 and D_2 as defined in the text. The same if both I banks have investment opportunities and the network structure is 3b or 3c.

When both $I \in \mathbb{I}_R$, there is possibility of diversification, and that some contagion can be avoided. I focus on parameters for which rent seeking incentives are stronger than diversification incentives, if they work against each other.

The gains from diversification is only through lower cost of default. Incremental diversification benefit for a bank with default cost V can at most be

$$(5) \quad q^2[(1 - (1 - p)^2) - p^2]V = 2q^2p(1 - p)V$$

Since if both projects are successful, regardless of interbank network every bank survives.

The minimum foregone intermeditation spread is

$$(6) \quad q(1 - q)\alpha(1 - \alpha)X$$

Comparing equations (5) and (6),

$$q < \bar{q}_1 = \frac{1}{1 + \frac{2p(1-p)V}{\alpha(1-\alpha)X}} = \frac{\alpha(1-\alpha)X}{\alpha(1-\alpha)X + 2p(1-p)V}$$

is sufficient to ensure gains from intermediation is larger than loss due to under-diversification. This condition also ensures that network structure in figure 9h is not an equilibrium.

Next I move to network structures that can arise in equilibrium.

Network structure 3a. Focus on when both I banks invest. The case where both projects fail or succeed are trivial. Consider the case where one project fails and one succeeds. Here two separate assumptions need to be considered about debt seniority as NI_1 has both in-network and outside debt.

In-network debt senior, case of Equity. Here the portion of return which is on outsider funding acts as equity, if NI_1 does not have enough resources to pay back all his obligation. As a result, one project payoff ($D_2 = \frac{\alpha(pR-1)+1}{p}$ the only asset of NI_1) is certainly sufficient to pay NI_2 in full, so NI_2 enjoys diversification benefits and fails only if both projects fail. It also implies $D_1 = \frac{\alpha^2(pR-1)+1}{p(2-p)}$.

Whether NI_1 enjoys diversification benefits or not as well depends on the parameters. For NI_1 to survive he has to be able to pay back his outsiders when only one project pays back which implies $D_{1h}^S = \frac{1}{p(2-p)}$, and NI_1 assets must exceeds his liabilities with only one repayment from I banks $D_2 \geq D_1 + D_{1h}^S$. Whether this inequality holds or not depends on α . Solving for the inequality leads two α 's:

$$\alpha_L = \frac{(2-p) - \sqrt{(2-p)^2 - 4p/(pR-1)}}{2}$$

$$\alpha_H = \frac{(2-p) + \sqrt{(2-p)^2 - 4p/(pR-1)}}{2}.$$

If $\alpha_L < \alpha < \alpha_H$ then NI_1 enjoys diversification benefits and fails only if both projects fail. Otherwise both projects must be successful in order for NI_1 to survive. In this case, face value of outside debt is given by $D_{1h}^F = \frac{p(3-2p)+2\alpha(pR-1)(1-p)(\alpha-(2-p))}{(2-p)p^2}$, as outsiders are only partially paid when one project succeeds. Clearly at the boundaries α_L and α_H , $D_{1h}^F = D_{1h}^S$.

This result is intuitive: the intermediation spread NI_1 receives acts as a cushion when asset returns are low, so high spreads have the positive effect of preventing failure. And spreads are equal to $\alpha(1 - \alpha)$, so they are high when α takes on intermediate values.

All debt pari-pasu. Either NI_1 pays all of his creditors in full, in which case he himself also survives, or he cannot pay either in full, in which case he also fails. The former case requires the exact same condition as in outside-debt-senior case. So regardless of debt seniority, if $\alpha_L < \alpha < \alpha_H$, when a single I project is successful both NI banks enjoy diversification benefits and survive.

Outside this range, NI_1 fails if only one project is successful. However, the partial payment NI_2 receives can still be sufficient to cover his obligations to outsiders, so NI_2 might survive. We have $D_1 = \frac{(1+\alpha^2(pR-1))(2p+\alpha(pR-1)(\alpha-2(1-p)))}{p^2(2+\alpha^2(pR-1))}$ and $D_{1h} = \frac{2p+\alpha(pR-1)(\alpha-2(1-p))}{p^2(2+\alpha^2(pR-1))}$. NI_2 survives with only one successful project iff $\frac{D_1}{D_1+D_{1h}}D_2 > \frac{1}{p(2-p)}$, where the rhs is the relevant face value to outsiders if they are paid in full unless both projects fail. This is intuitive as higher α provides NI_2 with a larger share of project return and acts as a cushion.

$\frac{D_1}{D_1+D_{1h}}D_2 - \frac{1}{p(2-p)} = 0$ is a cubic equation in α , and it's derivative is strictly positive for $0 < \alpha < 1$, so it has at most one root in this range, α^* . To sum up

1. $\alpha \in [\alpha_L, \alpha_H]$: both NI banks survive unless both projects fail.
2. $\alpha > \alpha^*$ and $\alpha \notin [\alpha_L, \alpha_H]$: NI_1 fails and NI_2 survives when one project is successful.
3. Otherwise: NI banks survive only if both projects succeed.

This implies the following sets of value functions for the banks under the two assumptions.

In-network debt senior.

$$\begin{aligned}\mathcal{V}_I^a &= (1 - q)^2V_I + q(1 - q)[p(V_I + (R - D))] + (1 - q)qV_I + q^2P(V_I + (R - D)) \\ \mathcal{V}_{NI_1}^a &= (1 - q)^2V_{NI} + 2(1 - q)q[p(V_{NI} + 2D - D_1) - 1] + q^2(\alpha(2 - \alpha)(pR - 1) + EV_{NI_1}^{a,S}) \\ \mathcal{V}_{NI_2}^a &= (1 - q)^2V_{NI} + 2(1 - q)q[p(V_{NI} + D_1) - 1] + q^2(\alpha^2(pR - 1) + EV_{NI_2}^{a,S})\end{aligned}$$

where

$$\begin{aligned}EV_{NI_1}^{a,S} &= \begin{cases} (1 - (1 - p)^2)V_{NI} & \text{if } \alpha_L < \alpha < \alpha_H \\ p^2V_{NI} & \text{otherwise} \end{cases} \\ EV_{NI_2}^{a,S} &= (1 - (1 - p)^2)V_{NI}\end{aligned}$$

All debt pari-pasu.

$$\begin{aligned}\mathcal{V}_I^a &= (1-q)^2V_I + q(1-q)[p(V_I + 2(R-D))] + (1-q)qV_I + q^2p(V_I + (R-D)) \\ \mathcal{V}_{NI_1}^a &= (1-q)^2V_{NI} + 2(1-q)q[p(V_{NI} + 2D - D_1) - 1] + q^2(\alpha(2-\alpha)(pR-1) + EV_{NI_1}^{a,PP}) \\ \mathcal{V}_{NI_2}^a &= (1-q)^2V_{NI} + 2(1-q)q[p(V_{NI} + D_1) - 1] + q^2(\alpha^2(pR-1) + EV_{NI_2}^{a,PP})\end{aligned}$$

where

$$\begin{aligned}EV_{NI_1}^{a,PP} &= \begin{cases} (1 - (1-p)^2)V_{NI} & \text{if } \alpha_L < \alpha < \alpha_H \\ p^2V_{NI} & \text{otherwise} \end{cases} \\ EV_{NI_2}^{a,PP} &= \begin{cases} (1 - (1-p)^2)V_{NI} & \text{if } \alpha_L < \alpha < \alpha_H \text{ or } \alpha > \alpha^* \\ p^2V_{NI} & \text{otherwise} \end{cases}\end{aligned}$$

Let $EV_{NI_i}^a = \max \{EV_{NI_i}^{a,PP}, EV_{NI_i}^{a,S}\}$.

Network structure 3b.

$$\begin{aligned}\mathcal{V}_{I_1}^b &= (1-q)V_I + q[p(V_I + 2(R-D))] \\ \mathcal{V}_{NI}^b &= (1-q)V_{NI} + q[p(V_{NI} + D) - 1]\end{aligned}$$

Although NI does not want to deviate from 3b to 3e but I_1 will unilaterally deviate and break $e_{I_1I_2}$ link if that increases his expected profit, which happens if $\kappa < \frac{1}{2}$.

Network structure 3c.

$$\begin{aligned}\mathcal{V}_I^c &= (1-q)V_I + q[p(V_I + (R-D))] \\ \mathcal{V}_{NI}^c &= (1-q)V_{NI} + q[p(V_{NI} + D) - 1]\end{aligned}$$

Network structure 3d. The first step is to determine the face value of debt when both I banks have an investment opportunity, and there can possibly be diversification. In accordance to assumption 1, each I bank lends the one unit he raises from the NI to the other I bank. So the balance sheet of each I bank is the following: on the asset side, there is (\tilde{R}, D_2) and on the liability side there is (D_1, D_2) . The sharing rule implies that when both investment opportunities are realized, in expectation, each NI bank gets $\mathbb{E}[NI] = \alpha^2X + 1$ and each I bank receives $\mathbb{E}[I] = (1 - \alpha^2)X$, from intermediating one unit and investing another unit.

