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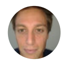
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Separating ambiguity and ambiguity attitude with mean-preserving capacities: Theory and applications

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Abstract

We introduce mean-preserving (MP) capacities for Choquet Expected Utility (CEU). MP-capacities disentangle ambiguity from ambiguity attitude and facilitate economically meaningful comparative statics. They rest on the complementary independence axiom for preferences over acts. We relate MP-capacities to other classes of capacities and discuss a variety of applications including the value of information, portfolio choice, self-insurance and self-protection, the value of a statistical life and precautionary saving. MP-capacities allow to derive results for ambiguity averters and ambiguity lovers, and simplify the analysis of greater ambiguity.

Keywords: Choquet Expected Utility · mean-preserving capacity · ambiguity · ambiguity aversion · comparative statics

JEL-Classification: D80 · D81

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"The greater the ambiguity, the greater the pleasure."

Milan Kundera

1 Introduction

Uncertainty is ubiquitous. Decision-makers experience uncertainty over their earnings, health, liability, physical assets, financial assets and longevity. Starting with the seminal works of David Schmeidler, a variety of decision models for choice under uncertainty have been brought forward in the past thirty years to accommodate Ellsberg-type phenomena (see Ellsberg, 1961). This has resulted in many models of choice under ambiguity, each featuring some violation of the classic independence axiom. The most prominent models are Choquet Expected Utility (CEU, Schmeidler, 1989), MaxMin Expected Utility (MEU, Gilboa and Schmeidler, 1989), alpha-MaxMin Expected Utility (Ghirardato et al., 2004) and the popular smooth ambiguity model (Klibanoff et al., 2005). These decision criteria generalize subjective expected utility (SEU) by featuring ambiguity and ambiguity attitude. Ambiguity refers to the decision-maker's uncertainty about probability, for example, due to missing information (Camerer and Weber, 1992); ambiguity attitude refers to decision-makers' sensitivity towards ambiguity.[1]

To obtain clean predictions in economic analysis, we need models that allow us to vary different aspects of the choice environment separately. While SEU has received acclaim for its ability to disentangle risk from risk preferences (see Pratt, 1964; Rothschild and Stiglitz, 1970), such a separation is harder to accomplish for ambiguity. The smooth model achieves this goal and allows to characterize ambiguity attitude (i.e., ambiguity aversion, ambiguity neutrality and ambiguity loving) and comparative ambiguity attitude (i.e., more or less ambiguity averse). It also admits comparative statics of greater ambiguity (see Jewitt and Mukerji, 2017). Other models such as CEU or alpha-MEU may still allow for comparative ambiguity aversion even though there can be difficulties in characterizing ambiguity attitude and greater ambiguity. Cubitt et al. (2020) developed a test that discriminates between models of ambiguity aversion and found greater support for the smooth model. Baillon and Bleichrodt (2015) and Chew et al. (2017), in contrast, observed that CEU and alpha-MEU could better explain the data.

In this paper, we introduce mean-preserving (MP) capacities and show that they facilitate a clean separation of ambiguity and ambiguity attitude for CEU preferences. Our approach represents a viable alternative to the smooth ambiguity model in those cases where CEU is better aligned with observed behavior. It also allows to scrutinize whether existing results are due to the choice of a specific decision-making framework or whether they hold more generally.[2] Our decision criterion lies at the intersection of several popular classes of preferences. We assume that preferences have

[1] See Gilboa and Marinacci (2016) for an overview of decision-making frameworks for ambiguity.

[2] Izhakian (2017) demonstrates that expected utility with uncertain probabilities also achieves a separation of ambiguity and ambiguity attitude. This model admits a risk-independent measure of ambiguity.

a CEU representation. This representation features MP-capacities when preferences satisfy the complementary independence axiom. The decision-maker's CEU then takes the form

$$V(f) = \mathbb{E}_m U(f) + \beta(f),$$

where $\mathbb{E}_m U(f)$ is the SEU of act f and $\beta(f)$ its associated measure of ambiguity.[3] The most general approach of this kind is Grant and Polak (2013) who characterize so-called mean-dispersion preferences axiomatically. An important feature of these models is that the sign of $\beta(f)$ characterizes the decision-maker's ambiguity attitude. Our preference representation is also a special case of the alpha-MEU model but we achieve a complete separation of ambiguity, reflected by the existence of a non-unique prior, and ambiguity attitude, represented by parameter alpha.

Our paper contributes to the literature in three respects. We identify MP-capacities as a class of capacities that disentangles ambiguity from ambiguity attitude and provides pragmatic handles for comparative static analysis. We characterize MP-capacities and relate them to other classes of capacities in the literature, specifically the neo-additive class (Chateauneuf et al., 2007). Second, our approach is particularly suitable for the analysis of ambiguity lovers. Recent empirical evidence suggests that ambiguity aversion is not universal because ambiguity loving prevails in a variety of decision contexts (see Kocher et al., 2018; Trautmann and Van De Kuilen, 2015; Wakker et al., 2007; Wakker, 2010). The smooth model has rarely been used to study ambiguity lovers; the introductory quote is meant to insinuate that ambiguity lovers should not be forgotten.[4] Third, the comparative statics of greater ambiguity are cumbersome in the smooth model and often require restrictions on the decision-maker's preferences (see Huang and Tzeng, 2018; Peter, 2019; Bleichrodt et al., 2019; Peter and Ying, 2019). MP-capacities simplify the analysis and may therefore be more broadly applicable.

We proceed as follows. The next section presents the decision theoretic set-up, introduces MP-capacities and their properties, characterizes them axiomatically and relates them to other classes of capacities. The third section presents typical applications to showcase the versatility of MP-capacities. We study the value of information, portfolio choice, self-insurance and self-protection, the value of a statistical life and precautionary saving. We compare our results for ambiguity averters with those obtained under the smooth ambiguity model. We also derive results for ambiguity lovers who often exhibit the reverse behavior. This opens up the possibility of null effects in the aggregate when different ambiguity attitudes coexist in the population. We provide comparative statics of greater ambiguity, which are simpler than in the smooth ambiguity model. A final section concludes.

[3] This representation is in line with Maccheroni et al. (2006), Siniscalchi (2009), Vicig and Seidenfeld (2012), Schneider and Nunez (2015) and Nunez and Schneider (2019).

[4] When an objective function is globally concave in its decision variables under SEU, it is also globally concave for ambiguity averse decision-makers in the smooth ambiguity model. Standard techniques from tackling questions involving risk can then be applied to the analysis of problems involving ambiguity (see Gollier, 2011). It might be for purely technical reasons that ambiguity lovers are often dismissed in applications.

2 Decision theoretic set-up

2.1 Choquet Expected Utility

Let $\Omega = \{\omega_1, \ldots, \omega_n\}$ be a finite set of mutually exclusive and collectively exhaustive states of the world and $\Sigma = \mathcal{P}(\Omega)$ be the power set of Ω. An element $E \in \Sigma$ is called an event and its complement is given by $E^c \equiv \Omega \backslash E$. An event other than $\emptyset$ and Ω is called nontrivial and we denote their collection by $\Sigma_{nt} = \Sigma \backslash \{\emptyset, \Omega\}$. Let X be an interval in $\mathbb{R}$. A simple act f is a function from Ω to X. Let $f^{-1}(x) = \{\omega \in \Omega : f(s) = x\}$ denote the preimage of x under f for $x \in X$; then $f^{-1}(x) \in \Sigma$ holds trivially so acts are measurable with respect to Σ. $\mathcal{F}$ denotes the collection of all acts. The range of act f is given by $f(\Omega) = \{f(\omega) : \omega \in \Omega\}$.

A binary relation $\succeq$ over $\mathcal{F}$ gives the decision-maker's preference over acts. As usual, $\sim$ denotes the symmetric part of $\succeq$. We assume throughout the paper that preferences can be represented by Choquet Expected Utility (CEU).[5] The decision-maker's *beliefs* are formalized by a so-called Choquet capacity $\nu : \Sigma \to \mathbb{R}$ in this model. The set function ν is

(i) normalized, $\nu(\Omega) = 1$ and $\nu(\emptyset) = 0$, and

(ii) monotonic with respect to set inclusion, $E \subseteq E' \Rightarrow \nu(E) \leq \nu(E')$.

We write $\nu(\omega)$ instead of $\nu(\{\omega\})$ for an elementary event consisting of a single state $\omega \in \Omega$. The decision-maker's preference representation also yields a continuous and strictly increasing utility function of outcomes, $U : X \to \mathbb{R}$. The CEU when choosing act $f \in \mathcal{F}$ is then given by the Choquet integral of $U(f)$ with respect to the capacity ν,

$$V(f) = \sum_{x \in f(\Omega)} U(x) \cdot [\nu(f \geq x) - \nu(f > x)] \,. \tag{1}$$

We write $\nu(f \geq x)$ to mean $\nu(\omega \in \Omega : f(\omega) \geq x)$ and likewise for other sets and set functions. We assume that the entire state space Ω is the only universal set, $\nu(E) = 1 \Rightarrow E = \Omega$, and that there is no null set other than the empty set, $\nu(E) = 0 \Rightarrow E = \emptyset$, to simplify the exposition. So we restrict our attention to capacities with $\nu(E) \in (0, 1)$ for any $E \in \Sigma_{nt}$.[6]

The capacity ν is called convex (concave) if $\nu(E) + \nu(E') \leq (\geq)\ \nu(E \cup E') + \nu(E \cap E')$ for all $E, E' \in \Sigma$. It is called additive if it is both convex and concave. In this case, ν is a subjective probability on Ω and $V(f)$ in Eq. (1) reduces to subjective expected utility (SEU),

$$\mathbb{E}_m U(f) = \sum_{\omega \in \Omega} m_\omega \cdot U(f(\omega)). \tag{2}$$

[5] Among others, Gilboa (1987), Schmeidler (1989), Wakker (1989), Sarin and Wakker (1992) and Ghirardato et al. (2003) provide axiomatic foundations of CEU.

[6] Alternatively, we could follow Chateauneuf et al.'s approach and partition Σ into null events, universal events, and essential events and focus on capacities that are exactly congruent with the set of null events in the sense of their Definition 3.1.

The expectation is taken with respect to the subjective probability m on Ω with $m_\omega = m(\omega) = \nu(\omega)$ for all $\omega \in \Omega$.

Ghirardato and Marinacci (2002) propose a definition of ambiguity attitude and comparative ambiguity aversion for the decision-maker's preference relation $\succeq$. They also provide characterizations when preferences have a CEU representation. We formulate their results as definitions of ambiguity attitude and comparative ambiguity aversion for CEU.

Definition 1 (Ambiguity attitude)**.** The decision-maker is *ambiguity averse* if there is a subjective probability m such that the capacity of his preference representation satisfies $m \geq \nu$. He is *ambiguity loving* if there is a subjective probability m such that his capacity satisfies $m \leq \nu$. He is *ambiguity neutral* if he is both ambiguity averse and ambiguity loving so that $\nu = m$.

Definition 2 (Comparative ambiguity aversion/loving)**.** Suppose that $\succeq_1$ and $\succeq_2$ are the preference relations of two decision-makers with CEU representations (U_1, ν_1) and (U_2, ν_2). The first decision-maker is *more ambiguity averse (more ambiguity loving)* than the second one if U_1 and U_2 differ by a positive affine transformation and $\nu_1 \leq \nu_2$ ($\nu_1 \geq \nu_2$).

The part about ambiguity aversion in Definition 1 is Ghirardato and Marinacci's Corollary 13 and ambiguity loving is defined symmetrically. Definition 2 is Ghirardato and Marinacci's Theorem 17(*i*) for comparative ambiguity aversion. We define comparative ambiguity loving by calling a decision-maker more ambiguity loving than another one if he is less ambiguity averse than the other one. Definition 1 is a special case of Definition 2 when one of the two decision-makers has an SEU representation so that his capacity is a subjective probability.

2.2 Mean-preserving capacities and their properties

We will now present a class of capacities that facilitate a sharp distinction between ambiguity and ambiguity attitude for CEU preferences. We provide the following definition.

Definition 3 (MP-capacity)**.** A *mean-preserving (MP) capacity* is defined via a triplet (m, a, α) with m being a subjective probability on Ω such that $m(E) \in (0,1)$ for $E \in \Sigma_{nt}$, $\alpha \in [0,1]$ a scalar and $a : \Sigma \to \mathbb{R}$ a nonnegative set function with the following properties:

(i) Normalization and symmetry: $a(\emptyset) = a(\Omega) = 0$ and $\forall E \in \Sigma : a(E) = a(E^c)$;

(ii) Submodularity: $\forall E, E' \in \Sigma : a(E) + a(E') \geq a(E \cup E') + a(E \cap E')$;

(iii) Bounded changes: $\forall E, E' \in \Sigma : E \subseteq E' \Rightarrow |a(E') - a(E)| \leq 2(m(E') - m(E))$ with a strict inequality for $E = \emptyset$ or $E' = \Omega$.

The MP-capacity ν is then given by

$$\nu(E) = m(E) + \left(\tfrac{1}{2} - \alpha\right) a(E) \quad \text{for } E \in \Sigma. \tag{3}$$

MP-capacities are centered around the subjective probability m, which motivates the terminology "mean-preserving." Denneberg (2000) refers to m as the central component and to the remainder as the ambiguity component. We can interpret α as a measure of comparative ambiguity attitude and a as the level of ambiguity. The CEU of act f for MP-capacity ν is

$$\begin{aligned} V(f) &= \sum_{x \in f(\Omega)} U(x) \cdot \left[m(f = x) + (\tfrac{1}{2} - \alpha) \cdot [a(f \geq x) - a(f > x)]\right] \\ &= \mathbb{E}_m U(f) + (\tfrac{1}{2} - \alpha) \cdot \sum_{x \in f(\Omega)} U(x) \cdot [a(f \geq x) - a(f > x)] \,. \end{aligned}$$

Property (*i*) in Definition 3 rules out ambiguity when the event is certain to happen or certain not to happen. The submodularity of set function a in Property (*ii*) means intuitively that the level of ambiguity for a larger event cannot exceed the sum of the ambiguity levels of its constituents. Properties (*i*) and (*ii*) correspond to Marinacci's Definition 1 of a being an ambiguity level, which justifies our interpretation. Property (*iii*) relates the level of ambiguity to the decision-maker's subjective probability. When comparing a nontrivial event E to certainty or impossibility, the level of ambiguity is bounded by $2\min\{m(E), 1 - m(E)\}$. For the comparison of two nontrivial events, Property (*iii*) restricts the change in ambiguity when moving from smaller to larger events. Three examples illustrate Definition 3.

Example 1 (Constant level of ambiguity)**.** If $a(E) = a > 0$ for $E \in \Sigma_{nt}$ and $a(E) = 0$ for $E \in \{\emptyset, \Omega\}$, Properties (*i*) and (*ii*) hold trivially. To satisfy Property (*iii*), notice that $|a(E') - a(E)| = 0$ for all $E, E' \in \Sigma_{nt}$. So we only need to ensure that (*iii*) holds as a strict inequality for $E = \emptyset$ and $E' = \Omega$, which yields $a < \min_{\omega \in \Omega} m_\omega$. The state that is subjectively perceived to be least likely determines the admissible level of ambiguity.[7]

Example 2 (Binary state space)**.** If $\Omega = \{\omega_1, \omega_2\}$, then any subjective probability with $m_{\omega_1} \in (0, 1)$ and set function a with $a(\emptyset) = a(\Omega) = 0$ and $a(\omega_1) \in [0, 2\min\{m_{\omega_1}, 1 - m_{\omega_1}\})$ yields an MP-capacity.