Let PP_Z^D denote the partial payment of a bank I whose project is in state $Z \in \{F, S\}$ (Failure, Success), when the nominal face value is D . In expectation, each I and NI bank gets

$$\begin{aligned}\mathbb{E}[I] &= p^2(R - D_1) + (1 - p)p \max\{0, PP_S^{D_2} - PP_F^{D_2} - PP_F^{D_1}\} \\ &\quad + p(1 - p) \max\{0, R + PP_F^{D_2} - PP_S^{D_2} - PP_S^{D_1}\} \\ \mathbb{E}[NI] &= p^2 D_1 + (1 - p)p PP_F^{D_1} + p(1 - p) PP_S^{D_1}\end{aligned}$$

where the second (third) term of each expression corresponds to the state where project i fails (succeeds) and project $-i$ succeeds (fails). The case where both projects fail or succeed are trivial. Consider the case where one project fails and one succeeds.

I_F , bank with failed project. This bank always fails, i.e. $PP_S^{D_2} - PP_F^{D_2} - PP_F^{D_1} \leq 0$, as the liabilities of bank with a failed project are $> D_2$, while his assets are $\leq D_2$. This also implies, $PP_F^{D_1} < D_1$.

I_S , bank with successful project. I_S fails if he does not pay back his obligations in full, which requires $PP_S^{D_2} < D_2$. Then D_1 and D_2 are given by

$$(7) \quad \mathbb{E}[I] = p^2(R - D_1)$$

$$(8) \quad \mathbb{E}[NI] = p^2 D_1 + (1 - p)p \frac{D_1}{D_1 + D_2} X_{SF} + p(1 - p) \frac{D_1}{D_1 + D_2} (R + X_{FS})$$

where X_{ij} is the gross payment of $I = I_i \in \{S, F\}$ to $I_j = I_{-i}$ in the sense of the equilibrium payment clearing vector and satisfy

$$\begin{aligned}X_{SF} &= \frac{D_2}{D_1 + D_2} (R + X_{FS}) = PP_S^{D_2} \\ X_{FS} &= \frac{D_2}{D_1 + D_2} X_{SF} = PP_F^{D_2}\end{aligned}$$

Solving the above system

$$\begin{aligned}X_{SF} &= \frac{D_2(D_1 + D_2)R}{D_2(2D_1 + D_1)} \\ X_{FS} &= \frac{D_2^2 R}{D_1(2D_2 + D_1)};\end{aligned}$$

where from equation (7)

$$D_1 = R - \frac{X(1 - \alpha^2)}{p^2}$$

Substituting back in equation (8), the equation is trivially satisfied, which implies D_2 is undetermined. However, it must also be the case that neither I bank has enough resources to pay back in full. First, $X_{SF} < D_1 + D_2$, which is satisfied as $X_{SF} < D_2$, otherwise I_S would be paying I_F in full and would not fail. Second, $R + X_{FS} < D_1 + D_2$, which requires $D_2 > \frac{D_1(R - D_1)}{2D_1 - R} = \frac{(pR - 1)(1 - \alpha^2)(1 - \alpha^2 + pR(-1 + p + \alpha^2))}{p^2(2 - 2\alpha^2 + pR(-2 + p + 2\alpha^2))}$, i.e. the inter- I bank obligations should be sufficiently large.

Alternatively, I_S survives if he can pay his obligations in full. Then D_1 and D_2 are given by

$$\begin{aligned}\mathbb{E}[I] &= p^2(R - D_1) + p(1 - p)(R - D_1 - D_2); \\ \mathbb{E}[NI] &= p^2D_1 + p(1 - p)\frac{D_1}{D_1 + D_2}D_2 + (1 - p)pD_1\end{aligned}$$

Solving the two equations jointly implies $D_1 = \frac{1 + \alpha^2(pR - 1)}{p}$ and $D_2 = 0$. This means I banks will not have cross-liabilities. Then each bank fails exactly when own project fails and in return keep the intermediation spread when own project survives. I have to pick one of the two equilibria in interbank rates. Since I banks prefer the latter set of rates, it makes sense to assume they jointly set the latter inter- I rate and D_1 will be set accordingly to ensure the rule for division of surplus holds.³²

Thus in expectation, an I bank and NI get the following, respectively:

$$\begin{aligned}\mathcal{V}_I^d &= (1 - q)^2V_I + q^2[p(V_I + R - D_1)] + q(1 - q)[p(V_I + 2(R - D))] + (1 - q)q[p(V_I + D - D_1)] \\ \mathcal{V}_{NI}^d &= (1 - q)^2V_{NI} + q^2[p(V_{NI} + D_1) - 1] + q(1 - q)[p(V_{NI} + D) - 1] + q(1 - q)[p(V_{NI} + D_1) - 1]\end{aligned}$$

³²Which equilibrium in face-values is picked only affects the probability of failure of I banks, and potentially the NI bank. The later would be the case only if the partial payment received by NI lender of I bank with failing project, allows the NI to pay the (equilibrium) rate to outside lenders in full and avoid failure. In that case the threshold for NI bank incentive joining the coalitional deviation might have to be slightly adjusted (depending on parameter values), but the mechanism remains the same.

The second alternative assumption is that when both I banks have a project, each invests his own borrowed unit and they don't lend to each other. This implies that the each NI bank gets $D = D_2$ when both I banks have a project. Again, the same threshold incentives for I and NI bank should be adjusted but the argument is intact.

Network structure 3e. I_1 has no outside claims so unlike in ??, debt seniority structure is irrelevant. When both banks have an investment opportunity, each invest one unit. If both projects fail, every bank fail. If both projects succeed, all banks survive. If I_1 survives when only I_2 project's succeed, it survives if only own project succeeds as well. This implies if

$$\alpha_L < \alpha_L^{e,1} = \frac{1}{2} \left(1 - p - \sqrt{(1-p)^2 - \frac{4p}{pR-1}} \right) < \alpha < \alpha_H^{e,1} = \frac{1}{2} \left(1 - p + \sqrt{(1-p)^2 - \frac{4p}{pR-1}} \right) < \alpha_H$$

then I_1 (and NI_1, NI_2) succeed if either project succeed. Alternatively, if

$$\alpha_L^{e,2} = \frac{1}{2} \left(-p - \sqrt{(p^2+4) - \frac{4p}{pR-1}} \right) < \alpha < \alpha_L^{e,1} \text{ or } \alpha_H^{e,1} < \alpha < \alpha_H^{e,2} = \frac{1}{2} \left(-p + \sqrt{(p^2+4) - \frac{4p}{pR-1}} \right)$$

then I_1 survives only if his own project survives and fails otherwise. Finally, if only one project succeeds, NI s survive if either the above conditions hold (depending on the successful project), or if

$$\alpha > \alpha^{*1} = \frac{p}{(2-p)(pR-1)}.$$

So to sum up

$$\begin{aligned} \mathcal{V}_{I_2}^e &= (1-q)V_I + q[p(V_I + (R-D))] \\ \mathcal{V}_{I_1}^e &= (1-q)^2V_I + (1-q)q[p(V_I + 2(D-D_1))] + q(1-q)[p(V_I + 2(R-D))] \\ &\quad + q^2[(1+\alpha)(1-\alpha)(pR-1) + EV_{I_1}^e] \\ \mathcal{V}_{NI}^e &= (1-q)^2V_{NI} + q(1-q)[p(V_{NI} + D_1) - 1] + (1-q)q[p(V_{NI} + D) - 1] \\ &\quad + q^2\left[\frac{1}{2}\alpha(2-\alpha)(pR-1) + EV_{NI}^e\right] \end{aligned}$$

where

$$EV_{I_1}^e = \begin{cases} (1-(1-p)^2)V_I & \text{if } \alpha_L^{e,1} < \alpha < \alpha_H^{e,1} \\ pV_I & \text{if } \alpha_L^{e,2} < \alpha < \alpha_L^{e,1} \text{ or } \alpha_H^{e,1} < \alpha < \alpha_H^{e,2} \\ p^2V_I & \text{otherwise} \end{cases}$$

$$EV_{NI}^e = \begin{cases} (1-(1-p)^2)V_{NI} & \text{if } \alpha_L^{e,1} < \alpha < \alpha_H^{e,1} \text{ or } \alpha > \alpha^{*1} \\ pV_{NI} & \text{if } \alpha_L^{e,2} < \alpha < \alpha_L^{e,1} \text{ or } \alpha_H^{e,1} < \alpha < \alpha_H^{e,2} \text{ and } \alpha < \alpha^{*1} \\ p^2V_{NI} & \text{otherwise} \end{cases}$$

Next consider all the possible deviations:

3a to 3d, 3d to 3c. Incentive constraints for I and NI_2 joining the deviation requires

$$\kappa = \frac{\alpha(1-\alpha)X}{(1-p)V_I} > \bar{\kappa} = \max \left\{ 1-q, \frac{q}{1-q} \frac{EV_{NI_2}^a - pV_{NI}}{(1-p)V_I} \right\}.$$

Moreover, if $\bar{\kappa} = 1-q > \frac{q}{1-q} \frac{EV_{NI_2}^a - pV_{NI}}{(1-p)V_I}$, then

$$\hat{\kappa} = \bar{\kappa} = \max \left\{ (1-q) \frac{\alpha}{1-q+\alpha}, \frac{q}{1-q} \frac{EV_{NI_2}^a - pV_{NI}}{(1-p)V_I} \right\} < \bar{\kappa},$$

and 3d is an equilibrium in $(\hat{\kappa}, \bar{\kappa})$ region. Here although I banks are willing to decrease their counterparty exposure if they continue to get funded by both NI banks, they cannot do so as NI_2 bank is not willing to join the deviation. $\hat{\kappa}$ is the level of spreads that either NI_2 is willing to join the deviation or I banks are willing to forgo investment to decrease counterparty risk.