Example 3 (Ellsberg's (1961) three-color urn)**.** Assume an urn with an unknown composition of red, black and yellow balls. Let $m_r, m_b, m_y \in (0, 1)$ denote the subjective probability of drawing a ball of a given color as abbreviated by the subscript. If a_r, a_b and a_y denote the ambiguity levels for the events "red ball", "black ball" and "yellow ball", the ambiguity level for the event "red or black ball" is given by a_y due to symmetry and likewise for the other two-color events. So the triplet (a_r, a_b, a_y) characterizes set function a completely. Property (*ii*) is then equivalent to the following three inequalities:

$$a_r + a_b \geq a_y, \quad a_r + a_y \geq a_b \quad \text{and} \quad a_b + a_y \geq a_r. \tag{4}$$

[7] MP-capacities are a special case of affine capacities when the level of ambiguity is constant. See Toquebeuf (2016) for an axiomatic characterization.

Property (*iii*) takes the form $a_i < 2\min\{m_i, 1 - m_i\}$ for $i \in \{r, b, y\}$ and $E = \emptyset$ or $E' = \Omega$, and yields the following inequalities for two nontrivial events:

$$|a_r - a_b| \leq 2m_y, \quad |a_r - a_y| \leq 2m_b \quad \text{and} \quad |a_b - a_y| \leq 2m_r \tag{5}$$

Take $m_r = m_b = m_y = 1/3$ as an example; we then need to require $a_i < 2/3$ for $i \in \{r, b, y\}$ and the inequalities in (5) are automatically satisfied. The remaining inequalities in (4) define a hexahedron in (a_r, a_b, a_y)-space, specifically a triangular bipyramid with vertices $(0, 0, 0)$, $(2/3, 2/3, 0)$, $(2/3, 0, 2/3)$, $(0, 2/3, 2/3)$ and $(2/3, 2/3, 2/3)$. The subjective probability $(1/3, 1/3, 1/3)$ is in the center of this triangular bipyramid.

The following proposition summarizes properties of MP-capacities.

Proposition 1. *Let m be a subjective probability on Ω with $m(E) \in (0, 1)$ for $E \in \Sigma_{nt}$ and $a : \Sigma \to \mathbb{R}$ a nonnegative set function satisfying Properties (i) - (iii) in Definition 3.*

(i) Set function ν defined in Eq. (3) *is a Choquet capacity for any $\alpha \in [0, 1]$.*

(ii) ν is convex for $\alpha \geq 1/2$, concave for $\alpha \leq 1/2$ and additive for $\alpha = 1/2$.

(iii) Preferences are ambiguity averse for $\alpha \geq 1/2$, ambiguity loving for $\alpha \leq 1/2$ and ambiguity neutral for $\alpha = 1/2$.

(iv) An increase in α increases ambiguity aversion in the sense of Definition 2.

Let $a_1, a_2 : \Sigma \to \mathbb{R}$ be two nonnegative set functions satisfying Properties (i) - (iii) in Definition 3 with $a_2 \geq a_1$.

(v) The change from a_1 to a_2 decreases the decision-maker's CEU for $\alpha \geq 1/2$, increases his CEU for $\alpha \leq 1/2$ and has no effect on CEU for $\alpha = 1/2$.

We provide a proof in Appendix A.1. According to Properties (*i*) and (*ii*), MP-capacities are Choquet capacities that are either convex, concave or both. Property (*iii*) facilitates a simple parametric handling of ambiguity attitude, which allows to determine extensive-margin effects of ambiguity aversion and ambiguity loving in applications. It is easy to see from Eq. (3) that $\nu \leq m$ for $\alpha \geq 1/2$ and $\nu \geq m$ for $\alpha \leq 1/2$ because a is nonnegative. For MP-capacities the definitions of ambiguity aversion proposed by Ghirardato and Marinacci (2002) and Schmeidler (1989) coincide. Similarly, the degree of uncertainty aversion proposed by Dow and da Costa Werlang (1992) is positive for $\alpha > 1/2$, zero for $\alpha = 1/2$ and negative for $\alpha \leq 1/2$. An increase in α lowers the capacity and Definition 2 applies. So MP-capacities allow to identify intensive-margin effects of ambiguity aversion and ambiguity loving via Property (*iv*). MP-capacities are also useful to analyze greater ambiguity due to Property (*v*). Moving to a set function with uniformly higher values does not affect ambiguity neutral decision-makers but makes all ambiguity averse decision-makers worse off. This is consistent with the underlying idea behind Jewitt and Mukerji's notion "more ambiguous (I)" and facilitates the analysis of intensive-margin effects of greater ambiguity.

For MP-capacities the behavior of an ambiguity neutral decision-maker is indistinguishable from the behavior of an ambiguity averse or ambiguity loving decision-maker who does not perceive any ambiguity. This is because the MP-capacity ν reduces to the subjective probability m either when $\alpha = 1/2$ or when $a = 0$. Our model shares this feature with Klibanoff et al.'s smooth ambiguity model with unbiased beliefs in the sense of Snow (2010, 2011).

2.3 Characterization

We will now analyze the restrictions imposed by MP-capacities on preferences and discuss characterizing features. We first return to our examples.

Example 2 (Binary state space)**.** If $\Omega = \{\omega_1, \omega_2\}$, any Choquet capacity ν is an MP-capacity. Set $m_{\omega_1} = (1 + \nu(\omega_1) - \nu(\omega_2)/2$ and $m_{\omega_2} = (1 - \nu(\omega_1) + \nu(\omega_2))/2$; then $m_{\omega_1}, m_{\omega_2} \in (0,1)$ because $\nu(\omega_1), \nu(\omega_2) \in (0,1)$ and $m_{\omega_1} + m_{\omega_2} = 1$ so m is a subjective probability. Select $\alpha \in [0,1]$ and $a \geq 0$ such that $(2\alpha - 1)a = 1 - \nu(\omega_1) - \nu(\omega_2)$. If ν is a subjective probability, either $\alpha = 1/2$ or $a = 0$; if ν is convex, then $\alpha \geq 1/2$, if ν is concave, then $\alpha \leq 1/2$.

In the two-state case MP-capacities do not impose any restriction on the kind of uncertainty under consideration other than those implied by biseparable preferences (see Ghirardato and Marinacci, 2002) and the nontrivial requirement $\nu(\omega) > 0$. Biseparable preferences reduce to CEU on binary acts. In applications with only two states of the world our results apply to a broader class of preferences, and the parameterization with MP-capacities allows for a simple distinction between ambiguity and ambiguity attitude. In the example uniqueness of m follows from the uniqueness of ν whereas different combinations of a and α are possible. This holds more generally.

Example 3 (Ellsberg's (1961) three-color urn)**.** Let ν be an arbitrary capacity for the three-color urn with values $\nu_r, \nu_b, \nu_y, \nu_{rb}, \nu_{ry}$ and $\nu_{by} \in (0,1)$ corresponding to the six different nontrivial events. If ν admits a representation as MP-capacity, then

$$\nu_i = m_i + (\tfrac{1}{2} - \alpha)a_i \quad \text{for } i \in \{r, b, y, rb, ry, by\}.$$

We obtain the subjective probabilities as $m_r = \frac{1}{2}(\nu_r + 1 - \nu_{by})$, $m_b = \frac{1}{2}(\nu_b + 1 - \nu_{ry})$ and $m_b = \frac{1}{2}(\nu_y + 1 - \nu_{rb})$. They are between zero and one because the ν_i are between zero and one. To add up to one, we need to impose the following restriction:

$$\nu_r + \nu_b + \nu_y + 1 = \nu_{rb} + \nu_{ry} + \nu_{by}. \tag{6}$$

We also obtain

$$\begin{aligned} 2(\nu_r - m_r) &= \nu_r + \nu_{by} - 1 = (1 - 2\alpha)a_r, \\ 2(\nu_b - m_b) &= \nu_b + \nu_{ry} - 1 = (1 - 2\alpha)a_b, \\ 2(\nu_y - m_y) &= \nu_y + \nu_{rb} - 1 = (1 - 2\alpha)a_y. \end{aligned}$$

So ν is necessarily convex or concave resulting in $\alpha \geq 1/2$ or $\alpha \leq 1/2$. Once we fix α, condition (6) ensures that a_r, a_b and a_y satisfy the inequalities in (4) and (5).

With more than two states of the world not all convex or concave capacities are mean-preserving. This raises the question which restrictions MP-capacities impose on the underlying preferences. To characterize CEU representations with an MP-capacity, we provide the following definition based on Siniscalchi (2009).

Definition 4. Two acts $f, f' \in \mathcal{F}$ are *complementary* if for any two states $\omega_i, \omega_j \in \Omega$,

$$\tfrac{1}{2}f(\omega_i) + \tfrac{1}{2}f'(\omega_i) \sim \tfrac{1}{2}f(\omega_j) + \tfrac{1}{2}f'(\omega_j).$$

The decision-maker's preferences $\succeq$ satisfy *complementary independence* if for any two complementary pairs of acts (f, f') and (g, g') in $\mathcal{F}$, and all $\lambda \in [0,1]$,

$$f \succeq f' \text{ and } g \succeq g' \quad \Rightarrow \quad \lambda f + (1-\lambda)g \succeq \lambda f' + (1-\lambda)g'.$$

Siniscalchi's Definition 3 introduces complementarity between acts. Complementary acts are mirror images of each other. A simple example based on the three-color urn is a prize of \$100 for drawing a red ball and zero otherwise for act f, and a prize of \$100 for drawing a black or yellow ball and zero otherwise for act f'. When facing the 50-50 mixture between f and f', the decision-maker is indifferent over the color of the ball drawn because he gets \$50 regardless. Siniscalchi's Axiom 7 is the independence axiom for complementary pairs of acts. If preferences satisfy complementary independence, the decision-maker assesses an act by its SEU baseline evaluation and its utility variability around this baseline. We then obtain the following result.

Proposition 2. *Let the decision-maker's preferences $\succeq$ be represented by CEU with utility function U and convex or concave capacity ν. The following are equivalent:*

(i) ν is an MP-capacity.

(ii) $\succeq$ satisfies complementary independence.

A proof is given in Appendix A.2. Siniscalchi (2009) explains that two acts f and f' are complementary if their utility profiles satisfy $U \circ f = c - U \circ f'$ for a constant c. In this case, their utility profiles have the same dispersion as measured, for example, with the standard deviation. The complementary independence axiom means that complementary acts receive the same adjustment in the decision-maker's preference representation. In the context of CEU, complementary independence introduces an additivity requirement on the capacity. Assume that preferences have a CEU representation with capacity ν; then they satisfy complementary independence if and only if, for all $E, E' \in \Sigma$,

$$\nu(E) - \nu(E^c) + \nu(E') - \nu(E'^c) = \nu(E \cup E') - \nu(E^c \cap E'^c) + \nu(E \cap E') - \nu(E^c \cup E'^c).$$

In the three-color urn this is precisely Eq. (6). For a geometric interpretation let $\Delta(\Sigma)$ be the set of probabilities with domain Σ and define the core of capacity ν as follows:

$$\mathcal{P}_\nu = \{p \in \Delta(\Sigma) : \nu \leq p\}.$$

A convex capacity has a nonempty core (see Shapley, 1971) and the underlying preferences are ambiguity averse. The converse is not true because not all capacities with a nonempty core are necessarily convex (see Ghirardato and Marinacci, 2002). The additivity requirement for a convex capacity ν is equivalent to a particular shape of its core.

Proposition 3. *Let ν be a convex capacity. The following are equivalent:*

(i) ν is an MP-capacity.

(ii) The core of ν is centrally symmetric: there exists $m_c \in \mathcal{P}_\nu$, called the center of $\mathcal{P}_\nu$, such that, for any $p \in \Delta(\Sigma)$, we have $p \in \mathcal{P}_\nu$ if and only if $2m_c - p \in \mathcal{P}_\nu$.

We provide a proof in Appendix A.3. A centrally symmetric subset of subjective probabilities also plays a role in the characterization of more ambiguous events in the alpha-MEU model, see Jewitt and Mukerji's Proposition 3.1. The set of probabilities for an event expands while retaining the same center as the event is substituted for a more ambiguous one. If ν is an MP-capacity, the center of $\mathcal{P}_\nu$ is given by $(\nu + \overline{\nu})/2$, where $\overline{\nu}$ denotes the dual capacity, $\overline{\nu}(E) = 1 - \nu(E^c)$ for $E \in \Sigma$. The proof of Proposition 3 shows that the center of $\mathcal{P}_\nu$ coincides with the subjective probability m in Definition 3. This demonstrates the uniqueness of m, that we already noted in the example with only two states of the world. The uniqueness of m is consistent with Siniscalchi's observation that the baseline prior is behaviorally identified from the decision-maker's preferences over complementary acts when the complementary independence axiom holds. Scalar α and set function a in Definition 3 are not unique.[8]

2.4 Relationship with JP-capacities

We will now discuss how MP-capacities are related to other classes of capacities. For capacity ν with dual capacity $\overline{\nu}$, we obtain that ν is convex if and only if $\overline{\nu}$ is concave. We now introduce a class of capacities that was first studied by Jaffray and Philippe (JP, 1997).

Definition 5 (JP-capacity)**.** A capacity ν on Σ is a JP-capacity if there exists a convex capacity w and a scalar $\alpha \in [0,1]$ such that $\nu = \alpha w + (1-\alpha)\overline{w}$.

A JP-capacity is a convex combination of a convex capacity and its dual. Jaffray and Philippe (1997) derive this class by requiring consistency between preferences under uncertainty modeled

[8] If (m, a_1, α_1) and (m, a_2, α_2) define the same MP-capacity ν, then $(\frac{1}{2} - \alpha_1)a_1(E) = (\frac{1}{2} - \alpha_2)a_2(E)$ for all $E \in \Sigma$. If there is at least one ambiguous event, then $\alpha_1 \geq 1/2$ if and only if $\alpha_2 \geq 1/2$, and $\alpha_1 \leq 1/2$ if and only if $\alpha_2 \leq 1/2$. Furthermore a_1 and a_2 only differ by a scalar.

by CEU and preferences under imprecise risk with objective upper and lower probabilities. Under this consistency requirement the capacity must be a convex combination of the upper and lower probabilities of the imprecisely probabilized events. Under certain conditions, this yields subjective upper and lower probabilities on all events. MP-capacities are a subclass of JP-capacities.

Proposition 4. *Let the triplet (m, a, α) define the MP-capacity ν according to Definition 3. Then $w = m - a/2$ is a convex capacity and $\nu = \alpha w + (1-\alpha)\overline{w}$ so that ν is a JP-capacity. Furthermore $m = (w + \overline{w})/2$ and $a = \overline{w} - w$.*

We prove Proposition 4 in Appendix A.4. The relationship between ν, w, m and a provides further intuition for the terminology mean-preserving. For an event E, $\nu(E)$ lies in the interval $[w(E), 1 - w(E^c)]$, $a(E)$ is the length of this interval and $m(E)$ its midpoint. So $a(E)$ measures the discrepancy between the highest and the lowest possible probability of E. The value of $a(E)$ indicates the perceived level of ambiguity surrounding the subjective likelihood of E. It is Marinacci's (2000) measure of vagueness for convex capacity w. When we increase $a(E)$, the range of possible probabilities of E becomes larger while the center of the interval $m(E)$ remains unaffected. This symmetry explains the term "mean-preserving"; MP-capacities allow us to manipulate the level of ambiguity without affecting the "mean" probability m.

When the CEU representation yields a JP-capacity ν, the underlying preferences have an alpha-MEU form. For $\alpha = 1$ we have $\nu = w$ and preferences are ambiguity averse. Schmeidler's (1986) Proposition 3 shows that the CEU of act f with respect to w is then given by the lowest expected utility of f over the core of w,

$$\int_\Omega U(f)\,\mathrm{d}w = \min_{m \in \mathcal{P}_w} \mathbb{E}_m U(f).$$

If $\alpha = 0$, the JP-capacity is concave and the core of its dual is nonempty. In this case it represents ambiguity loving preferences. The CEU of act f with respect to the dual capacity $\overline{w}$ is given by the highest expected utility of f over the core of w,

$$\int_\Omega U(f)\,\mathrm{d}\overline{w} = \max_{m \in \mathcal{P}_w} \mathbb{E}_m U(f).$$

Ambiguity lies in the fact that $\mathcal{P}_w$ may not be a singleton. When ν is a JP-capacity, the decision-maker's CEU of act f in Eq. (1) admits an alpha-MEU form,

$$V(f) = \alpha \min_{m \in \mathcal{P}_w} \mathbb{E}_m U(f) + (1-\alpha) \max_{m \in \mathcal{P}_w} \mathbb{E}_m U(f). \tag{7}$$

Per Proposition 4 also CEU representations with an MP-capacity have an alpha-MEU form because MP-capacities are a subclass of JP-capacities.

While it is easy to see that JP-capacities represent ambiguity aversion for $\alpha = 1$ and ambiguity loving for $\alpha = 0$, it is less clear how to obtain ambiguity neutrality. One might think that $\alpha = 1/2$ always represents ambiguity neutrality but this is not true. This difficulty is related to Klibanoff et al.'s remark on p. 1873 that *"the extent of separation of ambiguity and ambiguity attitude* *achieved in the alpha-MEU model is not strong enough."* We will show that such a separation can be achieved for CEU preferences if we restrict JP-capacities to be mean-preserving. We first show that an ambiguity neutral JP-capacity has $\alpha = 1/2$.

Proposition 5. *Let ν be a JP-capacity on Σ and assume that there is a nontrivial event $E \in \Sigma_{nt}$ with $\underline{w}(E) < \overline{w}(E)$. If the decision-maker is ambiguity neutral, then $\alpha = 1/2$.*

We provide a proof in Appendix A.5. The converse of Proposition 5 is not true because not all JP-capacities with $\alpha = 1/2$ are ambiguity neutrality. We will provide an explicit example in the next section. But if $\alpha = 1/2$ does not represent ambiguity neutrality, there is no unique value of α for which criteria (1) and (2) are equivalent when ambiguity is present. In this case α alone does not represent the decision-maker's ambiguity attitude because it is not the only component of the JP-capacity ν that governs ambiguity attitude. This undermines the separation of ambiguity and ambiguity attitude and complicates economic analysis. The class of JP-capacities is too broad to obtain a clean distinction between ambiguity and ambiguity attitude.

To characterize ambiguity attitude via α, we would like to have a unique value of α that represents ambiguity neutrality. Proposition 5 implies that $\alpha = 1/2$ is the *only possible candidate* for ambiguity neutrality in situations with a non-unique prior. We know from Proposition 1(*ii*) and (*iii*) that MP-capacities are ambiguity neutral if and only if $\alpha = 1/2$. We will now show that any JP-capacity for which $\alpha = 1/2$ implies ambiguity neutrality is already mean-preserving.

Proposition 6. *Let ν be a JP-capacity; if $\alpha = 1/2$ implies ambiguity neutrality, then ν is an MP-capacity.*

We provide a proof in Appendix A.6. Rogers and Ryan (2012) relate ambiguity neutrality of alpha-MEU preferences for $\alpha = 1/2$ to central symmetry of the associated set of probabilities. Given the link between alpha-MEU and CEU for JP-capacities, their result together with Proposition 3 renders the mean-preserving form for the capacity. While any ambiguity neutral JP-capacity exhibits $\alpha = 1/2$ per Proposition 5, some JP-capacities with $\alpha = 1/2$ are not ambiguity neutral. The focus on MP-capacities excludes them. MP-capacities are precisely those JP-capacities for which $\alpha = 1/2$ is equivalent to ambiguity neutrality.

2.5 Relationship with neo-additive capacities

Chateauneuf et al. (2007) introduce non-extreme-outcome-additive (neo-additive) capacities to model optimistic and pessimistic attitudes towards uncertainty under CEU. A neo-additive capacity is characterized by a triplet (m, δ, α) with $m \in \Delta(\Sigma)$ and $\delta, \alpha \in [0, 1]$ such that

$$\nu(E) = (1-\delta)m(E) + (1-\alpha)\delta \quad \text{for } E \in \Sigma_{nt},$$

$\nu(\emptyset) = 0$ and $\nu(\Omega) = 1$. Parameter α measures the degree of pessimism for neo-additive capacities. The CEU of act f in Eq. (1) then takes the following form:

$$V(f) = (1-\delta)\mathbb{E}_m U(f) + \delta\Big[\alpha \min_{x \in f(\Omega)} U(x) + (1-\alpha) \max_{x \in f(\Omega)} U(x)\Big].$$

Parameters δ and α are often interpreted as ambiguity and ambiguity attitude.

MP-capacities and neo-additive capacities have in common that they are subclasses of JP-capacities, see Proposition 4 in this paper and Remark 3.2 in Chateauneuf et al. (2007) or Eichberger et al. (2012). A neo-additive capacity ν can be written as $\alpha w + (1-\alpha)\overline{w}$ with convex capacity $w = (1-\delta)m + \delta \cdot \mathbf{1}_{\{\Omega\}}$, where $\mathbf{1}_{\{\Omega\}}$ denotes the indicator function of Ω. This implies that parameter α identifies comparative ambiguity aversion in the sense of Definition 2 because an increase in α uniformly lowers a given JP-capacity across all events.[9] An increase in α represents greater ambiguity aversion and a decrease in α greater ambiguity loving for MP-capacities and also for neo-additive capacities.

There are several differences between the two classes of capacities. The level of ambiguity is constant for neo-additive capacities, and parameter δ measures the length of the interval $[w(E), \overline{w}(E)]$ for any $E \in \Sigma_{nt}$, where w denotes the convex capacity in the JP-form of neo-additive capacity ν. For MP-capacities the level of ambiguity may vary across events. Second, for neo-additive capacities there is no unique value of α that represents ambiguity neutrality in the sense of Definition 1. A neo-additive capacity with $\alpha = 1/2$ is ambiguity neutral if and only if $\delta = 0$.[10] But then ν is a subjective probability regardless of the value of α. This reasoning illustrates that not all JP-capacities with $\alpha = 1/2$ are ambiguity neutral: take a neo-additive capacity with $\alpha = 1/2$ and $\delta > 0$. In contrast MP-capacities are ambiguity neutral if and only if $\alpha = 1/2$. Third, neo-additive capacities are in general neither convex nor concave, see Remark 3.2 in Chateauneuf et al. (2007), whereas MP-capacities are either convex or concave, see Proposition 1(*ii*). Chateauneuf et al. (2007) emphasize that neo-additive capacities exhibit uncertainty aversion for some events and uncertainty preference for other events. So while MP-capacities and neo-additive capacities both characterize *comparative* ambiguity aversion via α, they differ when it comes to identifying ambiguity attitude *per se*.

The difference between mean-preserving and neo-additive capacities becomes particularly apparent in the following proposition.

Proposition 7. *Consider a state space with at least three states of the world. A capacity that is both mean-preserving and neo-additive is a subjective probability.*

We give a proof in Appendix A.7. The class of mean-preserving capacities and neo-additive capacities intersect on the set of subjective probabilities. We conclude that both approaches extend SEU in different directions with different purposes in mind.

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[9] For $E \in \Sigma$, we obtain $\nu(E) = \alpha w(E) + (1-\alpha)\overline{w}(E) = (1 - w(E^c)) - \alpha[\overline{w}(E) - w(E)]$, where the square bracket is nonnegative from the convexity of w. So $\nu(E)$ is decreasing in α.

[10] The proof is similar to the proof of Proposition 7 in Appendix A.7 and requires a state space with at least three states of the world. With a binary state space every capacity is mean-preserving and neo-additive.

3 Applications

3.1 Preliminaries

To show the usefulness of CEU with MP-capacities we will now apply them to various economic problems of interest. We consider the value of information, portfolio choice, self-insurance and self-protection, the value of a statistical life and precautionary saving. In each application we discuss the comparative static effects of ambiguity attitude, the intensity of ambiguity aversion or ambiguity loving and the comparative static effects of greater ambiguity. We compare our results with existing results in the literature, which are often based on the smooth ambiguity model.

3.2 Value of information

In the absence of ambiguity information has nonnegative value because, as already noted by Savage (1954), *"the person is free to ignore the observation."* Information provides an opportunity to adjust behavior without requiring the decision-maker to do so. Camerer and Weber (1992) define ambiguity as *"uncertainty about probability, created by missing information that is relevant and could be known."* We would expect that ambiguity averse decision-makers value information that reduces or eliminates ambiguity and that the informational value increases in the degree of ambiguity aversion and the level of ambiguity.[11]

Let $\mathcal{F}' \subseteq \mathcal{F}$ be the collection of acts considered by the decision-maker in a given choice context. Let (m, a_1, α) define the decision-maker's MP-capacity ν_1. His objective function is

$$V_1(f) = \sum_{x \in f(\Omega)} U(x) \left[m(f = x) + \left(\tfrac{1}{2} - \alpha\right) \cdot \left[a_1(f \geq x) - a_1(f > x)\right]\right],$$

and he solves $\arg\max_{f \in \mathcal{F}'} V_1(f)$. We assume that a maximizer exists in $\mathcal{F}'$ and denote it by f_1^*. It may not be unique. Now consider a decrease in the level of ambiguity by moving to the MP-capacity ν_2 characterized by (m, a_2, α) with $a_2 < a_1$. Let $V_2(f)$ denote the associated objective function and let f_2^* be a maximizer of $V_2(f)$ in $\mathcal{F}'$. It is a simple envelope result that an ambiguity averse decision-maker is better off when the level of ambiguity is lower while an ambiguity loving decision-maker is better off when the level of ambiguity is higher. If $\alpha > 1/2$, then

$$V_2(f_2^*) \geq V_2(f_1^*) > V_1(f_1^*).$$

The first inequality holds because f_2^* maximizes $V_2(f)$ over $\mathcal{F}'$. The second inequality follows because $a_2 < a_1$ implies $\nu_2 > \nu_1$ for $\alpha > 1/2$ and because the Choquet integral is monotonic with respect to the capacity (see Denneberg, 1994; Wang and Klir, 1997). If the decision-maker

[11] We focus on situations where the choice set is unaffected by information acquisition. This may be violated, for example, for genetic testing in insurance where unfavorable test results may lead to more expensive contracts or rejection. Information may then have negative private value (see Doherty and Thistle, 1996; Peter et al., 2017).

is ambiguity loving ($\alpha < 1/2$), similar arguments show that

$$V_2(f_2^*) < V_1(f_2^*) \leq V_1(f_1^*).$$

Define the value of information that reduces ambiguity, denoted by $r \in \mathbb{R}$, as the willingness to pay to lower ambiguity from a_1 to a_2. For $f \in \mathcal{F}$, let $f - r$ denote the act that yields $f(\omega) - r$ for $\omega \in \Omega$. The value of information that reduces ambiguity is implicitly given by $V_2(f_2^* - r) = V_1(f_1^*)$, and our discussion establishes the following result.

Proposition 8. *The value of information that reduces ambiguity is positive for ambiguity averse decision-makers and negative for ambiguity loving decision-makers.*

Snow's (2010) Theorem 1 is based on the smooth ambiguity model, and Proposition 8 extends it to CEU with MP-capacities. Ambiguity averse decision-makers are willing to pay for information that reduces ambiguity whereas ambiguity loving decision-makers are willing to pay to avoid receiving such information. The value of information that reduces ambiguity may not be increasing in the degree of ambiguity aversion. A given level of ambiguity reduces the welfare of a more ambiguity averse decision-maker by more than that of a less ambiguity averse decision-maker. However, the value of information that reduces ambiguity is derived from the comparison of two different ambiguous situations. It is then possible that the change from a high to a low level of ambiguity has a stronger effect on the less ambiguity averse decision-maker than the more ambiguity averse decision-maker. There can be situations where only the less ambiguity averse decision-maker adjusts his behavior in response to the change in the level of ambiguity.

More can be said about the value of information that resolves ambiguity (i.e., $a_2 = 0$). Proposition 8 informs about its sign but the special case admits additional results.

Proposition 9. *The value of information that resolves ambiguity is:*

(i) Increasing (decreasing) with greater ambiguity aversion (ambiguity loving);

(ii) Increasing (decreasing) with greater ambiguity for ambiguity averters (ambiguity lovers).

Proposition 9 extends Snow's (2010) Theorems 2 and 3. The absence of ambiguity is represented by MP-capacity $\nu_0 = (m, 0, \alpha)$. If f_0^* denotes a maximizer of $V_0(f)$ over $\mathcal{F}'$, the value of information that resolves ambiguity is implicitly defined via $V_0(f_0^* - r) = V_1(f_1^*)$. An increase in ambiguity aversion decreases the right-hand side of this equation but leaves its left hand-side unchanged. But then r needs to increase to restore the equality. The reasoning is analogous for ambiguity loving decision-makers and changes in the level of ambiguity. Information that resolves risk has the same comparative statics as stated in Proposition 9 because it also resolves ambiguity. Under expected utility the value of information that resolves risk is not necessarily greater in riskier environments (see Gould, 1974) and is not necessarily greater for more risk averse agents (Blair and Romano, 1988).

We replicate Snow's results for the smooth ambiguity model in the case where preferences have a CEU representation with MP-capacity. This demonstrates their robustness. Ambiguity loving can explain information avoidance (see Golman et al., 2017) especially at high levels of ambiguity. Some papers argue that information acquisition may expose decision-makers to ambiguity. Take the example of a genetic test, which introduces message uncertainty because of uncertain test results. Ambiguity averse decision-makers may refrain from testing to avoid message uncertainty (Hoy et al., 2014). Recently Nocetti (2018) proposes a model that contains Snow's and Hoy et al.'s as special cases. He distinguishes between the multiplicity of posterior beliefs arising from the acquisition of a message service (unconditional ambiguity) and the ambiguity faced by a decision-maker once information is received (conditional ambiguity). The extension of his framework to MP-capacities is an interesting topic for further research.

3.3 Portfolio choice

The standard portfolio problem dates back to Arrow (1963) and Pratt (1964) and has received continued attention in the literature (see chapter 4 in Gollier, 2001, for a survey). A decision-maker faces the choice between a safe asset and an uncertain asset. The return of the safe asset is normalized to zero while the return of the uncertain asset is represented by random variable z. It takes values from the bounded interval $[z_-, z_+]$, with $z_- < 0 < z_+$, and we assume without loss of generality that the finite state space orders the possible returns in ascending order, $z_1 < \cdots < z_n$, where z_i is the return in state ω_i. If the decision-maker invests ρ in the uncertain asset, his final wealth will be $W_0 + \rho z_i$ if state ω_i prevails.

Let U denote the decision-maker's utility function and let the triplet (m, a, α) characterize his MP-capacity ν. The objective is then given by

$$V(\rho; a, \alpha) = \sum_{i=1}^{n} U(W_0 + \rho z_i) \left[m_i + \left(\tfrac{1}{2} - \alpha\right) \left[a(\omega_i^{\cup}) - a(\omega_{i+1}^{\cup})\right]\right],$$

where $\omega_i^{\cup}$ is shorthand for $\{\omega_i, \ldots, \omega_n\}$ with the convention that $\omega_{n+1}^{\cup} = \emptyset$. The decision-maker solves $\arg\max_\rho V(\rho; a, \alpha)$, and we denote his optimal investment by ρ^*. If U is concave, V is concave in ρ. There is a positive demand for the uncertain asset if and only if $V_\rho(0; a, \alpha) > 0$, where the subscript stands for the partial derivative. We find that

$$\begin{aligned} V_\rho(0; a, \alpha) &= U'(W_0) \cdot \left(\sum_{i=1}^{n} m_i z_i + (\tfrac{1}{2} - \alpha) \sum_{i=1}^{n} z_i \left(a(\omega_i^{\cup}) - a(\omega_{i+1}^{\cup})\right) \right) \\ &= U'(W_0) \cdot \left(\underbrace{\sum_{i=1}^{n} m_i z_i}_{\text{equity premium}} \underbrace{+ (\tfrac{1}{2} - \alpha) \sum_{i=1}^{n-1} a(\omega_{i+1}^{\cup}) (z_{i+1} - z_i)}_{\text{ambiguity adjustment}} \right). \end{aligned}$$

This shows the following result.