There is no deviation from 3d and 3e as NI s are weakly better off in 3e, and either I_1 is strictly better off in 3e in which case he will not agree to borrow from I_2 , or I_2 is strictly better off in 3e in which case I_1 unilaterally breaks the e_{I_1, I_2} in 3e.

3e to 3b deviation. If spreads are sufficiently low, I_1 unilaterally breaks the e_{I_1, I_2} in 3e. This happens if

$$\kappa < \check{\kappa} = \frac{1}{2} \frac{1 - q(1 + EV_{I_1}^e - pV_I)}{\left(1 - q^{\frac{(\alpha+1)}{2\alpha}}\right)}$$

Alternatively, in 3b, the two I banks jointly deviate and add e_{I_1, I_2} , which happens when $\kappa > \check{\kappa}$

3b to 3a deviation. NI_2 would do so if

$$\alpha X(\alpha(2-q) - 1) > V_{NI}(1-p)(1-q) + q(pV_{NI} - EV_{NI_2}^a)$$

which assuming $q < \bar{q}_1$, requires $\alpha > \frac{1}{2-q}$. So whenever $\check{\kappa} \leq \frac{1}{2-q}$, this deviation does not happen.

3c to 3e or 3a deviation. The former deviation always happens when 3e is an equilibrium, as long as $q < \bar{q}_1$. The latter deviation is identical to that in 3b above.

Efficiency. With the parametric assumption $pR - 1 > (1 - p)(V_I + V_{NI})$, the socially efficient structure is one of 3a, 3d, or 3e that reach the full scale of investment.

First, note that total default probability of NI_2 in 3a is weakly lower than that of 3d and 3e, regardless of debt seniority structure.

Total default cost of $\{I_1, I_2, NI_1\}$ in 3a is at most

$$(9) \quad 2q(1 - q)(1 - p)(V_I + V_{NI}) + q^2((1 - p)^2(2V_I + V_{NI}) + 2p(1 - p)(V_I + V_{NI}))$$

Total default cost of $\{I_1, I_2, NI_1\}$ in 3d is at least

$$(10) \quad 2q(1 - q)(1 - p)(2V_I + V_{NI}) + q^2((1 - p)^2(2V_I + V_{NI}) + 2p(1 - p)(V_I))$$

Total default cost of $\{I_1, I_2, NI_1\}$ in 3e is at least

$$(11) \quad 2q(1 - q)(1 - p)\left(\frac{3}{2}V_I + V_{NI}\right) + q^2((1 - p)^2(2V_I + V_{NI}) + 2p(1 - p)\left(\frac{1}{2}V_I\right))$$

Thus

$$(12) \quad q < \bar{q}_2 = \min \left\{ \frac{1}{p\frac{V_{NI}}{V_I} + 1}, \frac{1}{p\left(1 + \frac{2V_{NI}}{V_I}\right) + 1} \right\} = \frac{1}{p\left(1 + \frac{2V_{NI}}{V_I}\right) + 1}$$

is a sufficient condition for network structure 9a to be the only constraint efficient equilibrium.

Finally let

$$(13) \quad \bar{q} = \min \{\bar{q}_1, \bar{q}_2\}.$$

which completes the proof. □

Proof of Proposition 2.

For every cut C , parents of node b in \hat{G} are exactly the banks to whom b is lending to G . By construction of C , these parents are all included on the source side of C . So and node who is on the sink side of C only lends to banks on the source side. The total amount of funding which flows into any set of nodes cannot be more that total funding

raised by their direct and indirect lenders. The total flow is by construction $\text{Size}(C)$ and total funding raised at direct and direct lenders is $X_S(C)$, which is the number of banks on the sink side of C . So $\text{Size}(C) < X_S(C)$. When only leaf nodes are on the sink side, every edge in the cut set is on a shortest path, and each leaf node has exactly one unit of funding, so the inequality holds with equality.

For the second inequality, note that every edge with one end in $\text{In}\mathbb{I}_R$ and the other in \mathbb{NI} is on the shortest path of some NI to \mathbb{I}_R ,³³ so there is at least one unit lent over such edge in G . By construction the sum of flows of funding on such edges is $\text{Size}(C_o)$ which I just argued is at least as large as the number of such edges. \square

Proof of Lemma 1.

Consider a bank b who lends along a longest chain of length l_{max} with probability non zero.³⁴ There is no diversification so if the ultimate borrower I fails every bank who has lent to him through any chain fails. As a result when bank NI lends directly or to indirectly to a bank I then he fails with probability $(1 - p)$ regardless of the length of the intermediation chain. However, when he lends through his longest chain of length l_{max} in expectation he gets $\alpha^{l_{max}}X$. As a result l_{max} is the largest number for which b 's participation constraint is not violated, which means $\alpha^{l_{max}}X \geq (1 - p)V_{NI}$ and $\alpha^{l_{max}+1}X < (1 - p)V_{NI}$. \square

Proof of Theorem 1.

I will show that there is no feasible deviation for the relevant set of parameters. Let $\mathbb{C}(G)$ and s denote the core and the size of the core, respectively, so there are s I banks in $\mathbb{C}(G)$ and $k_I - s$ out of the core. First consider the unilateral deviation of $I_1 \in \mathbb{C}(G)$. Note that with sufficiently many peripheries (as described in the statement of the theorem), if an I lends to one other I he would lend to as many I 's as he can, since everything is linear; and similarly if he drops a lending he drops every lending. So I_1 's relevant unilateral deviation is to drop all of his links to I banks and stop intermediating. That is the case if intermediation rents that I_1 captures is not sufficient to cover his cost of default. With a core of size s , the division of peripheries which maximizes the profit of the worst-off member of the core is the equal division of NI peripheries, so that each $I \in \mathbb{C}(G)$ gets $\frac{k_{NI}}{s}$ lending to him. So I_1 deviates if $\frac{k_{NI}}{s}\alpha(1 - \alpha)X < (1 - p)V_I$ which determines a lower bound on κ : $M_s = \frac{s}{k_{NI}}$.

Next, consider other possible deviations. The first coalition consists of only $I \in \mathbb{C}(G)$. Each I who is in the core has maximum possible lending relationships so I 's at the core can

³³It is certainly on the shortest path of the NI at the end point, and maybe on the shortest path of others.

³⁴Note that b can lend over shorter paths to other banks I as well.

not form a blocking coalition alone. Second, there can be a coalition of a (proper) subset of I 's in the core \mathbb{C}_D , and NI banks lending to $I \in \mathbb{C}(G) \setminus \mathbb{C}_D$. In the current network, every NI gets an expected return of αX with probability q and $\alpha^2 X$ with probability $(1-q)(1-(1-q)^{k_I-1})$, and every single lending generates positive expected net profits (net of cost of default), so this is the maximum possible expected profit any bank can get without having any funds pledged from the inter-bank network. Simply becoming a periphery to a different core bank does not increase this payoff, so this is not a valid blocking deviation either.

Third, can a combination of I 's outside the core and NI 's form a profitable deviation? With the exact same argument as the last paragraph there is no such feasible deviation because it is not possible to make any NI_j better off than what they are without making some NI_k worse off (peripheral to NI_j). In this case, it is not even possible to make them as well off as before because the $I \in \mathbb{C}(G)$ bank(s) whose peripheral NI 's are part of the suggested deviation never agree to join the deviation and add links to borrow from the I banks who are part of the suggested deviation (currently out of the core). So NI banks who join such deviation would get intermediated spreads strictly less often than current structure (and the exact same unintermediated spreads), so they would be strictly worse off.

Forth, can $I \notin \mathbb{C}(G)$ deviate alone? It cannot add any links, and only loses by severing links, so there is no such deviation either.

Finally, can (a subset of) NI 's jointly deviate without any I 's in the coalition? Again the answer is no, for the following reason: Any such deviation implies that there is some NI at distance 2 to his closest I bank without any improvement in probability of being involved in the investment opportunity, which will be rejected by that NI .

The converse is simple. Assume $\kappa < M_s$. Then $\frac{k_{NI}}{s}\alpha(1-\alpha)X < (1-p)V_I$. Moreover, in any s -core network, at least one of $I \in \mathbb{C}(G)$ has $\frac{k_{NI}}{s}$ or less peripheries. This I bank would unilaterally deviate and sever all his potential lending contracts to all other I banks and strictly increase his expected surplus. \square

Proof of Proposition 3.

The proof is done in two steps. First I show that given any network G , realizations \mathbb{I}_R , and $\{R_k\}_{k \in \mathbb{I}_R}$, and face values of debt $\{D_{ij}\}_{i,j \in \mathbb{N}}$ and $\{D_i^h\}_{i \in \mathbb{N}_F}$ set at date $t = 1$, the system of inter-bank repayments (2) has a unique solution. This part of the proof is very similar to that of Acemoglu et al. [2015], proposition 1. The proof proceeds in multiple steps. First define the total liabilities of bank i to bank j by multiplying the per-unit payment by number of units lent and then define the share of each bank j in bank i liabilities.

Then I define an appropriate mapping function $\Phi(\cdot)$ which maps the min of partial and full payments to itself. It is straight forward to show that this mapping is a contraction which maps a convex and compact subset of Euclidean space to itself. As a result by Brouwer fixed point theorem, this contraction mapping has a fixed point which is the set of feasible inter-bank face values of debt and their relevant partial payments. For detail of generic uniqueness see Acemoglu et al. [2015].