Proposition 10. *The decision-maker invests a positive (resp. negative) amount in the uncertain asset if and only if the ambiguity-adjusted equity premium is positive (resp. negative). The ambiguity-adjusted equity premium is:*

(i) Decreasing (increasing) with greater ambiguity aversion (ambiguity loving);

(ii) Decreasing (increasing) with greater ambiguity for ambiguity averters (ambiguity lovers).

An increase in the degree of ambiguity aversion makes it less likely for the decision-maker to have a positive demand for the uncertain asset. The same is true for greater ambiguity when the decision-maker is ambiguity averse. This is in contrast to Gollier's (2011) Lemma 1, where the demand for the ambiguous asset is positive (zero, negative) if the unadjusted equity premium is positive (zero, negative). In our case the equity premium is part of the decision-maker's consideration but he also takes ambiguity into account, which distorts the criterion.

The reason for this difference is that Gollier (2011) uses the smooth ambiguity model, which is second-order ambiguity averse (see Lang, 2016). Then, a positive exposure to ambiguity is optimal if and only if there is a subjective belief such that the act's expected outcome is positive. Lang (2016) does not consider α-MEU preferences because not all such preferences admit an uncertainty-averse representation in the sense of Cerreia-Vioglio et al. (2011). According to Proposition 10 the behavior under CEU with MP-capacities is in line with first-order ambiguity aversion because ambiguity averse decision-makers prefer not to invest in the uncertain asset for some range of positive equity premia. The size of that range increases with their degree of ambiguity aversion and the level of ambiguity.[12]

To characterize investment behavior at the intensive margin, we derive the first-order condition for the decision-maker's objective function,

$$V_\rho(\rho^*; a, \alpha) = \sum_{i=1}^{n} z_i U'(W_0 + \rho^* z_i) + (\tfrac{1}{2} - \alpha) \sum_{i=1}^{n} z_i U'(W_0 + \rho^* z_i) \left(a(\omega_i^{\cup}) - a(\omega_{i+1}^{\cup}) \right) = 0,$$

and rearrange the second term as follows:

$$(\tfrac{1}{2} - \alpha) \sum_{i=1}^{n-1} a(\omega_{i+1}^{\cup}) \left[z_{i+1} U'(W_0 + \rho^* z_{i+1}) - z_i U'(W_0 + \rho^* z_i) \right].$$

As long as the square bracket is nonnegative, we can sign the comparative static effects of α and a on ρ^*. If $-WU''(W)/U'(W)$ denotes relative risk aversion, we find the following.

Proposition 11. *Assume the decision-maker's relative risk aversion is bounded by unity; then, his demand for the uncertain asset is:*

(i) Decreasing (increasing) in the degree of ambiguity aversion (ambiguity loving);

(ii) Decreasing (increasing) in the level of ambiguity for ambiguity averters (ambiguity lovers).

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[12] Our result is similar in spirit to Mukerji and Tallon's no trade result for ambiguity aversion with CEU preferences and idiosyncratic risk. Unlike the smooth ambiguity model, CEU preferences provide an ambiguity-based explanation for the nonparticipation puzzle. Empirical evidence supports the negative link between ambiguity aversion and nonparticipation in equities (see Dimmock et al., 2016).

The statement about ambiguity aversion in Result (*i*) extends Gollier's Proposition 2(1) to CEU. While Gollier requires relative risk aversion below unity and that priors are ranked according to first-order stochastic dominance (FSD), we only need the first assumption.[13] In our model the result can be easily extended to ambiguity loving decision-makers, who turn out to exhibit the reverse behavior. To sign the effects of changes in the level of ambiguity, Huang and Tzeng (2018) require relative prudence in ambiguity to be bounded by two in addition to Gollier's assumptions. Greater ambiguity then lowers the demand for the uncertain asset but no definitive result exists for relative prudence in ambiguity above two.[14] In our model greater ambiguity has the intuitive effect to lower demand for ambiguity averse decision-makers and to increase it for ambiguity loving decision-makers without any additional restrictions. MP-capacities simplify the comparative statics of greater ambiguity and yield intuitive results. Greater ambiguity aversion and greater ambiguity may lead to a higher demand for the uncertain asset in the smooth ambiguity model. This paradox does not arise under CEU with MP-capacities.

Let us provide some intuition for our results. Let

$$\widehat{m}_i = m_i + (\tfrac{1}{2} - \alpha)\left(a(\omega_i^{\cup}) - a(\omega_{i+1}^{\cup})\right)$$

denote the multiplier associated with utility $U(w_0+\rho z_i)$; we can then rewrite the objective function as $\sum_{i=1}^{n} \widehat{m}_i U(w_0 + \rho z_i)$. We can interpret $\widehat{m}_i$ as a distorted probability or decision weight for state ω_i. Property (*iii*) in Definition 3 implies

$$|a(\omega_i^{\cup}) - a(\omega_{i+1}^{\cup})| \leq 2(m(\omega_i^{\cup}) - m(\omega_{i+1}^{\cup})) = 2m_i,$$

with a strict inequality for $i = 1$ and $i = n$. These inequalities together with $\alpha \in [0, 1]$ ensure that the $\widehat{m}_i$ are nonnegative.[15] It also holds that

$$\sum_{i=1}^{n} \widehat{m}_i = \underbrace{\sum_{i=1}^{n} m_i}_{=1} + (\tfrac{1}{2} - \alpha)\left(\underbrace{\sum_{i=1}^{n} a(\omega_i^{\cup}) - \sum_{i=1}^{n} a(\omega_{i+1}^{\cup})}_{=0}\right) = 1.$$

This justifies our interpretation of the $\widehat{m}_i$ as distorted probabilities. If $M_i = \sum_{k=1}^{i} m_i$ denotes the cumulative probabilities for subjective probability m and likewise $\widehat{M}_i$ for subjective probability $\widehat{m}$,

[13] The overall evidence is more supportive of relative risk aversion above unity, see Meyer and Meyer (2005). The terms $z_i U'(w_0 + \rho^* z_i)$ are not necessarily ordered in that case and the comparative statics are indeterminate. Jouini et al. (2013) provide necessary and sufficient conditions for clear-cut risk effects under SEU.

[14] Baillon (2017) defines ambiguity prudence akin to Eeckhoudt and Schlesinger's (2006) behavioral definition of risk prudence based on simple lotteries. He analyzes ambiguity prudence in different ambiguity models. Berger and Bosetti's (2020) estimates suggest relative prudence in ambiguity below two but more research is needed.

[15] If $\alpha < 1/2$ and $a(\omega_i^{\cup}) - a(\omega_{i+1}^{\cup}) \geq 0$, then $\widehat{m}_i > m_i \geq 0$; if $\alpha < 1/2$ and $a(\omega_i^{\cup}) - a(\omega_{i+1}^{\cup}) < 0$, then $\widehat{m}_i > m_i + (\frac{1}{2} - \alpha) \cdot (-2m_i) = 2\alpha \cdot m_i \geq 0$; if $\alpha > 1/2$ and $a(\omega_i^{\cup}) - a(\omega_{i+1}^{\cup}) < 0$, then $\widehat{m}_i > m_i \geq 0$; finally, if $\alpha > 1/2$ and $a(\omega_i^{\cup}) - a(\omega_{i+1}^{\cup}) \geq 0$, then $\widehat{m}_i > m_i + (\frac{1}{2} - \alpha) \cdot 2m_i = 2(1 - \alpha) \cdot m_i \geq 0$.

it follows that

$$\widehat{M}_i = M_i - (\tfrac{1}{2} - \alpha)a(\omega^{\cup}_{i+1}).$$

$\widehat{M}_i$ and M_i differ by a positive term under ambiguity aversion and a negative term under ambiguity loving. The behavior of an ambiguity averse decision-maker is then observationally equivalent to the behavior of an SEU decision-maker who has deteriorated his subjective belief in the sense of FSD compared to m. Under ambiguity loving the distortion is an FSD improvement. Cheng et al. (1987) and Hadar and Seo (1990) analyze the effect of FSD shifts in the return distribution on investment behavior and identify the threshold value of one on relative risk aversion.

According to Proposition 11 ambiguity aversion can explain a low fraction of financial assets in stocks (Dimmock et al., 2016) and low foreign stock ownership if decision-makers feel more confident about the distributions of domestic versus foreign stocks (see also Bianchi and Tallon, 2019; Asano and Osaki, 2020). Due to the analogy between the portfolio problem and the coinsurance problem (see Schlesinger, 2013, p.170), Propositions 10 and 11 also have implications for insurance demand. Ambiguity aversion explains a demand for full insurance even for actuarially unfavorable prices. Lab experiments reveal a strong preference for full insurance (Shapira and Venezia, 2008) and in the field people are willing to pay large markups to lower their deductible (Sydnor, 2010). Mossin's Theorem holds in the smooth ambiguity model, and ambiguity averse decision-makers do not buy full insurance at unfair prices (see Alary et al., 2013; Bajtelsmit et al., 2015). CEU with MP-capacities can rationalize demand for full insurance at unfair prices

3.4 Self-insurance

Ehrlich and Becker (1972) introduced the distinction between self-insurance, a costly investment to reduce the severity of loss, and self-protection, a costly investment to reduce the probability of loss. Both activities mitigate risk but their comparative statics reveal marked dissimilarities. We will analyze the marginal willingness to pay (WTP) and optimal demand for each activity.

Willingness to pay. We use a simple binary loss model. Assume the decision-maker has initial wealth w_0, which is subject to a loss of $\ell < W_0$. His subjective probability of loss is p but he faces ambiguity represented by an MP-capacity. The level of ambiguity is $a \in (0, 2\min\{p, 1-p\})$, and $\alpha \in [0,1]$ denotes his degree of ambiguity aversion. His CEU is given by

$$V = \left[p - a\left(\tfrac{1}{2} - \alpha\right)\right] U(w_0 - \ell) + \left[1 - p + a\left(\tfrac{1}{2} - \alpha\right)\right] U(w_0).$$

$P(y)$ denotes the willingness to pay for an increase in wealth in the loss state by y:

$$\left[p - a\left(\tfrac{1}{2} - \alpha\right)\right] U(W_0 - \ell + y - P(y)) + \left[1 - p + a\left(\tfrac{1}{2} - \alpha\right)\right] U(W_0 - P(y)) = V.$$

So $P(0) = 0$ and a marginal increase in y is valued according to

$$P'(0) \quad = \quad \frac{\left[p - a\left(\tfrac{1}{2} - \alpha\right)\right] U'(W_0 - \ell)}{\left[p - a\left(\tfrac{1}{2} - \alpha\right)\right] U'(W_0 - \ell) + \left[1 - p + a\left(\tfrac{1}{2} - \alpha\right)\right] U'(W_0)} \tag{8}$$

$$= \left[1 + \frac{1-p+a\left(\frac{1}{2}-\alpha\right)}{p-a\left(\frac{1}{2}-\alpha\right)} \cdot \frac{U'(W_0)}{U'(W_0-\ell)}\right]^{-1}. \tag{9}$$

We can then derive the following result.[16]

Proposition 12. *The marginal WTP for self-insurance is:*

(i) Increasing (decreasing) in the degree of ambiguity aversion (ambiguity loving);

(ii) Increasing (decreasing) in the level of ambiguity for ambiguity averters (ambiguity lovers).

Result (*i*) for ambiguity averse decision-makers extends Alary et al.'s Proposition 1 to CEU. They consider more than two states and have to make an additional assumption on certainty equivalent wealth. Result (*i*) for ambiguity loving decision-makers and Result (*ii*) are new. Ambiguity increases the marginal WTP for self-insurance at the extensive and the intensive margin. Self-insurance shrinks the utility difference between the loss-state and the no-loss state. A higher level of ambiguity affects a self-insured decision-maker less than a decision-maker who is not self-insured. This makes self-insurance more valuable under greater ambiguity. Effects are reversed for ambiguity loving decision-makers. They have a lower marginal WTP for self-insurance than ambiguity neutral decision-makers and even more so the higher their degree of ambiguity loving. They appreciate being exposed to uncertainty, and without self-insurance ambiguity has a stronger effect on their welfare than when self-insured. This dichotomy can lead to situations where the aggregate effect of ambiguity is not perceptible because positive effects for ambiguity averse decision-makers are offset by negative effects for ambiguity loving decision-makers. Recent experimental evidence confirms such an aggregation bias (see Couture et al., 2019).

In the expected utility model the term $U'(W_0)/U'(W_0-\ell)$ in Eq. (9) is negatively associated with the degree of risk aversion (see Pratt, 1964). We can interpret Proposition 12 by saying that ambiguity aversion reinforces risk aversion while ambiguity loving attenuates it. This interaction is not linear. Changes in the ambiguity parameters have a stronger impact on $P'(0)$ when utility curvature is low. Ambiguity plays a greater role when risk aversion is moderate.

Optimal demand. We now analyze the demand for self-insurance. Let $y \in [0, \overline{y}]$ denote the level of the self-insurance activity. The loss is decreasing in y at a non-decreasing rate, $\ell(y) \geq 0$ with $\ell' < 0$ and $\ell'' \geq 0$, and self-insurance involves an upfront cost of $c(y)$, which is increasing and non-concave, $c' > 0$ and $c'' \geq 0$. Final wealth levels are $W_L = W_0 - \ell(y) - c(y)$ in the loss-state and $W_N = W_0 - c(y)$ in the no-loss state. Under ambiguity the decision-maker solves

$$\max_{y \in [0,\overline{y}]} V(y) = \left[p - a\left(\tfrac{1}{2}-\alpha\right)\right] U(W_L) + \left[1-p+a\left(\tfrac{1}{2}-\alpha\right)\right] U(W_N)$$

[16] The result also holds for large reductions in loss severity and not only infinitesimal ones. We state the result for the marginal WTP to facilitate a comparison with the literature.

with first-order condition

$$\left(-l'(y^*) - c'(y^*)\right)\left[p - a\left(\tfrac{1}{2} - \alpha\right)\right]U'(W_L) - c'(y^*)\left[1 - p + a\left(\tfrac{1}{2} - \alpha\right)\right]U'(W_N) = 0 \qquad (10)$$

for an interior solution y^*. The second-order condition holds. The next proposition summarizes.

Proposition 13. *The optimal level of self-insurance is:*

(i) Increasing (decreasing) in the degree of ambiguity aversion (ambiguity loving);

(ii) Increasing (decreasing) in the level of ambiguity for ambiguity averters (ambiguity lovers).