Next, I focus on network formation stage, and show that (at least) one of following three networks is an equilibrium for any parameter set: smallest member of the core-periphery family (single- I -core network), the star structure with an NI core (NI -star network), or a structure where every NI banks lend to a (potentially multiple) I bank(s) but I banks are not connected to each other (island network). Assume the NI -star is not an equilibrium. Either k_{NI} times intermediation spread is larger than $(1 - p)V_I$ (case 1) or it is smaller (case 2). The single- I -core is an equilibrium in case 1 (proof of theorem 1). Now consider case 2. Since NI -star is not an equilibrium, there is a coalitional deviation to block it. The deviation cannot be only breaking links since every banks is getting strictly positive expected net surplus from every transaction at $t = 1$, and solely breaking the link gives it zero net surplus. So the deviation involves adding links. For a peripheral NI to deviate, he needs to get strictly closer than one intermediary away, to at least one I bank, as in NI -star he is one-intermediary away from every I . So any deviation requires (at least) adding a link between a peripheral NI_j and one of the I banks, I_i .

Consider a potential deviation which is only NI_j breaking his link from the core NI and adding a link to I_i . In this deviation, NI_j trades off the spread he had to always pay the core NI with the lower probability of getting it only when I_i receives an investment opportunity. There are two possible cases: when this deviation is profitable for NI_j (case 2-1) and when it is not (case 2-2). First consider the former. Assume we start in the island network where every NI bank lends to I_i (single island). As we are in case 2-1, NI banks have no incentive to deviate and become peripheral to one of the NI s, and (at best) create the NI -star network in order to get the lower, intermediated rate of return, more often. I_i has no incentive to start intermediating as we are in case 2. The only remaining deviation is if (a subset of) other I 's deviate with (a subset of) NI 's and create a multi-core structure where I_i is completely left out. Note that I_i would not agree to be part of any deviating coalition. In the current structure he gets all the funding when he has a project and he is not willing to intermediate, so he cannot be better off than what he is in any other network). This structure is preferred by NI 's because they get the same high rate that they get in the single island, plus they sometimes get an intermediated rate of return,

so they would be willing to join such deviation. However, any link between two I banks, $e_{I_i I_k}$, will never be traversed because it is not individually rational for an I bank (I_j) to intermediate, which rules out this latter class of deviations. So the single-island network is an equilibrium in case 2-1.

Finally consider case 2-2. NI_j is only willing to deviate if he becomes peripheral to I_i who himself has a potential lending relationship to at least some other I_j . However by the exact same argument as above, such deviations are ruled out because traversing any link $e_{I_i I_k}$ violates individual rationality of I_i with probability one, so such links cannot be added in a coalitional deviation. So no NI_j bank would ever join a coalition, case 2-2 never happens, and NI -star is an equilibrium itself, which completes the existence proof. \square

Proof of Lemma 2. The size of the core is bounded by number of I banks, which justifies the second argument of the outer max. Next, for each periphery in the network, the corresponding core bank receives $\alpha(1 - \alpha)(pR - 1)$ expected spread when it intermediates, which has to cover his expected cost of default if he is willing to be in the core. As a result, each core bank requires $z = \lceil \frac{(1-p)V_I}{\alpha(1-\alpha)(pR-1)} \rceil$ attached peripheries. Moreover, each I bank in the core has to sustain links to every other I bank which is reflected in the second argument in the denominator of the ratio. The largest core is achieved when peripheries are most evenly distributed among core banks, which is captured by the ratio. \square

Proof of Proposition 4.

The proof is immediate from differentiating the equation that determines \bar{s} , equation 4, dropping the integer constraints. The integer constraint lead to the comparative statics being weak inequalities. \square

Proof of Theorem 2. Efficiency. First note that in this structure feasibility as well as the participation constraint of every bank are satisfied. Regardless of which bank receives the investment opportunity, all the funding will be channeled to some investment opportunity. Moreover, since every NI bank is lending to all I banks only through the same common intermediary, maximal concentration of risk is achieved. In other words, when multiple I banks receive investment opportunities, one and only one of them invests, which given the no diversification assumption 2 is welfare enhancing since it concentrates risk as much as possible and saves on expected cost of default of some I 's, while reaching the same scale of investment. Finally, for any realization of investment opportunities, aside from the single I bank who does the investment, every other bank with a realized lending and/or borrowing relationship provides funding for the investment, so removing him from the set of active

lenders decreases the scale of investment by one while also decreasing the expected cost of default by $(1 - p)V_{NI}$. Since a direct $NI \rightarrow I$ lending relationship is socially desirable, the former is larger, so this removal will be welfare destroying.

Tiered Network. For any (NI_j, I_i) , every path from NI_j to I_i goes through NI_c , so finding $SP(NI_j, I_i)$ reduces to finding $SP(NI_j, NI_c)$. As a result, any lending that NI_j does happens over his shortest path(s) to NI_c , and any potential lending of NI_j who is not part of (one of) $SP(NI_j, NI_c)$ is traversed with probability zero, and removed from the graph. So all the paths from NI_j to NI_c have the same length, $dist(i, j)$. So the socially efficient network is a tiered network.

Efficient Network Not Equilibrium. I will choose appropriate bounds \bar{K} and \bar{M} , and an appropriate group of NI banks, \mathbb{L}' such that \mathbb{L}' and \mathbb{I} form a blocking coalition.

Let $\hat{G} = (\hat{V}, \hat{E})$ be the subgraph consisting of all the NI banks and the edges among them in any efficient network G . Also, let \mathbb{L} denote the collection of leaf nodes in G . By Lemma 1, the length of the longest intermediation chain in \hat{G} is $l_{max} - 1$. Let $\tilde{G} = (\tilde{V}, \tilde{E})$, $\tilde{V} = \hat{V} \setminus NI_c$ and $\tilde{E} = \hat{E} \setminus \{e_{iNI_c} \mid \forall i \in \tilde{V}\}$, and \tilde{G} is undirected by removing all the directions of edges in E . \tilde{G} is a collection of connected components. For any fixed number of leafs z_l , there is only finitely many possible connected component which has z_l leaf nodes (because the length of each chain is limited). As a result, fixing k_I , for sufficiently large k_{NI} , $k_{NI} > \bar{K}_1$, there will be at least k_I^2 leafs in G , i.e. either there are many connected components in \tilde{G} with few leafs or some connected components with many leafs, or a combination of both. Let $\bar{K} = \bar{K}_1 + k_I$.

Let

$$(14) \quad \bar{M} = \frac{1}{k_I} \left(1 + \frac{q}{1-q} \frac{k_I - 1}{k_I} \right)$$

Choose k_I^2 of leaf NI banks ($NI \in \mathbb{L}$), and call it \mathbb{L}' . Then there is a joint deviation by all $NI \in \mathbb{L}'$ and all the I banks: groups of k_I of $NI \in \mathbb{L}'$ banks lend to each I bank, and all I banks start lending and borrowing from each other. The deviating NI banks are clearly better off.

Next consider the I banks. For any realization of investment opportunities, part of the surplus is generated by resources provided by $NI \in \mathbb{L}'$ and the rest by $NI \notin \mathbb{L}'$. Each I bank increases his share of surplus of the former part, which is insured by the first term on the right hand side of 14, i.e. the spreads that I captures by intermediating k_I banks when he does not have an investment opportunity, compensates him for his extra cost of default. As for the latter part, by definition of \bar{K} after the deviation there are still

more than k_I NI banks in G , so NI_c keeps all of his edges to all I banks. When $NI \in \mathbb{L}'$ deviates, his pre-deviation potential borrower, \hat{NI} might lose one potential lending due to feasibility, but since all of his intermediation chains to \mathbb{I} have the same length, there is no change in division of surplus generated by the remaining units, i.e. pre and post $dist(\hat{NI}, \mathbb{I})$ are the same.

Finally, we need to consider change in default cost of I banks. In NI -star network all the lending is done through NI_c , which corresponds to the highest level of concentration as argued previously: When more than one investment opportunity is realized only one I bank invests but at a high scale. Since no diversification is allowed this enhances each I bank's expected surplus because his expected investment remains the same while he fails less often. Maximum gains to concentration is attained when all I banks receive an investment opportunity, in which case each only incur a $\frac{1}{k_I}(1-p)V_I$ default cost for investing, as opposed to $(1-p)V_I$. This put a upper-bound of $\frac{k_I-1}{k_I}(1-p)V_I$ on how much an I bank would loose in default costs, by joining the deviation, when he does receive an investment opportunity. An I bank receives an investment opportunity with probability q and receives intermediation spreads with probability $(1-q)$, so we should have $(1-q)k_I\alpha(1-\alpha)X > q\frac{k_I-1}{k_I}(1-p)V_I$, which is insured by the second term on the right hand side of 14. So the proposed coalition is a blocking coalition and G is not an equilibrium. □

Proof of Proposition 5.

From the construction proposed in theorem 1, in every core-periphery equilibrium $G \in \mathbb{G}_{cp}$, each NI bank is lending to exactly one core I bank and each core I bank is lending to every other core I bank, so there is a path from every NI bank to every I bank, and the maximum scale of investment is always achieved regardless of the realization of investment opportunities. As a result, the size of inefficiency is purely determined by the expected number of defaults. From assumption 2 and that success probability of projects is iid, for any given draw of distribution of investment opportunities the expected number of NI banks who default are the same. However, every I bank in the core is exposed to exactly one investment in each configuration of active investment opportunities, and fails if either he is active and his project fails or with non-zero probability if he does not have an investment opportunity, given any realization of other I banks' project failure. On the other hand, every I bank outside the core fails iff he becomes an active investing bank and fails, less than a core bank, which implies $\frac{d\bar{n}_G}{ds_G} > 0$, which in turn implies $\frac{d\Gamma\bar{N}S_G}{ds_G} < 0$.