The proof is obtained from the Implicit Function Rule. The intuition derives from the effect of ambiguity on the marginal benefit and the marginal cost of self-insurance. The marginal benefit arises from the increase in consumption utility in the loss state,

$$MB(y) = -l'(y)\left[p - a\left(\tfrac{1}{2} - \alpha\right)\right]U'(W_L).$$

Ambiguity aversion raises the marginal benefit, and greater ambiguity raises it for ambiguity averse decision-makers and diminishes it for ambiguity loving decision-makers. The marginal cost measures the reduction in consumption utility from self-insurance expenditures,

$$MC(y) = c'(y)\left(\left[p - a\left(\tfrac{1}{2} - \alpha\right)\right]U'(W_L) + \left[1 - p + a\left(\tfrac{1}{2} - \alpha\right)\right]U'(W_N)\right).$$

Ambiguity aversion raises the marginal cost because marginal utility in the loss state is higher than in the no-loss state. Similarly, greater ambiguity raises the marginal cost for ambiguity averse decision-makers and diminishes it for ambiguity loving decision-makers. The net effect is therefore *a priori* indeterminate. But $MB(y^*) = MC(y^*)$ for an interior maximizer y^* so that

$$\begin{aligned} & a\left(\tfrac{1}{2} - \alpha\right)\left[l'(y^*)U'(W_L) - c'(y^*)\left(U'(W_N) - U'(W_L)\right)\right] \\ = \; & p(l'(y^*) + c'(y^*))U'(W_L) + (1-p)c'(y^*)U'(W_N). \end{aligned}$$

The right-hand side is the optimality condition for self-insurance under SEU evaluated at y^*. When $\alpha > 1/2$, first-order condition (10) implies that it is positive, and the effect of ambiguity on the marginal benefit preponderates its effect on the marginal cost. Therefore, self-insurance increases with greater ambiguity for ambiguity averse decision-makers. Likewise, when $\alpha < 1/2$, first-order condition (10) implies that the optimality condition under SEU is negative at y^*. Then the effect of ambiguity on the marginal benefit exceeds its effect on the marginal cost. This explains why self-insurance is decreasing with greater ambiguity for ambiguity loving decision-makers.

Our result for ambiguity averse decision-makers in Proposition 13(*i*) extends Snow's Proposition 2 and Alary et al.'s Proposition 2 to CEU. Our Result (*ii*) shows that Snow's Proposition 1 continues to hold at the intensive margin. Such an extension may fail to hold under the smooth ambiguity model (see Jewitt and Mukerji, 2017). For example, when using Neilson's simplified axiomatization of the smooth ambiguity model, Huang and Tzeng (2018) find that relative prudence in ambiguity preferences below 2 ensures that a second-degree increase in ambiguity lowers the optimal exposure to uncertainty in the standard portfolio problem, see their Corollary 1. The same restriction ensures higher insurance demand in the coinsurance problem under ambiguity or a higher demand for self-insurance. If preferences do not satisfy this condition, a countervailing Giffen-type effect may predominate, giving rise to the perplexing result that ambiguity averse decision-makers may decrease their optimal level of self-insurance as the level of ambiguity rises. This complication does not arise under CEU with MP-capacities. The bracketed statements in Results (*i*) and (*ii*) extend the analysis symmetrically to ambiguity loving decision-makers. They exhibit the reverse behavior compared to ambiguity averse decision-makers.

3.5 Self-protection

We will now investigate self-protection activities.

Willingness to pay. We modify the simple binary loss model from the previous section. Let $e \geq 0$ denote a reduction in the probability of loss and $P(e)$ the decision-maker's WTP for such a reduction. We obtain it from the following indifference condition:

$$\left[p - e - a\left(\tfrac{1}{2} - \alpha\right)\right] U(W_0 - \ell - P(e)) + \left[1 - p + e + a\left(\tfrac{1}{2} - \alpha\right)\right] U(W_0 - P(e)) = V.$$

It follows that $P(0) = 0$, and for $e = 0$ we obtain the marginal WTP as follows:

$$P'(0) = \frac{U(W_0) - U(W_0 - \ell)}{\left[p - a\left(\tfrac{1}{2} - \alpha\right)\right] U'(W_0 - \ell) + \left[1 - p + a\left(\tfrac{1}{2} - \alpha\right)\right] U'(W_0)}. \tag{11}$$

We can then derive our next result.

Proposition 14. *For a risk averse decision-maker (i.e., $U'' < 0$), the marginal WTP for self-protection is:*

(i) Decreasing (increasing) in the degree of ambiguity aversion (ambiguity loving);

(ii) Decreasing (increasing) in the level of ambiguity for ambiguity averters (ambiguity lovers).

For a risk loving decision-maker (i.e., $U'' > 0$), the marginal WTP for self-protection is:

(iii) Increasing (decreasing) in the degree of ambiguity aversion (ambiguity loving);

(iv) Increasing (decreasing) in the level of ambiguity for ambiguity averters (ambiguity lovers).

For a risk-neutral decision-maker (i.e., $U'' = 0$), ambiguity and ambiguity attitude are inconsequential for the marginal WTP for self-protection.

Proposition 14(*i*) for ambiguity averse decision-makers extends Alary et al.'s Proposition 3 to CEU. They consider several states and find that the effect of ambiguity aversion is *a priori* indeterminate. They need to specify restrictions on the decision-maker's utility function and the wealth level in the state that is being self-protected to sign the comparative static. We extend the analysis to ambiguity loving decision-makers and find a reversion of effects. Our framework also yields results for greater ambiguity, which are new in the literature. Results (*iii*) and (*iv*) provide an analysis for risk loving decision-makers. The synopsis of risk and ambiguity attitudes reveals that they co-determine the effect of greater ambiguity on the WTP for self-protection. Table 1 provides an overview.

	Risk averse ($U'' < 0$)	Risk loving ($U'' > 0$)
Ambiguity averse ($\alpha > 1/2$)	negative	positive
Ambiguity loving ($\alpha < 1/2$)	positive	negative

Table 1: Effect of greater ambiguity on the WTP for self-protection

When comparing Proposition 12 and Proposition 14, two observations deserve further explanation. For risk and ambiguity averse decision-makers, greater ambiguity aversion and greater ambiguity have a positive effect on the WTP for self-insurance but a negative effect on the WTP for self-protection. This is because self-insurance and self-protection affect the decision-maker's CEU differently. Self-insurance trades off an increase in consumption in the loss state against lower consumption in either state. Ambiguity and ambiguity aversion increase the marginal value of higher loss-state wealth (numerator in Eq. (8)) but also increase the marginal cost of lower expected consumption (denominator in Eq. (8)). The first effect preponderates the second effect because marginal utility is positive. As a result the marginal WTP for self-insurance is increasing in the degree of ambiguity aversion and the level of ambiguity. Self-protection instead trades off a reduction in the probability of loss against lower consumption in either state. Ambiguity and ambiguity aversion have no effect on the marginal value of a lower loss probability (numerator in Eq. (11)) because the uncertainty is about the level of the loss probability, and a reduction of this probability has the same marginal effect on CEU at whatever level it occurs. The only effect is an increase in the marginal cost (denominator in Eq. (11)) so that greater ambiguity aversion and greater ambiguity lower the marginal WTP for self-protection.

The second observation is the reversal of effects for self-protection when changing risk attitude from risk averse to risk loving. This reversal does not occur in case of self-insurance where no assumption on the sign of U'' is made. For self-insurance, greater ambiguity and greater ambiguity aversion have a negative effect on the marginal cost of lower expected consumption for risk loving decision-makers. They reinforce the positive effect on the marginal value of self-insurance, and the same holds true for self-protection. Changes in the degree of ambiguity attitude and greater ambiguity have the same effect on the marginal WTP for self-insurance and the marginal WTP for self-protection when risk lovers are considered. Risk aversion is a prerequisite for the dichotomy between self-insurance and self-protection when it comes to ambiguity.

Optimal demand. Let $e \in [0, \overline{e}]$ denote the level of the self-protection activity. The probability of loss is decreasing in self-protection at a non-decreasing rate, $p(e) \geq 0$ with $p' < 0$ and $p'' \geq 0$. Self-protection involves an upfront cost of $c(e)$, which we assume increasing and non-concave, $c' > 0$ and $c'' \geq 0$. Wealth levels are $W_L = W_0 - \ell - c(x)$ in the loss-state and $W_N = W_0 - c(x)$ in the no-loss state. To ensure nonnegative consumption, set $\overline{e} = c^{-1}(W_0 - \ell)$. With these specifications the decision-maker's objective is

$$\max_{e \in [0,\overline{e}]} V(e) = \left[p(e) - a(\tfrac{1}{2} - \alpha)\right] U(W_L) + \left[1 - p(e) + a(\tfrac{1}{2} - \alpha)\right] U(W_N).$$

We assume an interior maximizer e^*, which is characterized by the following first-order condition:

$$\begin{aligned} &-p'(e^*)\left[U(W_N) - U(W_L)\right] \\ &-c'(e^*)\left(\left[p(e^*) - a(\tfrac{1}{2} - \alpha)\right] U'(W_L) + \left[1 - p(e^*) + a(\tfrac{1}{2} - \alpha)\right] U'(W_N)\right) = 0. \end{aligned}$$

If the objective function is concave in e, the interior maximizer is unique and corresponds to the global maximizer of the decision-maker's objective.[17] If the objective function is globally concave, the comparative statics hold in the large, meaning for changes in the degree of ambiguity attitude and the level of ambiguity of arbitrary size. If global concavity fails, the results hold at least in the small but may fail to hold in the large. We obtain the following proposition.

Proposition 15. *The optimal level of self-protection is:*

(i) Decreasing (increasing) in the degree of ambiguity aversion (ambiguity loving);

(ii) Decreasing (increasing) in the level of ambiguity for ambiguity averters (ambiguity lovers).

The proof follows easily from the Implicit Function Rule. The marginal benefit of self-protection is not affected by ambiguity, only the marginal cost is. For ambiguity averse decision-makers the marginal cost is higher compared to an ambiguity neutral decision-maker. Uncertainty associated with the probability of loss puts more weight on the loss state where marginal utility is high. This also explains why greater ambiguity raises the marginal cost for ambiguity averse decision-makers and lowers it for ambiguity loving decision-makers.

Result (*i*) for ambiguity averse decision-makers is contrary to Snow's Proposition 3 where he finds a positive effect of ambiguity and greater ambiguity aversion on self-protection. Alary et al. (2013), however, find that greater ambiguity aversion may increase or decrease the optimal level of self-protection, depending how the wealth level in the state where ambiguity is concentrated relates to precautionary equivalent wealth. We find a definitive negative effect so that ambiguity

[17] It has long been recognized that convexity of p and concavity of U are not strong enough for global concavity of the self-protection problem in effort (see Dionne and Eeckhoudt, 1985; Jullien et al., 1999). Fagart and Fluet (2013) show that log-convexity of p and nonincreasing absolute risk aversion are jointly sufficient under SEU. In our framework the two conditions remain jointly sufficient for ambiguity averse decision-makers but may no longer be strong enough for ambiguity loving decision-makers.

and ambiguity aversion explain underprevention (see also Baillon et al., 2019). In our model ambiguity raises the marginal cost of spending money on self-protection for ambiguity averse decision-makers. Eeckhoudt and Gollier (2005) find a negative effect of prudence on self-protection; prudent individuals may prefer to lower self-protection expenditures it increases consumption in the loss state and mitigate downside risk. Proposition 15 does not require assumptions on the sign of U''' because high marginal utility in the loss state is purely driven by risk aversion.

Endogenous ambiguity. To explain why Snow (2011) finds the opposite, we will now endogenize the level of ambiguity. Self-protection affects the decision-maker's subjective probability and may also affect the level of ambiguity. Berger et al. (2016) provide an illustration for catastrophic climate risks and distinguish between a constant, increasing and decreasing degree of model uncertainty (see Figure 1 in their paper and also Hoy, 1989). Let the level of ambiguity be a continuously differentiable function of effort, $a = a(e)$ for $e \in [0, \overline{e}]$. As e changes, both the subjective probability of loss and the level of ambiguity change. We require $a(e) < 2\min\{p(e), 1 - p(e)\}$ for all $e \in [0, \overline{e}]$ for consistency with Definition 3. The decision-maker's objective is now

$$\max_{e \in [0,\overline{e}]} V(e) = \left[p(e) - a(e)(\tfrac{1}{2} - \alpha)\right] U(W_L) + \left[1 - p(e) + a(e)(\tfrac{1}{2} - \alpha)\right] U(W_N),$$

and an interior solution e^* is characterized by the first-order condition

$$\begin{aligned}&\left[-p'(e^*) + a'(e^*)(\tfrac{1}{2} - \alpha)\right] \cdot [U(W_N) - U(W_L)] \\ &-c'(e^*)\left(\left[p(e^*) - a(e^*)(\tfrac{1}{2} - \alpha)\right] U'(W_L) + \left[1 - p(e^*) + a(e^*)(\tfrac{1}{2} - \alpha)\right] U'(W_N)\right) = 0.\end{aligned}$$

With an endogenous ambiguity level, the marginal cost and the marginal benefit of self-protection both depend on the decision-maker's ambiguity attitude. If self-protection reduces the level of ambiguity (i.e., $a' < 0$), the marginal benefit is increasing in the degree of ambiguity aversion and decreasing in the degree of ambiguity loving. This introduces a countervailing effect on optimal self-protection compared to a constant level of ambiguity. If self-protection increases the level of ambiguity (i.e., $a' > 0$), the effects of changes in the degree of ambiguity attitude on the marginal benefit and the marginal cost are aligned. We obtain the following result.

Proposition 16. *Under endogenous ambiguity, the optimal level of self-protection is:*

(i) Decreasing (increasing) in the degree of ambiguity aversion (ambiguity loving) if

$$-\frac{a'(e^*)}{a(e^*)} \leq -c'(e^*)\frac{U'(W_N) - U'(W_L)}{U(W_N) - U(W_L)};$$

(ii) Increasing (decreasing) in the degree of ambiguity aversion (ambiguity loving) if

$$-\frac{a'(e^*)}{a(e^*)} \geq -c'(e^*)\frac{U'(W_N) - U'(W_L)}{U(W_N) - U(W_L)}.$$

To provide some interpretation, we construe the right-hand side as a measure of risk aversion. It follows from the Fundamental Theorem of Calculus that

$$-\frac{U'(W_N) - U'(W_L)}{U(W_N) - U(W_L)} = -\frac{\mathbb{E}U''(\widetilde{W})}{\mathbb{E}U'(\widetilde{W})},$$

where $\widetilde{W}$ is uniformly distributed on $[W_L, W_N]$. If U is exponential or quadratic, the term simplifies to Arrow-Pratt risk aversion. If U is risk vulnerable (see Gollier and Pratt, 1996), Arrow-Pratt risk aversion at the median wealth level $(W_L + W_N)/2$ is a lower bound. The term $-a'(e^*)/a(e^*)$ is the decay rate of the ambiguity level. If ambiguity increases in self-protection, we are in the situation of Proposition 16(*i*) because the left-hand side is negative and the right-hand side is positive. If ambiguity decreases in self-protection, both cases are possible, and an endogenous threshold on the agent's degree of risk aversion demarcates one case from another. Intuitively, if self-protection decreases ambiguity fast enough, greater ambiguity aversion leads to more self-protection while greater ambiguity loving lowers self-protection. If U is exponential with absolute risk aversion $\mathcal{A} > 0$, say $U(W) = 1 - \exp(-\mathcal{A}W)/\mathcal{A}$, the ambiguity level is exponential with decay rate ξ, say $a(x) = a \cdot \exp(-\xi x)$, and the cost function is linear, $c(e) = e$, the condition simplifies to $\xi \leq \mathcal{A}$ for Result (*i*) and $\xi \geq \mathcal{A}$ for Result (*ii*). So self-protection is increasing in the degree of ambiguity aversion if and only if the decay rate of the ambiguity level exceeds absolute risk aversion.

Snow (2011) uses a multiplicative specification for the self-protection technology to analyze the effect of ambiguity and greater ambiguity aversion. Under multiplicative separability, the level of ambiguity decreases as the self-protection investment increases. As shown in Proposition 16, this is a prerequisite for greater ambiguity aversion to increase optimal effort. What's more, the decay rate of the ambiguity level now determines whether self-insurance and self-protection react differently when the degree of ambiguity aversion changes. In the SEU model self-insurance is increasing in risk aversion (see Dionne and Eeckhoudt, 1985) whereas self-protection increases in risk aversion if and only if the probability of loss is below an endogenous threshold value (see Jullien et al., 1999).

3.6 The value of a statistical life

The value of a statistical life (VSL) is a concept to assess the economic value of mortality-reducing safety improvements. For example, the U.S. Department of Transportation uses a number of $9.6 million for the statistical value of a single life according to its latest memorandum from August 2016.[18] To estimate a decision-maker's willingness to pay to reduce mortality risk, economists typically multiply the change in the probability of death by the decision-maker's marginal rate of substitution between wealth and mortality risk (Drèze, 1962; Jones-Lee, 1974; Weinstein et al., 1980). Empirical estimates have been obtained in a number of different contexts (see Viscusi, 1993; Viscusi and Aldy, 2003).

[18] See `https://www.transportation.gov/`.

In terms of our model we distinguish between utility in the alive state, U_A, and utility in the deceased state, U_D. Typical assumptions are $U_A' > 0$ and $U_A'' < 0$ for the alive state, and $U_D' \geq 0$ (so that $U_D' = 0$ is possible) and $U_D'' \leq 0$ for the deceased state. Utility function U_D represents the decision-maker's preferences regarding bequests and consumption during the part of the period he survives. Furthermore $U_A(W) > U_D(W)$ and $U_A'(W) > U_D'(W)$ for all relevant wealth levels (see Eeckhoudt and Hammitt, 2001). If the triplet (m, a, α) defines the decision-maker's MP-capacity, his CEU is given by

$$V = [p - a(\tfrac{1}{2} - \alpha)]U_D(W_0) + [1 - p + a(\tfrac{1}{2} - \alpha)]U_A(W_0).$$

We interpret p as the individual's subjective baseline probability of death. VSL is defined as the marginal rate of substitution between wealth and mortality risk:

$$VSL = \left.\frac{\mathrm{d}W_0}{\mathrm{d}p}\right|_{V=const} = \frac{U_A(W_0) - U_D(W_0)}{[p - a(\frac{1}{2} - \alpha)]U_D'(W_0) + [1 - p + a(\frac{1}{2} - \alpha)]U_A'(W_0)}.$$

The strong analogy with Eq. (11) is directly apparent. Given that $U_A'(W_0) > U_D'(W_0)$, we obtain the following result as a corollary of Proposition 14.

Corollary 1. *The VSL is:*

(i) Increasing (decreasing) in the degree of ambiguity aversion (ambiguity loving);

(ii) Increasing (decreasing) in the level of ambiguity for ambiguity averters (ambiguity lovers).

Result (*i*) for ambiguity averse decision-makers extends Treich's finding based on the smooth ambiguity model to CEU. As usually we find the reverse effect for ambiguity loving decision-makers. Result (*ii*) says that greater ambiguity raises VSL for ambiguity averse decision-makers. In the smooth ambiguity model, greater ambiguity may increase or decrease VSL and restrictions on the decision-maker's ambiguity prudence are necessary to resolve the indeterminacy (see Bleichrodt et al., 2019). Our framework delivers the intuitive comparative static solely based on ambiguity aversion and extends it symmetrically to ambiguity loving.

As in case of optimal self-protection we can endogenize the ambiguity level. Let $a(p)$ denote ambiguity as a function of mortality. When adjusting the objective function accordingly, we find

$$VSL = \left.\frac{\mathrm{d}W_0}{\mathrm{d}p}\right|_{V=const} = \frac{[U_A(W_0) - U_D(W_0)] \cdot \left[1 - a'(p)(\frac{1}{2} - \alpha)\right]}{[p - a(p)(\frac{1}{2} - \alpha)]U_D'(W_0) + [1 - p + a(p)(\frac{1}{2} - \alpha)]U_A'(W_0)}.$$

Straightforward calculations then show the following result.

Proposition 17. *Under endogenous ambiguity, the VSL is:*

(i) Increasing (decreasing) in the degree of ambiguity aversion (ambiguity loving) if

$$\frac{p\,(U_A'(W_0) - U_D'(W_0))}{pU_D'(W_0) + (1-p)U_A'(W_0)} \geq -p\frac{a'(p)}{a(p)};$$

(ii) Decreasing (increasing) in the degree of ambiguity aversion (ambiguity loving) if

$$\frac{p\left(U'_A(W_0) - U'_D(W_0)\right)}{pU'_D(W_0) + (1-p)U'_A(W_0)} \leq -p\frac{a'(p)}{a(p)}.$$

If ambiguity is increasing in mortality (i.e., $a' > 0$), then mortality reductions are accompanied by less perceived ambiguity. In this case, the right-hand side is negative and Result (*i*) applies, extending our benchmark result with a fixed ambiguity level. However, if ambiguity is decreasing in mortality (i.e., $a' < 0$), both cases are possible. The necessary and sufficient condition compares the mortality-elasticity of expected marginal utility and the mortality-elasticity of the ambiguity level. If ambiguity increases faster than expected marginal utility as mortality goes down, higher ambiguity aversion lowers the VSL.

With endogenous ambiguity, mortality reductions have two effects. As mortality decreases, the VSL decreases because the expected marginal utility-cost of funds increases. Pratt and Zeckhauser (1996) call this the "dead-anyway" effect; higher mortality reduces the utility cost of funds spent on risk reduction since they are more likely to be drawn from the state with low marginal utility (see also Breyer and Felder, 2005). On the other hand, lower mortality may reduce perceived ambiguity, which increases the expected marginal utility-cost of funds for ambiguity averse decision-makers. The elasticities in Proposition 17 compare the relative magnitudes of both effects.

How are these results different from those in the smooth ambiguity model? Let φ denote the decision-maker's ambiguity function in the smooth ambiguity model. Bleichrodt et al. (2019) find that greater ambiguity increases the VSL for ambiguity averse decision-makers when $\varphi''' = 0$ (their Result 2(*ii*)); in case of $\varphi''' > 0$ they find a sufficient condition (their Result 2(*i*)) while the effect remains indeterminate for $\varphi''' < 0$ (their Result 2(*iii*)). Greater ambiguity lowers the VSL for ambiguity loving decision-makers when $\varphi''' = 0$ (their Result 5(*ii*)); in case of $\varphi''' < 0$ they find a sufficient condition (their Result 5(*i*)) while the effect remains indeterminate for $\varphi''' > 0$ (their Result 5(*iii*)). MP-capacities allow us to derive a single necessary and sufficient condition that is simple and applies to ambiguity averse and ambiguity loving decision-makers alike.

3.7 Precautionary saving

When does income risk generate a precautionary demand for saving? The early treatments of this question date back to Leland (1968) and Sandmo (1970) but it was not until Kimball (1990) that the theory of precautionary saving gained momentum (see Baiardi et al., 2020, for a survey). In the two-period consumption-saving model with time-separable utility function, income risk induces precautionary savings if and only if marginal utility is convex, a condition coined prudence.

We formulate the problem with nonseparable utility and derive results for separable utility as a special case. The decision-maker has a certain income of W_1 in the first period and W_2 in the second period. He can save an amount s (or borrow if s is negative) at the risk-free gross interest rate $R \geq 0$. He faces income risk represented by random variable ε with support $[\varepsilon_-, \varepsilon_+]$. We assume without loss that the finite state space orders the realizations of ε in ascending order,

$\varepsilon_1 < \cdots < \varepsilon_n$. U denotes the decision-maker's utility function over timed consumption pairs and the triplet (m, a, α) characterizes his MP-capacity ν. His objective function is then given by

$$V(s; a, \alpha) = \sum_{i=1}^{n} U(W_1 - s, W_2 + sR + \varepsilon_i) \left[m_i + (\tfrac{1}{2} - \alpha) \left[a(\omega_i^{\cup}) - a(\omega_{i+1}^{\cup}) \right] \right],$$

and he solves $\arg\max_s V(s; a, \alpha)$. If the bivariate function U is concave, then V is concave in s because the vector $(W_1 - s, W_2 + sR + \varepsilon)$ is linear in s. To ensure nonnegative consumption levels, s takes values in $[-(W_2 + \varepsilon_1)/R, W_1]$. We assume an interior solution s^*, which is then characterized by the corresponding first-order condition,

$$\begin{aligned} V_s(s^*; a, \alpha) \quad = \quad & -\sum_{i=1}^{n} U_1(W_1 - s^*, W_2 + s^*R + \varepsilon_i) \left[m_i + (\tfrac{1}{2} - \alpha) \left[a(\omega_i^{\cup}) - a(\omega_{i+1}^{\cup}) \right] \right] \\ & + R \sum_{i=1}^{n} U_2(W_1 - s^*, W_2 + s^*R + \varepsilon_i) \left[m_i + (\tfrac{1}{2} - \alpha) \left[a(\omega_i^{\cup}) - a(\omega_{i+1}^{\cup}) \right] \right] = 0. \end{aligned}$$

Subscripts 1 and 2 on the utility function denote the corresponding partial derivatives with respect to consumption in the first and second period. Let

$$H(s, \varepsilon) = -U_1(W_1 - s, W_2 + sR + \varepsilon) + RU_2(W_1 - s, W_2 + sR + \varepsilon),$$

be the first-order expression for an income shock of ε; this notation allows us to rewrite the first-order condition as follows:

$$V_s(s^*; a, \alpha) = \sum_{i=1}^{n} m_i H(s^*, \varepsilon_i) + (\tfrac{1}{2} - \alpha) \sum_{i=1}^{n-1} a(\omega_{i+1}^{\cup}) \left[H(s^*, \varepsilon_{i+1}) - H(s^*, \varepsilon_i) \right] = 0.$$

To sign the effects of greater ambiguity and changes in the degree of ambiguity aversion, we need to sign the expression in square brackets. If $H_\varepsilon(s^*, \varepsilon) < 0$, the bracketed terms are all negative. A sufficient condition is

$$-U_{12}(W_1 - s^*, W_2 + s^*R + \varepsilon) + RU_{22}(W_1 - s^*, W_2 + s^*R + \varepsilon) < 0, \qquad (12)$$

which holds if the decision-maker is correlation loving or correlation neutral (see Epstein and Tanny, 1980). The assumption $U_{12} \geq 0$ suffices because concavity of U implies $U_{22} < 0$. Sandmo (1970) shows that condition (12) is equivalent to consumption in the first period being increasing in first-period income, or said differently, that consumption in the first period is a normal good.[19] This proves the following result.

[19] When first-period income increases, there is a positive direct effect on consumption in the first period and a potentially conflicting indirect effect because the level of saving may increase. Inequality (12) ensures that the direct effect dominates when the demand for saving is normal. The second-order condition implies that no more than one consumption level can be inferior with respect to the corresponding income level.

Proposition 18. *If consumption in the first period is a normal good, the demand for saving is:*

(i) Increasing (decreasing) in the degree of ambiguity aversion (ambiguity loving);

(ii) Increasing (decreasing) in the level of ambiguity for ambiguity averters (ambiguity lovers).

If consumption in the first period is inferior, all effects are reversed.

So if consumption is a normal good, ambiguity and ambiguity aversion generate a precautionary demand for saving. As in the standard portfolio problem, the intuition is derived from observational equivalence. Similar arguments as in Section 3.3 allow us to define the distorted probabilities $\widehat{m}_i$ and the associated cumulative probabilities $\widehat{M}_i$. The behavior of an ambiguity averse CEU maximizer with MP-capacity ν is indistinguishable from that of an SEU maximizer who has deteriorated his subjective belief in an FSD sense relative to m. For an ambiguity loving decision-maker the belief distortion is an FSD improvement. Inequality (12) then allows to sign the effects of FSD shifts in future income on saving behavior.

The separable specification is a special case of our analysis. If $U_{12} = 0$, inequality (12) follows from the concavity of U. Berger (2014) uses Klibanoff et al.'s recursive version of the smooth ambiguity model to show that a convex marginal ambiguity function is not sufficient for precautionary saving. He identifies situations where the stronger notion of nonincreasing absolute ambiguity aversion guarantees the intuitive result (see also Osaki and Schlesinger, 2014). Wang and Li (2020) use Hayashi and Miao's (2011) time-separable specification, which generalizes Selden's (1978) ordinal certainty equivalence representation to ambiguity. They need to impose additional restrictions to obtain precautionary saving for greater ambiguity aversion. We find that ambiguity aversion alone generates precautionary saving, similar to Peter's (2019) application of the smooth ambiguity model with a nonseparable utility index. Our approach extends the results to ambiguity loving decision-makers and provides clean results for greater ambiguity.

According to Proposition 18 ambiguity may play a role in the understanding of low saving rates around the globe. The OECD reports saving rates of 6.0% in the U.S., 4.1% in the European Union and only 0.7% in Japan.[20] Recent survey evidence suggests that over half of the surveyed households could not come up with $2,000 in case of emergency, and that two-thirds of the respondents in the lowest income bracket had less than $2,000 in savings (see Lusardi et al., 2011). The general phenomenon of low saving rates is reported in many countries and especially among the young generation (e.g., Kirsanova and Sefton, 2007; Benartzi and Thaler, 2007). If ambiguity averse decision-makers underestimate the level of uncertainty, they accumulate less savings than they should. Our model also predicts lower saving rates for ambiguity loving decision-makers compared to SEU-based models.

[20] They define net household savings as household disposable income minus household consumption expenditures plus the change in net equity of household pension fund holdings (see `https://data.oecd.org/`). Nowadays saving rates are substantially lower compared to the time period from the 1960s to the 1990s (Maddison, 1992).

4 Conclusion

In this paper, we introduce mean-preserving (MP) capacities in the context of Choquet Expected Utility (CEU). MP-capacities separate ambiguity from ambiguity attitude and provide a tool for clean comparative statics. They rest on the complementary independence axiom for preferences, which states intuitively that decision-makers assess acts according to their SEU baseline evaluation and their utility variability around this baseline. MP-capacities are precisely those JP-capacities for which $\alpha = 1/2$ is necessary and sufficient for ambiguity neutrality. They extend SEU in a direction orthogonal to neo-additive capacities.

We showcase the versatility of MP-capacities in several applications including the value of a information, portfolio choice, self-insurance and self-protection, the value of a statistical life and precautionary saving. We derive results for ambiguity averse and ambiguity loving decision-makers and compare them to the literature. Existing applications based on the smooth ambiguity model focus almost exclusively on ambiguity aversion although recent evidence suggests that ambiguity loving arises in several decision contexts Kocher et al. (2018); Trautmann and Van De Kuilen (2015); Wakker (2010); Li et al. (2018). MP-capacities put us in a position to extend results symmetrically to ambiguity loving decision-makers, who often exhibit the reverse behavior. This cautions against aggregation bias when different ambiguity attitudes coexist in the population. MP-capacities also simplify the analysis of greater ambiguity. The smooth ambiguity model leads to the same indeterminacy that arises under SEU for the comparative statics of risk. Unless restrictions are imposed on the decision-maker's ambiguity function, greater ambiguity can lead to counterintuitive effects due to conflicting Giffen-type effects (see Gollier, 2011). There is no evidence to date that such perplexing behavior materializes in actuality. MP-capacities yield intuitive comparative statics without additional presuppositions. This simplification has the potential to inform applied research in more intricate settings than those discussed in this paper.

References

Alary, D., Gollier, C., and Treich, N. (2013). The effect of ambiguity aversion on insurance and self-protection. *The Economic Journal*, 123(573): 1188–1202.

Arrow, K. J. (1963). Liquidity preference. In *Lecture Notes for Economics 285: The Economics of Uncertainty*, pages 33–53. Stanford University.

Asano, T. and Osaki, Y. (2020). Portfolio allocation problems between risky and ambiguous assets. *Annals of Operations Research*, 284(1):63–79.