Finally note that by definition, $\forall G \in \mathbb{G}_{cp}^s$ have the same size core, so they have the same degree of inefficiency and same expected number of defaults. □

Proof of Proposition 6.

We will prove the result for R , and the other two comparative statics follow the exact same argument.

Let $\phi(p, R, V_I, \alpha) \equiv \frac{(1-p)V_I}{\alpha(1-\alpha)(pR-1)}$, and assume $\phi > k_I$ and is an integer. Moreover, assume $\frac{k_{NI}}{\phi(p, R, V_I, \alpha)} = m - \epsilon$, for an integer $m < k_I$ and $\epsilon \rightarrow 0$. As a result, $\bar{s}(p, R, V_I, \alpha) = m - 1$ in this economy.

There exists a finite positive constant K such that increasing R by $\hat{\epsilon} = K\epsilon$ to \hat{R} implies $m < \frac{k_{NI}}{\phi(p, \hat{R}, V_I, \alpha)} < m + 1$, which leads to $\bar{s}(p, \hat{R}, V_I, \alpha) = m$. Note that since $\epsilon \rightarrow 0$, $\hat{\epsilon} \rightarrow 0$, and the economy is finite, so the gross expected surplus added by a marginal increase in R goes to zero. However, size of the core discretely jumps, which implies a discrete jump in expected cost of default, which completes the proof. \square

Proof of Theorem 3.

Theorem 1. Note that the only place in proof of Theorem 1 where the value of the spreads are used is in determining the cut-offs. Analogous to that proof, in an s -core core-periphery network, the min expected profit of a core bank I who intermediates is $\frac{k_{NI}}{s} \bar{\mathcal{L}}(2; 3, X)$, which should cover his cost of default $(1-p)V_I$, which in turn determines the lower bound $M_s = \frac{s}{k_{NI}}$.

Every other deviational argument goes through directly from the properties of the rule for division of surplus defined in section 4.1: Adding to the chain weakly decreases the share of every member of the chain and strictly decreases the share of initial lender, which implies that everything else equal, every NI bank unambiguously prefers to be lending to I bank through as few intermediaries as possible. Moreover, the rule is anonymous and holding the surplus fixed, does not depend on the identity of banks in the chain.

Theorem 2. Similar to the previous part, the only place where the level of intermediation spreads are referenced are exactly the same as Theorem 1, so the same argument as above applies. There can be an extra force here from the borrower, when borrower share of surplus also decreases as a function of the length of the intermediation chain. Let \mathbb{NI}_i denote the set of k_I NI banks who lend to I_i in the coalitional deviation by \mathbb{L}' and \mathbb{I} . I_i bank requires even less intermediation spreads to cover his extra expected cost of default due to intermediation, because he receives some extra share of surplus of own investment, by having \mathbb{NI}_i lending directly to him rather than indirectly, through NI_c . Moreover, the intermediation chain to I_i from any other $NI \in \mathbb{L}' \setminus \mathbb{NI}_i$ is at least as short as before: they are one intermediary away post deviation and they were at least one intermediary away pre-deviation. For $NI \notin \mathbb{L}'$ the proof is the same as 2, as the socially efficient network is tiered.

The above arguments all rely on anonymity, i.e. the division of surplus does not depend on the identity of banks in the chain. Appendix 9.3.3 provides an example where the rule does depend on the identities by incorporating the default costs into \mathcal{L} , and proves similar results. □

Proof of Lemma 3. j_1 is connected to at least z_2 of $I \in \mathbb{I}$, through “pointwise” weakly shorter paths, as defined in the lemma. Call this set $\mathbb{I}_{j_2}^{z_2}$. When any $I \in \mathbb{I}_{j_2}^{z_2}$ is in \mathbb{I}_R , the expected rate that j_1 (and consequently any lender to j_1) receives on their (indirect) lending is independent from distance of any $I \notin \mathbb{I}_{j_2}^{z_2}$ but $I \in \mathbb{I}_R$ to whom j_1 is connected. As a result the expected return that j_1 (and his lenders) receive conditional on realization of an investment opportunity at $I \in \mathbb{I}_{j_2}^{z_2}$ is larger than what j_2 (and his lenders) receive when what of the I banks j_2 is connected to is in \mathbb{I}_R . The above two events happen with exactly same probability (equal to at least one out of z_2 binomial random variables being one). Conditional the former event not happening j_1 still earns positive rents when $I \in \mathbb{I} \setminus \mathbb{I}_{j_2}^{z_2}$ is in \mathbb{I}_R which more than covers his expected cost of default³⁵, while j_1 earns no rents. So in expectation over all realizations of investment opportunities, j_1 and his lenders are better off than j_2 and his lenders, respectively. □

Proof of Proposition 7. Equilibrium.

All the references to figures in this proof are to Figure 8.

First, solve for the face values payable to NI peripheries, D_{11} and D_{22} . Failure probability of I_2 determines the face value payable to its NI peripheries to be $D_{22} = \frac{1+\alpha X}{p}$. As a result, the only remaining equilibrium object is D_{11} . D_{11} depends on the share of surplus that goes to a direct lender, the endogenous probability of (partial) repayment by I_1 , as well as Y_1 and Y_2 .

The structure of equilibrium and the face value of debt from I_1 to his NI peripheries are jointly determined in equilibrium, based on which of the following regions the total liabilities of the net lender I_1 lies in:

$$\left\{ \begin{array}{ll} Y_1 D_{11} \geq \frac{Y_1 + Y_2}{2} R & I_1 \text{ survives with probability } p^2 \\ \frac{Y_1 - Y_2}{2} R \leq Y_1 D_{11} < \frac{Y_1 + Y_2}{2} R & I_1 \text{ survives with probability } p \\ Y_1 D_{11} < \frac{Y_1 - Y_2}{2} R & I_1 \text{ survives with probability } 1 - (1 - p)^2 \end{array} \right.$$

³⁵Because I assume participation constraint must be satisfied for each realization of lending.

First note that liabilities can be high for two reasons: either α is high so that a large share of surplus goes to the lenders, or default probability of borrower is high. In the first region above liabilities are so high that unless both assets pay, I_1 fails. In the middle region I_1 fails if his asset investment fails and survives otherwise, and in the last region I_1 survives unless both assets fail. In the first two regions there will be partial payments. Let $\hat{D} = D_{22} = \frac{1+\alpha X}{p}$, which is the face value of debt which corresponds to the case where a bank fails exactly when his own investment fails.

Region One ($Y_1 D_{11} > \frac{Y_1+Y_2}{2} R$).

$$p^2 D_{11} + p(1-p) \frac{Y_1 + Y_2}{2Y_1} R + (1-p)p \frac{Y_1 - Y_2}{2Y_1} R = \alpha X + 1$$

$$D_{11} = \frac{1}{p} (\hat{D} - (1-p)R)$$

In order for the total liabilities with the above face value to be in region one it must be that

$$\frac{Y_2}{Y_1} < \frac{2}{pR} \hat{D} - \frac{2-p}{p}$$

Region Two ($\frac{Y_1-Y_2}{2} R \leq Y_1 D_{11} < \frac{Y_1+Y_2}{2} R$).

$$p D_{11} + (1-p)p \frac{Y_1 - Y_2}{2Y_1} R = \alpha X + 1$$

$$D_{11} = \hat{D} - (1-p) \frac{R}{2} \left(1 - \frac{Y_2}{Y_1}\right)$$

In order for the total liabilities with the above face value to be in region two it must be that

$$(15) \quad \frac{Y_2}{Y_1} > \frac{2}{pR} \hat{D} - \frac{2-p}{p}$$

$$(16) \quad \frac{Y_2}{Y_1} > 1 - \frac{2}{R(2-p)} \hat{D}$$

Region Three ($Y_1 D_{11} < \frac{Y_1-Y_2}{2} R$).

$$(1 - (1-p)^2) D_{11} = \alpha X + 1$$

$$D_{11} = \frac{1}{2-p} \hat{D}$$

In order for the total liabilities with the above face value to be in region two it must be that

$$\frac{Y_2}{Y_1} < 1 - \frac{2}{R(2-p)}\hat{D}$$

Let $y = \frac{Y_2}{Y_1} \leq 1$ denote the ratio of the *NI* peripheries of I_2 to I_1 . The inequality holds because I_1 is assumed to have more peripheries. The two inequalities defined in 15 characterize the three regions in which I_1 fails with different probabilities; where each region characterizes the set of (α, y) for which the probability of I_1 failure is the same.

The two lines cross each other and zero, if they do so, at $(\bar{\alpha}, 0)$ such that

$$1 = \frac{2}{R(2-p)} \frac{1 + \bar{\alpha}X}{p}$$

However, the two lines will not cross zero (and each other) at any $\alpha \geq 0$ if even at $\alpha = 0$ I_1 's own investment must survive for him to survive. This happens if

$$\frac{2}{p\bar{R}} - \frac{2-p}{p} > 0$$

Let $\bar{R} = \frac{2}{p(2-p)}$. The above inequality holds if

$$(17) \quad R < \bar{R}$$

This happens in panel 8b. Recall that $R > \frac{1}{p}$ for the project to be positive NPV. The intuition is that if the project is positive NPV but the upside is not sufficiently high, I_1 fails if its own project, i.e. its larger asset, does not pay off. In other words, there are different combinations of (p, R) with the same NPV, that is, constant pR . I_1 prefers the combinations with higher R because it provides I_1 with sufficient resources to be able to pay its lenders, even if only I_1 's smaller asset pays back. In this case $\bar{\alpha} < 0$.