Baiardi, D., Magnani, M., and Menegatti, M. (2020). The theory of precautionary saving: an overview of recent developments. *Review of Economics of the Household*, 18: 513–542.

Baillon, A. (2017). Prudence with respect to ambiguity. *The Economic Journal*, 127(604): 1731–1755.

Baillon, A. and Bleichrodt, H. (2015). Testing ambiguity models through the measurement of probabilities for gains and losses. *American Economic Journal: Microeconomics*, 7(2):77–100.

Baillon, A., Bleichrodt, H., Emirmahmutoglu, A., Jaspersen, J. G., and Peter, R. (2019). When risk perception gets in the way: Probability weighting and underprevention. *Operations Research (forthcoming)*.

Bajtelsmit, V., Coats, J. C., and Thistle, P. (2015). The effect of ambiguity on risk management choices: An experimental study. *Journal of Risk and Uncertainty*, 50(3): 249–280.

Benartzi, S. and Thaler, R. (2007). Heuristics and biases in retirement savings behavior. *Journal of Economic Perspectives*, 21(3): 81–104.

Berger, L. (2014). Precautionary saving and the notion of ambiguity prudence. *Economics letters*, 123(2): 248–251.

Berger, L. and Bosetti, V. (2020). Are policymakers ambiguity averse? *The Economic Journal*, 130(626): 331–355.

Berger, L., Emmerling, J., and Tavoni, M. (2016). Managing catastrophic climate risks under model uncertainty aversion. *Management Science*, 63(3): 749–765.

Bianchi, M. and Tallon, J.-M. (2019). Ambiguity preferences and portfolio choices: Evidence from the field. *Management Science*, 65(4): 1486–1501.

Blair, R. D. and Romano, R. E. (1988). The influence of attitudes toward risk on the value of forecasting. *The Quarterly Journal of Economics*, 103(2): 387–396.

Bleichrodt, H., Courbage, C., and Rey, B. (2019). The value of a statistical life under changes in ambiguity. *Journal of Risk and Uncertainty*, 58(1): 1–15.

Breyer, F. and Felder, S. (2005). Mortality risk and the value of a statistical life: The dead-anyway effect revis(it)ed. *The Geneva Risk and Insurance Review*, 30(1): 41–55.

Camerer, C. and Weber, M. (1992). Recent developments in modeling preferences: Uncertainty and ambiguity. *Journal of Risk and Uncertainty*, 5(4): 325–370.

Cerreia-Vioglio, S., Maccheroni, F., Marinacci, M., and Montrucchio, L. (2011). Uncertainty averse preferences. *Journal of Economic Theory*, 146(4): 1275–1330.

Chateauneuf, A., Eichberger, J., and Grant, S. (2007). Choice under uncertainty with the best and worst in mind: Neo-additive capacities. *Journal of Economic Theory*, 137(1): 538–567.

Cheng, H.-C., Magill, M. J., and Shafer, W. J. (1987). Some results on comparative statics under uncertainty. *International Economic Review*, 28(2): 493–507.

Chew, S. H., Miao, B., and Zhong, S. (2017). Partial ambiguity. *Econometrica*, 85(4): 1239–1260.

Couture, S., Lemarié, S., Teyssier, S., and Toquebeuf, P. (2019). Reduction of ambiguity, value of information and use of pesticides: A theoretical and experimental approach. *University of Grenobles-Alpes (Working Paper)*.

Cubitt, R., Van De Kuilen, G., and Mukerji, S. (2020). Discriminating between models of ambiguity attitude: A qualitative test. *Journal of the European Economic Association*, 18(2):708–749.

Denneberg, D. (1994). Conditioning (updating) non-additive measures. *Annals of Operations Research*, 52(1): 21–42.

Denneberg, D. (2000). Non-additive measure and integrals, basic concepts and their role for applications. In Grabisch, M., Murofushi, T., and Sugeno, M., editors, *Fuzzy Measures and Integrals. Theory and Applications*, pages 289–313. Physica Verlag.

Dimmock, S. G., Kouwenberg, R., Mitchell, O. S., and Peijnenburg, K. (2016). Ambiguity aversion and household portfolio choice puzzles: Empirical evidence. *Journal of Financial Economics*, 119(3): 559–577.

Dionne, G. and Eeckhoudt, L. (1985). Self-insurance, self-protection and increased risk aversion. *Economics Letters*, 17(1-2): 39–42.

Doherty, N. A. and Thistle, P. D. (1996). Adverse selection with endogenous information in insurance markets. *Journal of Public Economics*, 63(1): 83–102.

Dow, J. and da Costa Werlang, S. R. (1992). Uncertainty aversion, risk aversion, and the optimal choice of portfolio. *Econometrica*, 60(1):197–204.

Drèze, J. (1962). L'utilité sociale d'une vie humaine. *Revue Française de Recherche Opérationnelle*, 6: 93–118.

Eeckhoudt, L. and Gollier, C. (2005). The impact of prudence on optimal prevention. *Economic Theory*, 26(4): 989–994.

Eeckhoudt, L. and Schlesinger, H. (2006). Putting risk in its proper place. *American Economic Review*, 96(1): 280–289.

Eeckhoudt, L. R. and Hammitt, J. K. (2001). Background risks and the value of a statistical life. *Journal of Risk and Uncertainty*, 23(3): 261–279.

Ehrlich, I. and Becker, G. S. (1972). Market insurance, self-insurance, and self-protection. *Journal of Political Economy*, 80(4): 623–648.

Eichberger, J., Grant, S., and Kelsey, D. (2012). When is ambiguity–attitude constant? *Journal of Risk and Uncertainty*, 45(3): 239–263.

Ellsberg, D. (1961). Risk, ambiguity, and the Savage axioms. *The Quarterly Journal of Economics*, 75(4): 643–669.

Epstein, L. G. and Tanny, S. M. (1980). Increasing generalized correlation: a definition and some economic consequences. *Canadian Journal of Economics*, 13(1): 16–34.

Fagart, M.-C. and Fluet, C. (2013). The first-order approach when the cost of effort is money. *Journal of Mathematical Economics*, 49(1): 7–16.

Ghirardato, P., Maccheroni, F., and Marinacci, M. (2004). Differentiating ambiguity and ambiguity attitude. *Journal of Economic Theory*, 118(2): 133–173.

Ghirardato, P., Maccheroni, F., Marinacci, M., and Siniscalchi, M. (2003). A subjective spin on roulette wheels. *Econometrica*, 71(6): 1897–1908.

Ghirardato, P. and Marinacci, M. (2002). Ambiguity made precise: A comparative foundation. *Journal of Economic Theory*, 102(2): 251–289.

Gilboa, I. (1987). Expected utility with purely subjective non-additive probabilities. *Journal of Mathematical Economics*, 16(1): 65–88.

Gilboa, I. and Marinacci, M. (2016). Ambiguity and the Bayesian paradigm. In *Readings in formal epistemology*, pages 385–439. Springer.

Gilboa, I. and Schmeidler, D. (1989). Maxmin expected utility with non-unique prior. *Journal of Mathematical Economics*, 18(2): 141–153.

Gollier, C. (2001). *The Economics of Risk and Time*. The MIT Press, Cambridge, MA.

Gollier, C. (2011). Portfolio choices and asset prices: The comparative statics of ambiguity aversion. *The Review of Economic Studies*, 78(4): 1329–1344.

Gollier, C. and Pratt, J. W. (1996). Risk vulnerability and the tempering effect of background risk. *Econometrica*, 64(5):1109–1123.

Golman, R., Hagmann, D., and Loewenstein, G. (2017). Information avoidance. *Journal of Economic Literature*, 55(1): 96–135.

Gould, J. P. (1974). Risk, stochastic preference, and the value of information. *Journal of Economic Theory*, 8(1): 64–84.

Grant, S. and Polak, B. (2013). Mean-dispersion preferences and constant absolute uncertainty aversion. *Journal of Economic Theory*, 148(4): 1361–1398.

Hadar, J. and Seo, T. K. (1990). The effects of shifts in a return distribution on optimal portfolios. *International Economic Review*, 31(3): 721–736.

Hayashi, T. and Miao, J. (2011). Intertemporal substitution and recursive smooth ambiguity preferences. *Theoretical Economics*, 6(3): 423–472.

Hoy, M. (1989). The value of screening mechanisms under alternative insurance possibilities. *Journal of Public Economics*, 39(2): 177–206.

Hoy, M., Peter, R., and Richter, A. (2014). Take-up for genetic tests and ambiguity. *Journal of Risk and Uncertainty*, 48(2): 111–133.

Huang, Y.-C. and Tzeng, L. Y. (2018). A mean-preserving increase in ambiguity and portfolio choices. *Journal of Risk and Insurance*, 85(4):993–1012.

Izhakian, Y. (2017). Expected utility with uncertain probabilities theory. *Journal of Mathematical Economics*, 69: 91–103.

Jaffray, J.-Y. and Philippe, F. (1997). On the existence of subjective upper and lower probabilities. *Mathematics of Operations Research*, 22(1): 165–185.

Jewitt, I. and Mukerji, S. (2017). Ordering ambiguous acts. *Journal of Economic Theory*, 171: 213–267.

Jones-Lee, M. (1974). The value of changes in the probability of death or injury. *Journal of Political Economy*, 82(4): 835–849.

Jouini, E., Napp, C., and Nocetti, D. (2013). Economic consequences of nth-degree risk increases and nth-degree risk attitudes. *Journal of Risk and Uncertainty*, 47(2): 199–224.

Jullien, B., Salanié, B., and Salanié, F. (1999). Should more risk-averse agents exert more effort? *The Geneva Risk and Insurance Review*, 24(1): 19–28.

Kimball, M. S. (1990). Precautionary saving in the small and in the large. *Econometrica*, 58(1): 53–73.

Kirsanova, T. and Sefton, J. (2007). A comparison of national saving rates in the UK, US and Italy. *European Economic Review*, 51(8): 1998–2028.

Klibanoff, P., Marinacci, M., and Mukerji, S. (2005). A smooth model of decision making under ambiguity. *Econometrica*, 73(6): 1849–1892.

Klibanoff, P., Marinacci, M., and Mukerji, S. (2009). Recursive smooth ambiguity preferences. *Journal of Economic Theory*, 144(3): 930–976.

Kocher, M. G., Lahno, A. M., and Trautmann, S. T. (2018). Ambiguity aversion is not universal. *European Economic Review*, 101: 268–283.

Lang, M. (2016). First-order and second-order ambiguity aversion. *Management Science*, 63(4): 1254–1269.

Leland, H. E. (1968). Saving and uncertainty: The precautionary demand for saving. *The Quarterly Journal of Economics*, 82(3): 465–473.

Li, Z., Müller, J., Wakker, P. P., and Wang, T. V. (2018). The rich domain of ambiguity explored. *Management Science*, 64(7): 3227–3240.

Lusardi, A., Schneider, D. J., and Tufano, P. (2011). Financially fragile households: Evidence and implications. Technical report, National Bureau of Economic Research.

Maccheroni, F., Marinacci, M., and Rustichini, A. (2006). Ambiguity aversion, robustness, and the variational representation of preferences. *Econometrica*, 74(6): 1447–1498.

Maddison, A. (1992). A long-run perspective on saving. *The Scandinavian Journal of Economics*, 94(2): 181–196.

Marinacci, M. (2000). Ambiguous games. *Games and Economic Behavior*, 31(2): 191–219.

Meyer, D. J. and Meyer, J. (2005). Relative risk aversion: What do we know? *Journal of Risk and Uncertainty*, 31(3): 243–262.

Mossin, J. (1968). Aspects of rational insurance purchasing. *Journal of Political Economy*, 76(4, Part 1): 553–568.

Mukerji, S. and Tallon, J.-M. (2001). Ambiguity aversion and incompleteness of financial markets. *The Review of Economic Studies*, 68(4): 883–904.

Neilson, W. S. (2010). A simplified axiomatic approach to ambiguity aversion. *Journal of Risk and Uncertainty*, 41(2): 113–124.

Nocetti, D. C. (2018). Ambiguity and the value of information revisited. *The Geneva Risk and Insurance Review*, 43(1): 25–38.

Nunez, M. and Schneider, M. (2019). Mean-dispersion preferences with a specific dispersion function. *Journal of Mathematical Economics*, 84: 195–206.

Osaki, Y. and Schlesinger, H. (2014). Precautionary saving and ambiguity. *Working Paper (Waseda University)*.

Peter, R. (2019). Revisiting precautionary saving under ambiguity. *Economics Letters*, 174: 123–127.

Peter, R., Richter, A., and Thistle, P. (2017). Endogenous information, adverse selection, and prevention: Implications for genetic testing policy. *Journal of Health Economics*, 55: 95–107.

Peter, R. and Ying, J. (2019). Do you trust your insurer? Ambiguity about contract nonperformance and optimal insurance demand. *Journal of Economic Behavior & Organization (forthcoming)*.

Pratt, J. W. (1964). Risk aversion in the small and in the large. *Econometrica*, 32(1–2): 122–136.

Pratt, J. W. and Zeckhauser, R. J. (1996). Willingness to pay and the distribution of risk and wealth. *Journal of Political Economy*, 104(4): 747–763.

Rogers, A. and Ryan, M. (2012). Additivity and uncertainty. *Economics Bulletin*, 32(3): 1858–1864.

Rothschild, M. and Stiglitz, J. E. (1970). Increasing risk: I. A definition. *Journal of Economic Theory*, 2(3): 225–243.

Sandmo, A. (1970). The effect of uncertainty on saving decisions. *The Review of Economic Studies*, 37(3): 353–360.

Sarin, R. R. and Wakker, P. P. (1992). A simple axiomatization of nonadditive expected utility. *Econometrica*, 60(6): 1255–1272.

Savage, L. (1954). *The foundation of statistics*. Wiley, New York.

Schlesinger, H. (2013). The theory of insurance demand. In Dionne, G., editor, *Handbook of Insurance*, chapter 7, pages 167–184. Springer.

Schmeidler, D. (1986). Integral representation without additivity. *Proceedings of the American Mathematical Society*, 97(2): 255–261.

Schmeidler, D. (1989). Subjective probability and expected utility without additivity. *Econometrica*, 57(3): 571–587.

Schneider, M. A. and Nunez, M. A. (2015). A simple mean–dispersion model of ambiguity attitudes. *Journal of Mathematical Economics*, 58: 25–31.

Selden, L. (1978). A new representation of preferences over "certain x uncertain" consumption pairs: The "ordinal certainty equivalent" hypothesis. *Econometrica*, 46(5):1045–1060.

Shapira, Z. and Venezia, I. (2008). On the preference for full-coverage policies: Why do people buy too much insurance? *Journal of Economic Psychology*, 29(5): 747–761.

Shapley, L. S. (1971). Cores of convex games. *International Journal of Game Theory*, 1(1): 11–26.

Siniscalchi, M. (2009). Vector expected utility and attitudes toward variation. *Econometrica*, 77(3): 801–855.

Snow, A. (2010). Ambiguity and the value of information. *Journal of Risk and Uncertainty*, 40(2): 133–145.

Snow, A. (2011). Ambiguity aversion and the propensities for self-insurance and self-protection. *Journal of Risk and Uncertainty*, 42(1): 27–43.

Sydnor, J. (2010). (Over)insuring modest risks. *American Economic Journal: Applied Economics*, 2(4): 177–99.

Toquebeuf, P. (2016). Choquet expected utility with affine capacities. *Theory and Decision*, 81(2): 177–187.

Trautmann, S. T. and Van De Kuilen, G. (2015). Ambiguity attitudes. *The Wiley Blackwell handbook of judgment and decision making*, 1: 89–116.

Treich, N. (2010). The value of a statistical life under ambiguity aversion. *Journal of Environmental Economics and Management*, 59(1): 15–26.

Vicig, P. and Seidenfeld, T. (2012). Bruno de Finetti and imprecision: Imprecise probability does not exist! *International Journal of Approximate Reasoning*, 53(8): 1115–1123.

Viscusi, W. K. (1993). The value of risks to life and health. *Journal of Economic Literature*, 31(4): 1912–1946.

Viscusi, W. K. and Aldy, J. E. (2003). The value of a statistical life: A critical review of market estimates throughout the world. *Journal of Risk and Uncertainty*, 27(1): 5–76.

Wakker, P. (1989). Continuous subjective expected utility with non-additive probabilities. *Journal of Mathematical Economics*, 18(1): 1–27.

Wakker, P. P. (2010). *Prospect theory: For risk and ambiguity*. Cambridge University Press.

Wakker, P. P., Timmermans, D. R., and Machielse, I. (2007). The effects of statistical information on risk and ambiguity attitudes, and on rational insurance decisions. *Management Science*, 53(11): 1770–1784.

Wang, J. and Li, J. (2020). Comparative ambiguity aversion in intertemporal decisions. *Journal of Risk and Insurance*, 87(1):195–212.

Wang, Z. and Klir, G. J. (1997). Choquet integrals and natural extensions of lower probabilities. *International Journal of Approximate Reasoning*, 16(2): 137–147.

Weinstein, M. C., Shepard, D. S., and Pliskin, J. S. (1980). The economic value of changing mortality probabilities: A decision-theoretic approach. *The Quarterly Journal of Economics*, 94(2): 373–396.

Appendix A Proofs

A.1 Proof of Proposition 1

(*i*) ν is normalized because $a(\Omega) = a(\emptyset) = 0$ implies $\nu(\Omega) = m(\Omega) = 1$ and $\nu(\emptyset) = m(\emptyset) = 0$. To show monotonicity with respect to set inclusion, take $E \subseteq E'$ with $E, E' \in \Sigma_{nt}$.[21] We obtain

$$\nu(E') - \nu(E) = m(E') - m(E) + \left(\tfrac{1}{2} - \alpha\right)\left(a(E') - a(E)\right).$$

Property (*iii*) ensures that $|a(E') - a(E)| \leq 2(m(E') - m(E))$ for all $E, E' \in \Sigma$. If $\alpha < 1/2$ and $(a(E') - a(E)) \geq 0$, then $\nu(E') - \nu(E) \geq m(E') - m(E) \geq 0$; if $\alpha < 1/2$ and $(a(E') - a(E)) < 0$, then $\nu(E') - \nu(E) \geq 2\alpha\,(m(E') - m(E)) \geq 0$; if $\alpha > 1/2$ and $(a(E') - a(E)) < 0$, then $\nu(E') - \nu(E) \geq m(E') - m(E) \geq 0$; if $\alpha > 1/2$ and $(a(E') - a(E)) \geq 0$, then $\nu(E') - \nu(E) \geq 2(1-\alpha)\,(m(E') - m(E)) \geq 0$. The desired inequality $\nu(E') \geq \nu(E)$ follows in any of these cases. The strict inequality in Property (*iii*) for $E = \emptyset$ or $E' = \Omega$ guarantess that the entire state space is the only universal set and that the empty set is the only null set, $\nu(E) \in (0,1)$ for $E \in \Sigma_{nt}$.

(*ii*) For $E, E' \in \Sigma$,

$$m(E) + m(E') = m(E \cup E') + m(E \cap E')$$

because m is a subjective probability. Due to Property (*ii*) in Definition 3, we have

$$a(E) + a(E') \geq a(E \cup E') + a(E \cap E').$$

from the submodularity of a. The statement

$$\nu(E) + \nu(E') \leq \nu(E \cup E') + \nu(E \cap E')$$

is then equivalent to

$$\begin{aligned} & m(E) + \left(\tfrac{1}{2} - \alpha\right) a(E) + m(E') + \left(\tfrac{1}{2} - \alpha\right) a(E') \\ \leq \quad & m(E \cup E') + \left(\tfrac{1}{2} - \alpha\right) a(E \cup E') + m(E \cap E') + \left(\tfrac{1}{2} - \alpha\right) a(E \cap E'), \end{aligned}$$

which, in turn, is equivalent to

$$\left(\tfrac{1}{2} - \alpha\right) \cdot \left[a(E) + a(E') - a(E \cup E') - a(E \cap E')\right] \leq 0.$$

ν is convex for $\alpha \geq 1/2$ and concave for $\alpha \leq 1/2$ by reversing the last three inequalities. It is additive if it is both convex and concave, which holds for $\alpha = 1/2$.

(*iii*) Eq. (3) reveals that $\nu \leq m$ for $\alpha \geq 1/2$, $\nu \geq m$ for $\alpha \leq 1/2$ and $\nu = m$ for $\alpha = 1/2$.

[21] For $E = \emptyset$ or $E' = \Omega$, monotonicity with respect to set inclusion is trivial.

(*iv*) This is also directly evident from Eq. (3) because a is nonnegative.

(*v*) Let ν_1 and ν_2 be two MP-capacities that only differ by the level of ambiguity such that $a_2(E) \geq a_1(E)$ for all $E \in \Sigma$. It follows from Eq. (3) that $\nu_2 \leq \nu_1$ for $\alpha \geq 1/2$ and $\nu_2 \geq \nu_1$ for $\alpha \leq 1/2$. The Choquet integral is monotonic with respect to the capacity (see Denneberg, 1994; Wang and Klir, 1997) so that

$$\int_\Omega U(f)\,\mathrm{d}\nu_2 \leq (\geq) \int_\Omega U(f)\,\mathrm{d}\nu_2 \quad \text{for } \alpha \geq (\leq)\, 1/2.$$

A.2 Proof of Proposition 2

(*i*) ⇒ (*ii*): For $E \in \Sigma$, let $\overline{\nu}(E) = 1 - \nu(E^c)$ denote the dual capacity associated with ν. If ν is an MP-capacity, then $\overline{\nu}(E) = m(E) - (\frac{1}{2} - \alpha)a(E)$. It follows that $(\nu + \overline{\nu})/2 = m$ so $(\nu + \overline{\nu})/2$ is a subjective probability. According to Siniscalchi's Table II, preferences with a CEU representation satisfy complementary independence in this case.

(*ii*) ⇒ (*i*): Siniscalchi (2009) shows in Section 4.6 that, if a preference relation with a CEU representation satisfies complementary independence, there is a subjective probability m such that $1 - \nu(E^c) = 2m(E) - \nu(E)$ for all $E \in \Sigma$. So $(\nu + \overline{\nu})/2$ defines a subjective probability. Set $\beta = \nu - m$; if ν is convex, then $\nu \leq \overline{\nu}$ so that $\nu \leq m \leq \overline{\nu}$ and therefore $\beta \leq 0$. If we set $\alpha = 1$ and $a = -2\beta$, we can rewrite ν in the desired form,

$$\nu = m + (\nu - m) = m + \beta = m + (\tfrac{1}{2} - \alpha)a.$$

We now show that set function a satisfies Properties (*i*) - (*iii*) in Definition 3. We obtain $a(\Omega) = -2\beta(\Omega) = -2\nu(\Omega) + 2m(\Omega) = 0$ and similarly $a(\emptyset) = 0$ so a is normalized. For $E \in \Sigma$,

$$\beta(E) = \nu(E) - m(E) = \nu(E) - \tfrac{1}{2}(\nu(E) + \overline{\nu}(E)) = \tfrac{1}{2}(\nu(E) + \nu(E^c) - 1)$$

and

$$\beta(E^c) = \nu(E^c) - m(E^c) = \nu(E^c) - \tfrac{1}{2}(\nu(E^c) + \overline{\nu}(E^c)) = \tfrac{1}{2}(\nu(E) + \nu(E^c) - 1)$$

so $\beta(E) = \beta(E^c)$ for all $E \in \Sigma$ and a is symmetric. It is also submodular because ν is convex and m is a subjective probability. It has bounded changes because capacity ν is monotonic with respect to set inclusion.

If ν is concave, then $\overline{\nu} \leq \nu$ so that $\overline{\nu} \leq m \leq \nu$ and therefore $\beta \geq 0$. In this case, set $\alpha = 0$ and $a = 2\beta$ to obtain $\nu = m + (\frac{1}{2} - \alpha)a$. As before set function a satisfies the required properties in Definition 3.

A.3 Proof of Proposition 3

(*i*) ⇒ (*ii*): Let the triplet (m, a, α) define the MP-capacity ν according to Definition 3. We have $\alpha \geq 1/2$ because ν is assumed convex. Let $p \in \Delta(\Sigma)$ be an element of the core of ν so that

$\nu \leq p$. For $E \in \Sigma$ it follows that $\overline{\nu}(E) = 1 - \nu(E^c) \geq 1 - p(E^c) = p(E)$. But $\overline{\nu} \geq p$ is equivalent to $m - (\frac{1}{2} - \alpha)a \geq p$, which is equivalent to $2m - p \geq m + (\frac{1}{2} - \alpha)a = \nu$. So for any $p \in \Delta(\Sigma)$ we obtain $2m - p \in \Delta(\Sigma)$. The core of ν is centrally symmetric with center m.

(ii) $\Rightarrow$ *(i)*: Let the core of convex capacity ν be centrally symmetric with center $m_c \in \mathcal{P}_\nu$. Define $m = (\nu + \overline{\nu})/2$ and let $p \in \Delta(\Sigma)$ be an element of the core of ν so that $\nu \leq p$. As before this implies $\overline{\nu} \geq p$, which is equivalent to $\nu + \overline{\nu} - p \geq \nu$ or $2m - p \geq \nu$. In other words, $(\nu + \overline{\nu})/2$ is the center of $\mathcal{P}_\nu$ and therefore coincides with subjective probability m_c. But if $(\nu + \overline{\nu})/2$ is a subjective probability, we can proceed as in the only-if part of Appendix A.2 to show that ν is an MP-capacity in the sense of Definition 3.

A.4 Proof of Proposition 4

Let the triplet (m, a, α) define the MP-capacity ν according to Definition 3. Then $w = m - a/2$ is obtained by setting $\alpha = 1$ in Eq. (3). Property (*iii*) in Definition 3 ensures that any $\alpha \in [0, 1]$ yields a set function on Σ that is normalized and monotonic with respect to set inclusion and therefore a capacity. Furthermore w is convex due to Proposition 1(*ii*), and its dual is $\overline{w} = m + a/2$ due to Property (*i*) in Definition 3. But then

$$\alpha w + (1 - \alpha)\overline{w} = \alpha(m - \tfrac{a}{2}) + (1 - \alpha)(m + \tfrac{a}{2}) = m + (\tfrac{1}{2} - \alpha)a = \nu$$

so that ν is a JP-capacity. We also obtain

$$\tfrac{1}{2}(w + \overline{w}) = \tfrac{1}{2}(m - \tfrac{a}{2} + m + \tfrac{a}{2}) = m \quad \text{and} \quad \tfrac{1}{2}(\overline{w} - w) = \tfrac{1}{2}(m + \tfrac{a}{2} - (m - \tfrac{a}{2})) = a.$$

A.5 Proof of Proposition 5

By assumption, there is $E \in \Sigma_{nt}$ such that $w(E) < \overline{w}(E) = 1 - w(E^c)$. If the decision-maker is ambiguity neutral, then by Definition 1, there is a subjective probability m such that $\nu = m$. By virtue of ν being a JP-capacity, we then obtain

$$m(E) = \nu(E) = \alpha w(E) + (1 - \alpha)(1 - w(E^c))$$

and

$$m(E^c) = \nu(E^c) = \alpha w(E^c) + (1 - \alpha)(1 - w(E)).$$

If we add the two equations and use that m is a subjective probability, we find

$$\alpha w(E) + (1 - \alpha)(1 - w(E^c)) + \alpha w(E^c) + (1 - \alpha)(1 - w(E)) = 1,$$

or equivalently

$$(1 - 2\alpha)(1 - w(E) - w(E^c)) = 0.$$

The second round bracket is strictly positive because $w(E) < \overline{w}(E)$. Therefore $\alpha = 1/2$ follows.

A.6 Proof of Proposition 6

Let $\nu = \alpha w + (1-\alpha)\overline{w}$ be a JP-capacity with convex capacity w. Assume that there is a subjective probability m such that $\alpha = 1/2$ implies $\nu = m$. In this case we obtain $(w + \overline{w})/2 = m$. Define $a = \overline{w} - w$; then JP-capacity ν has the desired form because

$$m + (\tfrac{1}{2} - \alpha)a = \tfrac{1}{2}(w + \overline{w}) + (\tfrac{1}{2} - \alpha)(\overline{w} - w) = \alpha w + (1-\alpha)\overline{w} = \nu.$$

We will now show that a satisfies Properties (*i*) - (*iii*) in Definition 3. First, a is normalized because w is normalized and a is symmetric because for $E \in \Sigma$,

$$a(E^c) = \overline{w}(E^c) - w(E^c) = 1 - w((E^c)^c) - w(E^c) = 1 - w(E^c) - w(E) = a(E).$$

Second, a is submodular because w is convex so that $\overline{w}$ is concave. For $E, E' \in \Sigma$ we then have

$$w(E) + w(E') \leq w(E \cup E') + w(E \cap E')$$

and

$$\overline{w}(E) + \overline{w}(E') \geq \overline{w}(E \cup E') + \overline{w}(E \cap E'),$$

and therefore

$$a(E) + a(E') \geq a(E \cup E') + a(E \cap E')$$

because $a = \overline{w} - w$. Third, a has bounded changes. For $E, E' \in \Sigma$ with $E \subseteq E'$, we obtain

$$a(E') - a(E) = w(E) - w(E') + w(E^c) - w(E'^c)$$

and

$$2(m(E') - m(E)) = -(w(E) - w(E')) + w(E^c) - w(E'^c).$$

Now w is a capacity and therefore monotonic with respect to set inclusion so that $w(E) \leq w(E')$ and $w(E^c) \geq w(E'^c)$. Hence $|a(E') - a(E)| \leq 2(m(E') - m(E))$ is satisfied. The inequality is strict for $E = \emptyset$ or $E' = \Omega$ because the empty set is the only null set for capacity w; so for any $E'' \in \Sigma_{nt}$ we obtain $a(E'') < 2m(E'')$ from $w(E'') > 0$.

A.7 Proof of Proposition 7

Let the triplet (m, δ, α) define the neo-additive capacity ν. Set $\mu = (\nu + \overline{\nu})/2$; if ν is mean-preserving, μ is a subjective probability, see Propositions 2 and 3, and satisfies

$$\mu(E) + \mu(E') = \mu(E \cup E') + \mu(E \cap E') \quad \text{for all } E, E' \in \Sigma. \tag{13}$$

The neo-additive capacity ν is defined as $(1-\delta)m + (1-\alpha)\delta$ on Σ_{nt} and its dual is given by $\overline{\nu} = (1-\delta)m + \alpha\delta$ on Σ_{nt}. Therefore $\mu = (\nu + \overline{\nu})/2 = (1-\delta)m + \delta/2$ on Σ_{nt} with $\mu(\emptyset) = 0$ and

$\mu(\Omega) = 1$. Let $E, E' \in \Sigma_{nt}$ with $E \cup E' = \Omega$ and $E \cap E' \neq \emptyset$. We can always find such events when the state space has at least three different states of the world. Eq. (13) becomes

$$(1-\delta)m(E) + \frac{\delta}{2} + (1-\delta)m(E') + \frac{\delta}{2} = 1 + (1-\delta)m(E \cap E') + \frac{\delta}{2}.$$

Now m is a subjective probability so that $m(E) + m(E') = 1 + m(E \cap E')$. But then the above equation simplifies to $1 = 1 + \delta/2$, which implies $\delta = 0$. In this case the neo-additive capacity ν is a subjective probability, $\nu = m$.