In the left panel, 8a, $\bar{\alpha} > 0$. When $0 \leq \alpha < \bar{\alpha}$, I_1 bank prefers to have many peripheries to lie below the red line, which would imply an unbalanced core-periphery structure, while for $\bar{\alpha} < \alpha \leq 1$ it prefers to have similar number of peripheries as I_2 has, which will be a more balanced core-periphery structure.

The equilibria in the two case defined by 17 should be studied separately. For now ignore the constraint that α should be such that intermediation rents are high enough so that either one or both of the I banks agree to intermediate, i.e. ignore the participation

constraint.³⁶

When 17 does not hold, the two lines defined in 15 cross at $\alpha = \bar{\alpha}$ in 8a. Recall that peripheries of net borrower fail with probability p and we need to consider incentives of NI s peripheral to net lender. These incentives are not necessarily aligned with that of the I banks. NI incentives about which I bank to lend to is purely driven by their default probability, and are determined at $\alpha = 0$, as explained in the text. Here at $\alpha = 0$ there is a range of positive y for which NI banks (and I_i) survive as often as possible, i.e. unless both projects fail. So at those y 's NI 's will survive at higher values of α as well, since the (partial) payments they receive from I_i only increases in α .

To see this, consider two different economies; L and H , with two different levels of α ; $\alpha_L = 0$ and $\alpha_H > \bar{\alpha}$.³⁷ Denote the NI banks in economy L and H , NI_L and NI_H , respectively. First consider economy L and assume Y_1 and Y_2 are such that y lies below the solid red line. For this level of y , if at least one of the assets held by I_1 pays back (probability $(1 - (1 - p)^2)$), NI_L peripheries of I_1 are payed back in full. They pay all of what they get to households³⁸, and they survive with probability $(1 - (1 - p)^2)$, the same probability as I_1 survives.

Now consider economy H . Here I_1 survives only if both of its assets pay back, that is, if both investments are successful, because its liabilities are too high. This happens with probability p^2 . However, when I_1 fails it makes partial payments if either of his assets pay back. As a result, for every state of the world, what each NI_H bank gets in the H economy, is at least as high as what each NI_L bank gets in the L economy. As NI_L and NI_H banks have the same expected liabilities, NI_H cannot fail more often than NI_L . This implies that for each (p, R, V_I, V_{NI}) , and each level of y , the probability of default for an NI periphery of I_1 , for any α , is the same as probability of default of an NI with $\alpha = 0$.

For $\alpha < \bar{\alpha}$, every NI lenders of I_2 prefers to instead lend to I_1 and save on the expected cost of default. I_1 likes that too. So every NI periphery of I_2 deviates to I_1 as long as I_2 has one periphery. If I_2 loses its last periphery, when both I banks have an investment opportunity, even if I_1 lends to I_2 and I_2 invests, I_2 does not receive a share of his own investment's net surplus, because I_1 absorbs all the returns. However, I_2 still incurs the expected cost of default. As a result, participation constraint of I_2 is violated and $I_1 \rightarrow I_2$

³⁶Note that I have assumed participation constraint must be satisfied case by case. When only one bank get the investment opportunity diversification does not come in, so this argument does not affect the range of α for which either one or both I s are willing to intermeditate. The final equilibria are the ones which are consistent with both sets of conditions.

³⁷This example is purely for illustration, so ignore the fact that NI_L 's participation constraint is violated at $\alpha = 0$.

³⁸Because $\alpha = 0$.

will not happen when both banks have the investment opportunity. Consequently, I_1 's probability of default would rise to p , and I_2 's last periphery would be indifferent between deviating or not, which by definition of equilibrium implies it does not deviate.³⁹

On the other hand, when $\alpha > \bar{\alpha}$, I_1 fails more often below the dashed blue line while NI lenders to I_1 still fail less often. As a result, NI peripheries of I_2 want to deviate and lend to I_1 . Interestingly, I_1 does agree to this deviation although it increases its probability of default. The reason is that the return it gets from investing this extra unit, more than covers the incremental cost of default, $\alpha(1 - \alpha)X > (1 - p)V_I > p(1 - p)V_I$.

The above argument requires a minor adjustment. Note that the 2- I core-periphery equilibrium never features $y = 0$, instead $y = \frac{1}{k_{NI}-1}$, which must be in the Region Three at $\alpha = 0$ for the above argument to work. As a result \bar{R} needs to be updated to adjust for this:

$$(18) \quad \bar{R} = \frac{2}{p(2-p)}z$$

where $z = \frac{k_{NI}-1}{k_{NI}-2}$. Note that $\bar{R} \rightarrow \frac{2}{p(2-p)}$ as $k_{NI} \rightarrow \infty$. Moreover, instead of $\bar{\alpha}$ there are two relevant thresholds, $\bar{\alpha}_l$ and $\bar{\alpha}_h$, one on each line defining the borders of the three regions, which replace α

$$\begin{aligned} \bar{\alpha}_l &= \left(\frac{p(2-p)R}{2} \left(1 - \frac{1}{k_{NI}-1} \right) - 1 \right) (pR - 1)^{-1} \\ \bar{\alpha}_h &= \left(\frac{pR}{2} \left(\frac{p}{k_{NI}-1} + 2 - p \right) - 1 \right) (pR - 1)^{-1} \end{aligned}$$

Note that as $k_{NI} \rightarrow \infty$, $\bar{\alpha}_l \rightarrow \alpha$ and $\bar{\alpha}_h \rightarrow \alpha$.

In the region where 17 does not hold (with adjusted \bar{R} defined in 18), if $\alpha < \bar{\alpha}_l$, then I_1 survives with probability $1 - (1 - p)^2$. If $\bar{\alpha}_l < \alpha < \bar{\alpha}_h$, then I_1 survives with probability p . If $\alpha > \bar{\alpha}_h$, then I_1 survives with probability p^2 . So a small region is added in the middle for I_1 . All NI s who lend to I_1 still survive with probability $1 - (1 - p)^2$.

Next consider the case where 17 holds. As a result, Region Three disappears. Here the realized return of the project, R , is so low that even at $\alpha = 0$, regardless of level of y , I_1 fails if its larger asset, namely, its own investment, does not pay back. However, depending on the level of y and α , I_1 may need its second asset to also pay back in order to survive. Specifically, if α is high I_1 survives only if both assets pay back.

³⁹The fact that I_2 remains with a single NI periphery is simply because I assumed intermediation rents are high enough so that intermediating a single unit of funding covers I 's extra cost of default. If intermediating c units is necessary to keep I_2 intermediating, then it will end up with c peripheries.

Now consider default probability of NI banks who are peripheral to I_1 . Again, the relevant range of the parameters for NI peripheries, to prefer one borrower to the other, is determined only at $\alpha = 0$, but for different reasons. First note that the highest (partial) payments that an NI bank receives is at $\alpha = 1$, where NI receives R for the proportion of his portfolio invested in the successful project(s), and has to pay lenders who only have to break even, i.e. they in turn have $\alpha = 0$ effectively. This is the exact same problem that I_1 faces when his NI lenders have $\alpha = 0$.

Two different scenarios must be considered separately. First, can NI fail only with probability $1 - (1 - p)^2$, given that we know this is not possible for I_1 ? As I argued, the best an NI can do is at $\alpha = 1$, and for him to survive unless the two projects fail we should have

$$\frac{1}{Y_1} \frac{Y_1 - Y_2}{2} R > \frac{1}{1 - (1 - p)^2}$$

which boils down to the boundary of Region three at $\alpha = 0$ as argued above, which we know is negative when 17 holds. So this case never happens (regardless of how often I_1 survives).

When I_2 survives with probability π , NI does also survive with probability at least as high as π . So the only remaining case is when I_1 survives with probability p^2 but his peripheries survive with probability p .

Let D_{1h} denote the face value of debt payable to households lending to an NI bank peripheral to I_1 . The trick is to realize that when I_1 fails, he pays all the proceeds from his project as partial payment, *as if* NI has $\alpha = 1$, and when NI fails himself he pays all of those proceeds to his households. As a result the equation which defines D_{1h} boils down to the same equation which defines D_{11} in Region two, at $\alpha = 0$:

$$pD_{1h} + (1 - p)p \frac{Y_1 - Y_2}{2Y_1} R = 1$$

Which in turns implies that the boundary for this case is the same as the boundary in Region two at $\alpha = 0$, \bar{y} in 8b. So in this case when $y > \bar{y} = \frac{2}{p^2 R} - \frac{2-p}{p}$, NI peripheries of either I bank are indifferent between moving around since they have no room to improve on their default probability. However, when $y < \bar{y}$, NI peripheries of I_1 deviate to I_2 until $y \geq \bar{y}$. Such deviation pushes y up and above \bar{y} . Any $y > \bar{y}$ is an equilibrium because NI peripheries of I_1 has no incentive to deviate to I_2 , because they fail with the same probability in both places.

Finally, one should consider $y = 0$, where only I_1 lends to I_2 , separately. As long as intermediation rents are sufficiently high, $y = 0$ is also an equilibrium. The reason is that NIs would not benefit from any joint deviation with I_2 unless I_1 agrees to the deviation and adds the $e_{I_2I_1}$ potential relationship, which would require I_1 to lose at least one of its peripheries to I_2 , and I_1 does not agree to be part of such deviation even if it improves his survival probability, as explained above.

Efficiency.

I will show that in the range provided in the proposition, the 2- I core periphery equilibrium is dominated by NI -star, and cannot be efficient. This does not necessarily means NI -star itself is efficient.

Consider NI -star, and let NI_c be the NI who lends to all I banks. NI_c survives either with probability p^2 or $1 - (1 - p)^2$ because his two assets are symmetric. Assume NI_c fails only if both projects fail. So if each of his assets pay back, he must be able to pay his liabilities in full

$$\frac{k_{NI}}{2} \frac{\alpha X + 1}{p} \geq (k_{NI} - 1) \frac{\alpha^2 X + 1}{p(2 - p)} + \frac{1}{p(2 - p)}$$

The first term on right hand side is his total liabilities from other NIs assuming he pays back with probability $1 - (1 - p)^2$, and the second term to his households. With some algebra we get

$$k_{NI} \left[\frac{2 - p}{2} (\alpha X + 1) - (\alpha^2 X + 1) \right] > -\alpha^2 X$$

This is a similar condition to what I_1 faces, with a few adjustments. Unlike I_1 , total assets available to NI_c when an investment survives is lower than full value, R . His liabilities are also lower, and are not fully symmetric. A sufficient condition for the above inequality is

$$(19) \quad \alpha^2 X + 1 < \frac{2 - p}{2} (\alpha X + 1)$$

This is now very similar to the condition for I_1 , except that both assets and liabilities decrease with α , so for instance at $\alpha = 0$, NI_c fail: his liabilities are low, but the same with his assets. It does not hold at $\alpha = 1$ either. So the corresponding quadratic equation has two roots, $0 < \hat{\alpha}_l < \hat{\alpha}_h < 1$, and the above inequality holds if $\hat{\alpha}_l < \alpha < \hat{\alpha}_h$.

In this case, every NI survives with probability $1 - (1 - p)^2$. NI_c diversifies the risk that NIs face very well, but not the risk that I_1 faces.

Note that

$$\alpha^2 X + 1 < \frac{p(2-p)}{2} \frac{(\alpha X + 1)}{p} < \frac{p(2-p)}{2} R$$

where the last inequality holds simply because face value paid to NI_c is less than R as final borrower gets positive share of surplus. As a result when 17 holds, the total assets are too low and NI_c survives only if both asset pay.

When $\alpha > \hat{\alpha}_l$, all peripheral NI s can still survive with probability $1 - (1 - p)^2$ if their partial payment, when only one project pays off, is sufficiently large

$$\frac{\alpha^2 X + 1}{(k_{NI} - 1)(\alpha^2 X + 1) + 1} \frac{k_{NI} \alpha X + 1}{2} \frac{1}{p} > \frac{1}{p(2-p)}$$

The left hand side is increasing in α , so there is a constant $\tilde{\alpha}$ such that for $\alpha > \tilde{\alpha}$ it holds.

Next I compare the difference between the expected loss in NI -star and core-periphery equilibria. Let Δ denote the difference, so $\Delta > 0$ implies that the core-periphery network is inefficient (but not the reverse).

- $R > \bar{R}$, $\hat{\alpha}_l < \alpha < \hat{\alpha}_h$, $\alpha > \bar{\alpha}_l$

$$\Delta = q^2[(1-p)pV_{NI} + \mathbb{1}[\alpha > \bar{\alpha}_h]p(1-p)V_I] + 2q(1-q)(1-p)V_I > 0$$

The first term is when there are two investment opportunities. In NI -star, all NI s and one I survive if only one project pays off. In the core-periphery if only project of I_1 pays off, I_2 and his periphery fail. If $\alpha > \bar{\alpha}_h$, I_1 fails unless both projects payoff. The last term corresponds to states where only one I get the investment. So the NI -star is strictly better.

- $R > \bar{R}$, $\hat{\alpha}_l < \alpha < \hat{\alpha}_h$, $\alpha < \bar{\alpha}_l$

$$\Delta = q^2[(1-p)pV_{NI} - p(1-p)V_I] + 2q(1-q)(1-p)V_I$$

Here if only I_1 's project survive there is a gain of one extra NI being saved in NI -star, but if only I_2 's project survive there is a cost of I_1 failing in NI -star. Intermediation costs are the same. If $q < \hat{q} = \frac{V_I}{V_I + 0.5p(V_I - V_{NI})}$, $\Delta > 0$.

- $R < \bar{R}$: Here project payoff in case of success is low, so NI -star does poorly in terms

of diversification.

$$\Delta > q^2[-p(1-p)k_{NI}V_{NI}] + 2q(1-q)(1-p)V_I$$

the first inequality comes from the fact that there are equilibria here where I_1 fail unless both projects payoff, in which case NI -star saves on that. However, no matter which project fail all NI s fail, which is not the case in the core-periphery equilibrium as NI banks reorganize themselves to improve on survival probability. This is the first term on right hand side. A sufficient condition for the above is

$$q < \hat{q} = \frac{V_I}{V_I + 0.5\frac{p}{1-p}k_{NI}V_{NI}}$$

□

9.2 Discussion of Assumption

In the model, potential lending relationships are formed before realization of investment opportunities. There is ample evidence that banks interact through long term relationships. Afonso et al. [2011] documents that in the federal funds market, approximately 60% of the funds an individual bank borrows in one month persistently comes from the same lender. Di Maggio et al. [2015] finds that in the inter-dealer market, banks with longer term relationships get access to better terms. When funding and investment opportunities arrive at different points in time, and the cost of finding, verifying, and matching with borrowers is sufficiently high, a lender prefers to be intermediated through its current connections to a bank that has an investment opportunity, as opposed to searching and switching. In addition, because investment happens at $t = 1$ and non-contractible return is realized at $t = 2$, the borrower cannot commit to pay the lender a side payment above and beyond the face value of debt enforceable by the contract. Note that in the period during which actual lending happens, no extra funding is available to make an early side payment. As such ruling out side payments is a reasonable assumption.

Next, the feasibility assumption is consistent with the hypothesis that establishing relationship lending is costly (information, trust, etc), and with the observation that hedge funds, even large ones, typically maintained only one or two prime brokerage relationships and did not frequently switch.⁴⁰ It can be explicitly micro-founded by an appropriate choice

⁴⁰<https://www.wellsfargo.com/downloads/pdf/com/securities/hedge-fund-risk.pdf>

of upfront fixed or declining cost of link formation. The cost should be such that with j units of available funds, the expected marginal gains from $j + 1^{\text{th}}$ potential lending relationship is below the cost, while it covers the cost with $j + 1$ units. However, the motivation for this assumption is not to capture a fixed cost.

Requiring banks to have ex-ante relationships in order to lend, along with feasibility, leads to market incompleteness and implies that banks cannot spontaneously reallocate their funding, or borrow on the interbank market, which in turn makes intermediation necessary to allocate resources within the financial sector. Contingent debt avoids addition market incompleteness.

Furthermore, I have assumed that banks earn positive intermediation spreads. Intermediation spreads have been documented in different interbank markets. Di Maggio et al. [2015], Li and Schürhoff [2014], Adrian [2011] and Bech and Atalay [2010] provide evidence for intermediation and positive intermediation spreads in different markets. Moreover, section 9.3.1 provides a micro-foundation to endogenize the prices/spreads along with the network structure without altering the main results.

The assumption that households break even, i.e. they earn zero rate of return, is a normalization. Alternative constant positive rates of return leads to the same outcomes.

I assume only NI banks raise funding from households to get stark normative predictions. The positive predictions are invariant to whether I banks also raise funding from households. The normative results remain the same if an I bank's contribution to scale of investment is not sufficient to justify its risk-taking behavior. To be more precise, assume I_1 raises $\epsilon < 1$ funds from households. Without intermediation, the participation constraint of a direct I_1 lender requires $\epsilon\alpha(pR - 1) \geq (1 - p)V_I$. Let $\hat{\epsilon}$ be the amount of funds for which the above inequality holds with equality. Then for any $\epsilon < \hat{\epsilon}$, it is more efficient that an NI bank with one unit raised from households do the intermediation as opposed to I_1 .

Next, the assumption that project returns are iid leads to maximum room for diversification. Section 9.3.2 discusses how changing this assumption to perfectly correlated project returns strengthens my results.

Lastly, I have assumed lenders cannot default on their promises. As a result the contagion in my model spreads only from borrowers to lenders. An interesting extension would be to allow lenders to default on their contingent promises if several borrowers demanded liquidity at once. This extension would enrich the model and open the possibility of contagion from lenders to borrowers. Moreover, how financial institution restructure the interbank network in the face of failure of some banks is an important avenue for future research.

9.3 Extensions

9.3.1 Endogenous Price Microfoundation

This section drops the rule for division of surplus, and jointly endogenizes equilibrium network and prices.

Consider the same environment as section 5, with two modifications. There is no rule for division of surplus. Instead, in any lending relationship the lender makes a take-it-or-leave-it offer to the borrower, subject to amoral hazard friction. We assume that there is limited commitment, and that borrower banks can renege on obligations. Due to limited commitment, banks can pledge only a fraction of profit they make on their borrowing to the corresponding lender. Since this is a one shot game, the limited commitment corresponds to the case that borrower bank can steal the profits at the cost of α -fraction of profit being destroyed. As a result, he needs to capture a per-unit rent of $(1 - \alpha)$ out of his profits from each lender to make him indifferent between stealing or paying back.

Theorem 3 characterized the equilibria in the economy where α -rule is replaced with this moral hazard friction and prices are endogenously determined in equilibrium. The endogenous prices are exactly the same as those implied by α -rule.

9.3.2 Perfectly Correlated Project Outcomes

Here I solve an extension of the model where lenders lend to all eligible borrowers, i.e. assumption 2 is relaxed⁴¹, and the project outcomes are perfectly correlated across banks. This exhibit the extreme opposite case of having iid return realizations for projects, and shows how social planner and individual incentives to intermediate and diversify, vary as a function of this degree of correlation.

Perfectly correlated project returns implies that there is no room for diversification. All active investment opportunities fail or succeed together. However, as 2 is relaxed a lender has to lend at least one unit to each of his eligible borrowers. The first implication is that from the social planner's perspective, gains from lending to one extra I bank is decreasing while the cost is constant. To see this, assume a bank is lending to x I banks. The net benefit from lending to the $x + 1^{\text{th}}$ bank is that the lender is now able to lend as much funds as he is able to raise, when bank $x + 1$ receives an investment opportunity while none of the first x banks did, net of cost of default of the borrower and lenders. However, there is an extra cost. Everything else equal, when any (subset) of the first x I banks, as well as bank $x + 1$ receive an investment opportunity, the scale of investment remains fixed,

⁴¹Assumption 3 is maintained.

but bank $x + 1$ also invests and is now exposed to failure (of his own project). In other words, with multiple realized investment opportunities there is gain to *concentrating* the risk, which is lost here. Let $Z(x; K)$ denote the total net surplus from an NI bank, with K unites of funds (raised from his households and $K - 1$ other NI banks), lending to x I banks ($K > x$).

$$Z(x; K) = (1 - (1 - q)^x)K \left((pR - 1) - (1 - p)V_{NI} \right) - (1 - p)qV_I x$$

$$\Delta(x; K) = Z(x + 1; K) - Z(x; K) = q \left[(1 - q)^x K \left((pR - 1) - (1 - p)V_{NI} \right) - (1 - p)V_I \right]$$

Let $c = V_I \left(K \left(\frac{pR-1}{1-p} - V_{NI} \right) \right)^{-1}$. Note that from the assumption that one unit $NI \rightarrow I$ is efficient we know $c < 1$. The marginal gain turns negative when

$$x > x^* = \frac{\log(c)}{\log(1 - q)}$$

First assume $k_I < x^*$, so the social planner prefers to lend to every I bank. The efficient solution requires investing every unit of funding whenever there is at least one realized investment opportunity, i.e. there should be a path from every NI bank to every I bank. Note that there is no room for concentration as Assumption 2 is relaxed. Moreover, all the intermediation must be done by NI banks, so no I bank lends.

In terms of equilibrium structure, the analogue of Theorem 1 holds here, with the exact same proof. This is the case because as long as there are no diversification effects, a lender only cares about level of rents, not where (or from how many borrowers) they come from, or what risk is undertaken to generate them. Moreover, the efficient structure is not an equilibrium when intermediation spreads are sufficiently high.

More interestingly, assume $k_I > x^*$. Now the social planner prefer to keep some investment opportunities unfunded because the marginal benefit is too small. In other words, reaching optimal scale of investment in one low-probability state requires destroying surplus in many states. This is the case when q is large, while k_{NI} is not too large. However, the same family of equilibria as defined in 1 still exist. A lender and/or intermediary wants to get as high a return as possible, as often as possible, so he prefers to be connected (directly or indirectly) to as many I banks as possible. Each I bank wants to invest as often as he gets an investment opportunity, so he would want to be connected to all units of funding. In this case, not only redistributinal effects within a state are not internalized by individual players, but also redistributinal effect across states are ignored.

This appendix manifests that incentives of banks to intermediate are the same with and

without Assumption 2. In the extreme case where project returns are perfectly correlated, the core-periphery equilibrium remains inefficient because there is no gain to diversification. As section 7 shows, even with iid projects the core-periphery structure is inefficient under certain parameter restrictions. As the correlation across project returns rises,⁴² the gain to diversification falls but gain to intermediate remains the same, so the space of parameters for which the core-periphery equilibrium is inefficient grows.⁴³

9.3.3 Incorporating Cost of Default into Rule for Division of Surplus

Here I solve the 4 bank model of section 3 with a variation of α -rule which incorporates the default cost of banks along the intermediation chain. In this variation, the net surplus divided between the members of an intermediation chain is net of expected cost of default, and each agent receives his expected default cost plus his share. Let L , B , and In denote lender, borrower and intermediary respectively and let V_k be the cost of default of agent $k \in \{L, B, In\}$. Let X_k be the expected net surplus associated with a unit of investment intermediated through a chain of length k .⁴⁴

$$\begin{aligned} X_1(V_B, V_L, V_{In}) &= X - (1 - p)(V_B + V_L) \\ X_2(V_B, V_L, V_{In}) &= X - (1 - p)(V_B + V_L + V_{In}) \end{aligned}$$

I suppress arguments to simplify the notation to X_1 and $X_2(V_{In})$, as the rest of the arguments do not change ($V_B = V_I$ and $V_L = V_{NI}$). Note that in each chain, each agent is compensated for the risk he takes as if this unit was the only unit he is involved in. This rule does not satisfy anonymity. Nevertheless, considering it reveals more insight from the model.

The new rule implies that agents are always compensated for the risk that they take (and maybe over-compensated). Now consider the deviation analogous to the one depicted in Figure 4. Let \hat{x} denote variables in the right panel, i.e. the core-periphery structure.

$$\begin{aligned} \hat{V}_{NI_2} &= q\alpha X_1 + (1 - q)q\alpha^2 X_2(V_I) + V_{NI} \\ \hat{V}_I &= q^2(1 - \alpha)X_1 + q(1 - q)(1 - \alpha)[X_1 + X_2(V_I)] + q(1 - q)\alpha(1 - \alpha)X_2(V_I) + V_I \end{aligned}$$

⁴²keeping project expectations the same.

⁴³Solving for the most efficient structure with interim levels of return correlation is not straightforward, and is left for future work.

⁴⁴ k is the number of edges along the chain.

while

$$\begin{aligned}\mathcal{V}_{NI_2} &= (1 - (1 - q)^2)\alpha X_2(V_{NI}) + V_{NI} \\ \mathcal{V}_I &= (q(1 - q) + \frac{1}{2}q^2)2(1 - \alpha)X_2(V_{NI}) + V_I\end{aligned}$$

Let $\Delta\mathcal{V}_j = \hat{\mathcal{V}}_j - \mathcal{V}_j$, $j = I, NI$. With some algebra we get

$$\begin{aligned}\Delta\mathcal{V}_{NI} &= q\alpha^2\left[\frac{1 - \alpha}{\alpha}X_1 + (1 - p)V_{NI}\right] + q(1 - q)\alpha^2(1 - p)(V_{NI} - V_I) \\ \Delta\mathcal{V}_I &= q^2(1 - \alpha)(1 - p)V_{NI} + q(1 - q)(1 - \alpha)\alpha X_2(V_I) + q(1 - q)(1 - \alpha)(1 - p)(2V_{NI} - V_I)\end{aligned}$$

The sign of the last term in both expressions is ambiguous. The first observation is that if $V_I = V_{NI}$, both the peripheral lender and the I banks want to *unconditionally* deviate: I bank is now compensated for the excessive risk that he can take, and the cost is born by NI_1 (recall that the expected length of chains is *the same* in both network structures). Moreover, $\forall V_I \exists \bar{C}$ such that for $X > \bar{C}$, both $\Delta\mathcal{V}_{NI} > 0$ and $\Delta\mathcal{V}_I > 0$ even if $V_I > 2V_{NI}$. This condition is similar to what we have in section 3: if surplus of a unit investment is sufficiently large, the share of it which used to go to the NI intermediary before the deviation, and post deviation is divided between the peripheral NI and the new intermediators, I banks, is sufficiently large to cover the extra cost that they have to each bear by deviating. The higher cost is due to the fact that a *costlier* I banks intermediates in the new network, which is directly incorporated in the rule of division of surplus.

Now let me make an even more extreme assumption, and assume $V_{NI} = 0$, so if an NI intermediates it is costless. Then we have

$$\begin{aligned}\Delta\mathcal{V}_{NI} &= q\alpha^2\left[\frac{1 - \alpha}{\alpha}X_1\right] - q(1 - q)\alpha^2(1 - p)V_I \\ \Delta\mathcal{V}_I &= q(1 - q)(1 - \alpha)\left[\alpha X_2(V_I) - (1 - p)V_I\right]\end{aligned}$$

comparing the two pair of expressions, it is clear that it is more difficult to satisfy the latter two. However, still $\exists \bar{C} > \bar{C}$ for which the same argument goes through.

This appendix shows that the intuition for role of intermediation in formation of financial networks is quite general and beyond the sufficient conditions provided in section 4.1, leading to a core-periphery interbank equilibrium. The crucial assumption is that there are positive intermediation spreads, and longer intermediation chains are associated with lower spreads per bank involved.